

8. For each of the following, define P such that the given function is continuous at 3.

$$\text{a. } f(x) = \begin{cases} \frac{3(x^4 - 81)}{x^2 - 9}, & \text{if } x \neq 3 \\ Px + 9, & \text{if } x = 3 \end{cases} \quad \text{b. } g(x) = \begin{cases} \frac{x^3 - 27}{x^2 - 9}, & \text{if } x \neq 3 \\ P, & \text{if } x = 3 \end{cases}$$

9. Determine the intervals on which the functions defined below are continuous.

$$\text{a. } f(x) = \begin{cases} 8 - 7x, & \text{if } x \leq 4 \\ -x - 16, & \text{if } x > 4 \end{cases} \quad \text{b. } g(x) = \begin{cases} x^2 + 1, & \text{if } x \leq -3 \\ 5 - x, & \text{if } x > -3 \end{cases}$$

10. Identify all asymptotes of the following.

$$\text{a. } y = \frac{x-2}{x-1} \quad \text{b. } y = \frac{x^2 - 3x + 2}{x^2 - 4} \quad \text{c. } y = \frac{\sqrt{x^2 + 4}}{x} \quad \text{d. } y = \frac{x^2 - 9}{x^2 - 5x + 6}$$

11. Give a specific example to show that it is possible for $\lim_{x \rightarrow a} f(x)$ to exist if $f(a)$ is undefined.

12. Determine y' when $x = 2$ if $y = \frac{7}{\sqrt{x^4 - 15}}$.

13. Determine $f'(5)$ if $f(x) = \sqrt[3]{(2x + 17)^2}$.

14. Determine y' when $x = 1$ if $y = \frac{8x}{3} - \frac{3}{8x}$.

15. Determine y' when $x = 1$ if $y = (2 - x)\sqrt{x^2 + 8}$.

16. Find the equation of the tangent line to the curve defined by $y = 2x^2 - 5x + 8$ when $x = 2$.

17. At what point (x, y) is the tangent line to the curve $y = 2x^2 - 5x + 8$ parallel to the line $y = 3x - 7$?

18. Find the equation of the line tangent to the graph of $x^2 + 2xy^2 + 3y = 31$ at the point $(2, -3)$.

19. Let $y = \sqrt{x^2 - 1}$. Find y'' when $x = 2$.

23. Determine y'' at $x = 1$ if $y = 3\sqrt[3]{x^4} - \frac{1}{3x^3}$.

20. If $f(x) = \sqrt{x + \sqrt{x}}$, find $f'(1)$.

24. Let $y = [\cos(2x - \pi)]^3$. Find y' at $x = \frac{\pi}{6}$.

21. Determine $f'(1)$ if $f(x) = \sqrt[3]{\frac{x}{x^3 + 1}}$.

25. Determine $f'(\frac{\pi}{4})$ if $f(x) = \frac{\tan x}{1 + \cos x}$.

22. Determine $f'(3)$ if $f(x) = \frac{1}{x - \sqrt{x^2 - 5}}$.

26. If $f(x) = \sin 3x \cos 2x$, find $f'(\frac{\pi}{6})$.

27. If $y^4 + x^4 - 2x^2y + 9x = 9$. Find y' at $(1, 1)$.

41. A rectangular open tank is to have a square base, and its volume is to be 125 yd^3 . The cost per square yard for the base is \$8 and for the sides is \$4. Find the dimensions of the tank in order to minimize the cost of the material.

42. A power station is on one side of a river which is $\frac{1}{2}$ mile wide, and a factory is 1 mile downstream on the other side of the river. It costs \$300 per foot to run power lines overland and \$500 per foot to run them under water. Find the most economical way to run the power lines from the power station to the factory.

43. A cardboard box manufacturer wishes to make open boxes from pieces of cardboard 12 in square by cutting equal squares from the four corners and turning up the sides. Find the length of the side of the square to be cut out in order to obtain a box of the largest possible volume. What is the largest possible volume?

44. A train leaves a station traveling north at the rate of 60 mph. One hour later, a second train leaves the same station traveling east at the rate of 45 mph. Find the rate at which the trains are separating 2 hours after the second train leaves the station.

45. A street light hangs 24 ft above the sidewalk. A man 6 ft tall walks away from the light at the rate of 3 ft/sec. At what rate is the length of his shadow increasing?

46. A barge is pulled toward a dock by means of a taut cable. If the barge is 20 ft below the level of the dock, and the cable is pulled in at the rate of 36 ft/min, find the speed of the barge when the cable is 52 ft long.

47. Find the values of Δy and dy if $y = 2x^2 - x$, $x = 2$, and $dx = \Delta x = .01$.

48. Use differentials to approximate the maximum possible error that can be produced when calculating the volume of a cube if the length of an edge is known to be $2 \pm .005$ ft.

49. Approximate $\sqrt{50}$ using each of the following.

a. Differentials

b. A linearization

50. The moment of inertia of an annular cylinder is $I = .5M(R_2^2 - R_1^2)$, where M is the mass of the cylinder, R_2 is its outer radius, and R_1 is its inner radius. If $R_1 = 2$, and R_2 changes from 4 to 4.01, use differentials to estimate the resulting change in the moment of inertia.

51. The range of a shell shot from a certain ship is $R = 300 \sin(2\theta)$ meters, where θ is the angle above horizontal of the gun when it is shot. If the gun is intended to be fired at an angle of $\frac{\pi}{6}$ radians to hit its target, but due to waves it actually shot .05 radians too low, use differentials to estimate how far short of its target the shell will fall.

52. For each of the following, determine whether the Intermediate Value Theorem guarantees the equation has a solution in the specified interval.

a. $x^5 + 2x^2 - 10x + 5 = 0$; $[1, 2]$

c. $x = \cos x$; $[0, \pi]$

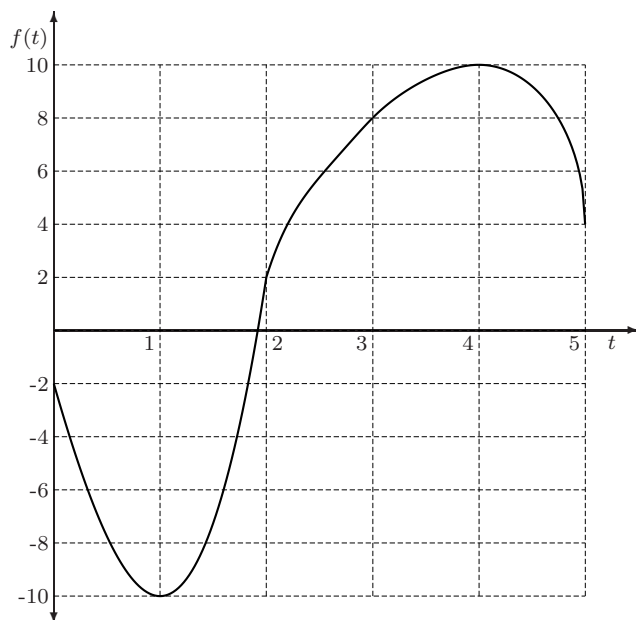
b. $x - \sqrt{x} = 5$; $[4, 9]$

d. $x = \tan x$; $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

53. Find $f(x)$ if $f''(x) = 4x + 3$, $f(1) = 2$, and $f'(1) = -3$.

54. Find $f(x)$ if $f'(x) = x^2 + 3x + 2$ and $f(-3) = -\frac{3}{2}$.

For problems 55 and 56, let $f(t)$ be the function defined by the graph shown.

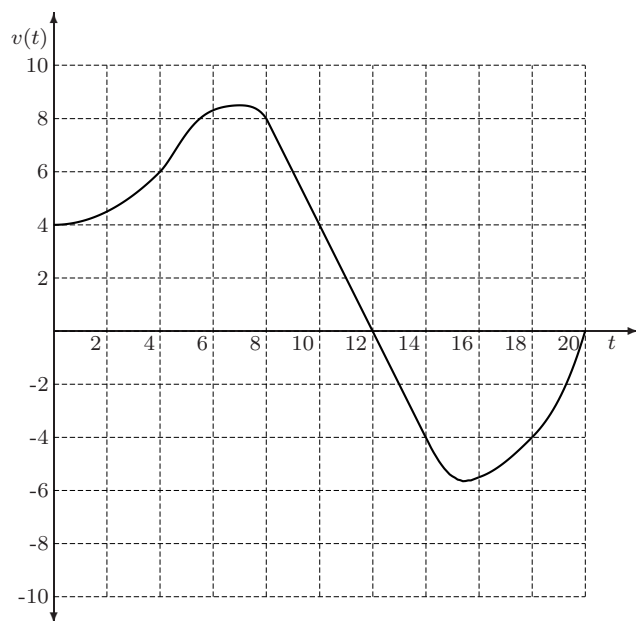


55. Estimate each of the following. Round numbers to the nearest integer.

- The instantaneous rate of change of f at $t = 3$.
- The average rate of change of f over the interval $[0, 4]$.
- The intervals where $f(t)$ is increasing and where it is decreasing.
- The inflection point or points of $f(t)$.

56. Find the intervals where $f'(t)$ is increasing and where it is decreasing. Round numbers to the nearest integer.

57. Let $v(t)$, as shown in the graph below, be the velocity of a car in meters per second at time t in seconds, where positive velocity means the car is moving forward. Round your answers to the nearest integer.



- a. When did the car stop?
- b. Approximately how far did the car travel in the time interval 8 to 12 seconds?
- c. Approximately how far did it travel in the time interval 12 to 14 seconds?
- d. Approximately how far did it travel in the time interval 8 to 14 seconds?
- e. At the time 2 seconds, is the car moving forward or backward? Is the driver's foot on the gas or the brake?
- f. At the time 16 seconds, is the car moving forward or backward? Is the driver's foot on the gas or the brake?

58. Integrate.

- | | | |
|---|---|---|
| a. $\int x(3x^2 + 4)^4 dx$ | e. $\int_3^8 \sqrt{12 - x} dx$ | i. $\int 3x^2 \sqrt{6x^3 + 1} dx$ |
| b. $\int (2x^4 + 4x)^3 (2x^3 + 1) dx$ | f. $\int (3x - 1) \sqrt{3x^2 - 2x + 1} dx$ | j. $\int x \sqrt{2 - x} dx$ |
| c. $\int_1^2 \frac{2x^3 + 3x + 5}{x^5} dx$ | g. $\int \frac{1}{(x + 5)^4} dx$ | k. $\int \frac{\sqrt[3]{3 + z^{-1}}}{z^2} dz$ |
| d. $\int \frac{3x^2 + 6x + 2}{x^4} dx$ | h. $\int \frac{5x + \frac{1}{2}}{(10x^2 + 2x + 40)^4} dx$ | l. $\int \sec x \tan x \cos(\sec x) dx$ |
| m. $\int \sin \frac{x}{3} \left(\cos \frac{x}{3} \right)^3 dx$ | n. $\int \tan x (\sec x)^2 dx$ | |

59. Differentiate.

a. $f(x) = \int_3^x (t^2 - 8) dt$

b. $g(x) = \int_4^{x^2} (t^2 + 6)^{\frac{5}{2}} dt$

60. Find the area of the region bounded by $y = x^2$ and $y = x^4$.

61. Find the area of the region bounded by $y = x^2$ and $y = 2 - x$.

62. Find the area of the region bounded by $y = x^4$ and $y = 8x$.

63. Find the volume of the solid generated by revolving the region bounded by $y = x^{\frac{3}{2}}$, the x -axis, and the line $x = 4$ about the x -axis.

64. Find the volume of the solid generated by revolving the region bounded by $y^2 = 4x$ and $x^2 = 4y$ about the x -axis.

65. Find the volume of the solid generated by revolving the region in the first quadrant bounded by $y = x^2$, $y = 4$, and the y -axis about each of the following.

a. the y -axis

b. $y = 4$

66. Find the volume of the solid generated by revolving the region bounded by $y = x + \frac{4}{x}$, the x -axis, and the lines $x = 1$ and $x = 3$ about the y -axis.

67. A solid has as its base the region in the first quadrant bounded by $x^2 + y^2 = 25$. Every plane section of the solid taken perpendicular to the x -axis is a square. Find the volume of the solid.

68. A solid has as its base the region in the xy -plane bounded by the graphs of $y = x$ and $y^2 = x$. Find the volume of the solid if every cross section by a plane perpendicular to the x -axis is a semicircle with diameter in the xy -plane.

69. Find the average value of the function $f(x) = x^2 + x + 1$ on the interval $[-1, 2]$.

70. Find the average value of the function $f(x) = \sin x$ on the interval $[0, \pi]$.

71. Assume that the density of water is 62.5 lb/ft^3 . A cylindrical water tank with a circular base has radius 3 feet and height 10 feet. Find the work required to empty the tank by pumping the water out of the top for each of the following situations.

a. The tank is full.

b. The tank is half full.

72. A bucket with 24 lb of water is raised 30 feet from the bottom of a well. Find the work done in each of the following cases.

a. The weight of the empty bucket is 4 lb and the weight of the rope is negligible.

b. The bucket weighs 4 lb and the rope weighs 4 oz/ft.

c. The bucket weighs 4 lb, the rope weighs 4 oz/ft, and water is leaking out of the bucket at a constant rate so that only 18 lb of water remain in the bucket when it reaches the top.

73. Match each numbered item with a lettered item. (There are more lettered items than numbered items. Some lettered items do not match any numbered item.)

- | | |
|---|---|
| <p>1. Definition of $\lim_{x \rightarrow a} f(x) = L$.</p> <p>2. Definition of $\lim_{x \rightarrow a^+} f(x) = L$.</p> <p>3. Definition of $\lim_{x \rightarrow \infty} f(x) = L$.</p> <p>4. Definition of “f is continuous at a”.</p> <p>5. The Intermediate Value Theorem.</p> <p>6. Definition of the derivative of f at a.</p> <p>7. Definition of a function f being differentiable at a.</p> <p>8. Theorem relating differentiability and continuity.</p> <p>9. The power rule for differentiation.</p> <p>10. Definition of the differential.</p> <p>11. Definition of a function f having an absolute maximum at c.</p> <p>12. Definition of a function f having a local maximum at c.</p> <p>13. The Extreme Value Theorem.</p> <p>14. Definition of a function that is increasing on an interval I.</p> <p>15. The Mean Value Theorem.</p> <p>16. Definition of an antiderivative of f on an interval I.</p> | <p>a. $\lim_{x \rightarrow a} f(x) = f(a)$.</p> <p>b. For every $\varepsilon > 0$ there is a corresponding number N such that $f(x) - L < \varepsilon$ whenever $x > N$.</p> <p>c. If f is continuous on the closed interval $[a, b]$, and N is a number strictly between $f(a)$ and $f(b)$, then there exists a number c in (a, b) such that $f(c) = N$.</p> <p>d. The limit $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ exists.</p> <p>e. If f is differentiable at a, then f is continuous at a.</p> <p>f. If f is continuous at a, then f is differentiable at a.</p> <p>g. $\frac{d}{dx} x^n = nx^{n-1}$.</p> <p>h. The function g has the property $g'(x) = f(x)$ for all x in I.</p> <p>i. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$</p> <p>j. $f(c) \geq f(x)$ for all x in the domain of f.</p> <p>k. For every $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $f(x) - L < \varepsilon$ whenever $a < x < a + \delta$.</p> <p>l. For every $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $f(x) - L < \varepsilon$ whenever $0 < x - a < \delta$.</p> <p>m. There is an open interval I containing c such that $f(c) \geq f(x)$ for all x in I.</p> <p>n. $f'(x) > 0$ for all x in I.</p> <p>o. If f is continuous on $[a, b]$, then there are numbers c and d in $[a, b]$ such that $f(c)$ is an absolute maximum for f in $[a, b]$ and $f(d)$ is an absolute minimum for f in $[a, b]$.</p> <p>p. If f is differentiable, $dy = f'(x)dx$.</p> <p>q. $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ and x_1 and x_2 are in I.</p> <p>r. If f is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.</p> |
|---|---|

MATHEMATICS 1571

Answers to Final Examination Review Problems

1. **a.** $2a^2 + 4ab + 2b^2 - 5a - 5b$

b. $4x^2$

2. **a.** Dom: $(-\infty, -1] \cup [4, \infty)$

b. Dom: $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$

3. Left 3 units and down 4 units.

4. **a.** $(f \circ g)(x) = \frac{1}{x-1}$

Dom: $[0, 1) \cup (1, \infty)$

$$(g \circ f)(x) = \frac{1}{\sqrt{(x+1)(x-1)}}$$

Dom: $(-\infty, -1) \cup (1, \infty)$

b. $(f \circ g)(x) = \frac{x^2}{(x-1)^2} + 1$

Dom: $(-\infty, 1) \cup (1, \infty)$

$$(g \circ f)(x) = \frac{x^2 + 1}{x^2}$$

Dom: $(-\infty, 0) \cup (0, \infty)$

5. **a.** y -intercept: $(0, 0)$

x -intercept: $(0, 0)$

Symmetry: none

b. y -intercept: none

x -intercepts: none

Symmetry: origin

c. y -intercept: $(0, 2)$

x -intercepts: $(-2, 0), (2, 0)$

Symmetry: y -axis

6. **a.** DNE

b. 3

7. **a.** -1

b. $-\frac{1}{2}$

c. 4

d. $-\infty$

e. ∞

f. DNE

g. 1

h. -1

i. DNE

j. 0

k. $\frac{1}{3}$

l. DNE

m. 0

n. 1

o. $\frac{1}{3}$

p. 4

q. $\frac{\sqrt{3}}{2}$

r. $-\sin x$

8. **a.** 15

b. $\frac{9}{2}$

9. **a.** $(-\infty, \infty)$

b. $(-\infty, -3], (-3, \infty)$

10. a. HA: $y = 1$

VA: $x = 1$

OA: none

b. HA: $y = 1$

VA: $x = -2$

OA: none

c. HA: $y = -1, y = 1$

VA: $x = 0$

OA: none

d. HA: $y = 1$

VA: $x = 2$

OA: none

11. $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$

$a = 3$

There are infinitely many correct answers.

12. -112

13. $\frac{4}{9}$

14. $\frac{73}{24}$

15. $-\frac{8}{3}$

16. $y = 3x$

17. (2, 6)

18. $y = \frac{22}{21}x - \frac{107}{21}$

19. $\frac{-1}{3\sqrt{3}}$

20. $\frac{3}{4\sqrt{2}}$

21. $-\frac{\sqrt[3]{4}}{12}$

22. $\frac{1}{2}$

23. $-\frac{8}{3}$

24. $\frac{3\sqrt{3}}{4}$

25. $\sqrt{2}$

26. $-\sqrt{3}$

27. $-\frac{9}{2}$

28. Inc: $\left[\frac{3}{4}, \infty\right)$

29. Dec: $\left(-\frac{3}{2}, 1\right]$

30. CU: $(-4, 0), (3, \infty)$

31. CD: $(-\infty, -2), (0, 2)$

32. IP: $\left(-4, -\frac{256}{3}\right), \left(3, -\frac{171}{4}\right)$

33. IP: none

34. Local max: $13 + \frac{32}{3}\sqrt{2}$ at $-3 - 2\sqrt{2}$

Local min: $13 - \frac{32}{3}\sqrt{2}$ at $-3 + 2\sqrt{2}$

35. Local max: 3 at 0

Local min: $-\frac{13}{4}$ at $-\sqrt{5}$, $-\frac{13}{4}$ at $\sqrt{5}$

36. a. Max: 0

Min: -16

b. Max: 16

Min: 0

37. a. 192 ft

b. 6 sec

c. -128 ft/sec

38. a. 64 ft/sec

b. 288 ft

39. 5000 ft²

40. 80

41. Dimensions: $5 \times 5 \times 5$

42. Overland $\frac{5}{8}$ mile

Underwater $\frac{5}{8}$ mile

43. 2×2 square

Volume of 128 in³

44. $33\sqrt{5}$ mph

45. 1 ft/sec

46. 39 ft/min

47. $\Delta y = .0702$

$dy = .07$

48. $\pm .06$ ft³

49. a. $7 + \frac{1}{14}$

b. $7 + \frac{1}{14}$

50. $dI = .04M$

51. 15 meters

52. a. Yes

b. Yes

c. Yes

d. No

53. $f(x) = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 8x + \frac{47}{6}$

54. $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x$

55. a. 4

b. 3

c. Dec: $[0, 1], [4, 5]$

Inc: $[1, 4]$

d. IP: $(2, 2)$

56. Dec: $[2, 5]$

Inc: $[0, 2]$

57. a. 12 seconds

b. 16 meters

c. 4 meters

d. 20 meters

e. Forward

Accelerator

f. Backward

Brake

58. a. $\frac{1}{30}(3x^2 + 4)^5 + C$

b. $\frac{1}{16}(2x^4 + 4x)^4 + C$

c. $\frac{195}{64}$

d. $-\frac{3}{x} - \frac{3}{x^2} - \frac{2}{3x^3} + C$

e. $\frac{38}{3}$

f. $\frac{1}{3}\sqrt{(3x^2 - 2x + 1)^3} + C$

g. $\frac{-1}{3(x+5)^3} + C$

h. $\frac{-1}{12(10x^2 + 2x + 40)^3} + C$

i. $\frac{1}{9}\sqrt{(6x^3 + 1)^3} + C$

j. $\frac{2}{5}\sqrt{(2-x)^5} - \frac{4}{3}\sqrt{(2-x)^3} + C$

k. $-\frac{3}{4}\sqrt[3]{(3+z^{-1})^4} + C$

l. $\sin(\sec x) + C$

m. $-\frac{3}{4}\left(\cos \frac{x}{3}\right)^4 + C$

n. $\frac{1}{2}\tan^2 x + C$

59. a. $f'(x) = x^2 - 8$

b. $g'(x) = (x^4 + 6)^{\frac{5}{2}}(2x)$

60. $\frac{4}{15}$

61. $\frac{9}{2}$

62. $\frac{48}{5}$

63. 64π

64. $\frac{96\pi}{5}$

73.

1. l

2. k

3. b

4. a

5. c

6. i

7. d

8. e

9. g

10. p

11. j

12. m

13. o

14. q

15. r

16. h

65. a. 8π

b. $\frac{256\pi}{15}$

66. $\frac{100\pi}{3}$

67. $\frac{250}{3}$

68. $\frac{\pi}{240}$

69. $\frac{5}{2}$

70. $\frac{2}{\pi}$

71. a. 28125π foot-pounds

b. 21093.75π foot-pounds

72. a. 840 foot-pounds

b. 952.5 foot-pounds

c. 862.5 foot-pounds

1. Let $f(x) = 2x^2 - 5x$. Find each of the following.

a. $f(a + b)$

$$= 2(a + b)^2 - 5(a + b)$$

$$= 2(a^2 + 2ab + b^2) - 5a - 5b$$

$$= 2a^2 + 4ab + 2b^2 - 5a - 5b$$

$$= 2a^2 + 4ab + 2b^2 - 5a - 5b$$

b. $f(2x) - 2f(x)$

$$= 2(2x)^2 - 5(2x) - 2(2x^2 - 5x)$$

$$= 8x^2 - 10x - 4x^2 + 10x$$

$$= 4x^2$$

2. Find the domain of each function.

a. $f(x) = \sqrt{x^2 - 3x - 4}$

$$f(x) = \sqrt{(x - 4)(x + 1)}$$

$$\text{Dom: } (-\infty, -1] \cup [4, \infty)$$

b. $g(x) = \frac{x + 2}{x^3 - x}$

$$g(x) = \frac{x + 2}{x(x^2 - 1)} = \frac{x + 2}{x(x + 1)(x - 1)}$$

$$\text{Dom: } (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$$

3. The graph of $f(x) = \sqrt{x + 3} - 4$ is the same as the graph of $g(x) = \sqrt{x}$ except that it is moved how?

Since $f(x) = g(x + 3) - 4$, the graph of f is the graph of g shifted left 3 units and down 4 units.

4. For the functions f and g , find $f \circ g$, $g \circ f$, and the domains of each.

a. $f(x) = \frac{1}{x^2 - 1}$ and $g(x) = \sqrt{x}$

$$(f \circ g)(x)$$

$$= f(g(x))$$

$$= f(\sqrt{x})$$

$$= \frac{1}{\sqrt{x^2} - 1}$$

$$= \frac{1}{x - 1}$$

$$\text{Dom: } [0, 1) \cup (1, \infty)$$

$$(g \circ f)(x)$$

$$= g(f(x))$$

$$= g\left(\frac{1}{x^2 - 1}\right)$$

$$= \sqrt{\frac{1}{x^2 - 1}}$$

$$= \frac{1}{\sqrt{x^2 - 1}}$$

$$= \frac{1}{\sqrt{(x + 1)(x - 1)}}$$

$$\text{Dom: } (-\infty, -1) \cup (1, \infty)$$

$$\text{Dom: } (-\infty, 1) \cup (1, \infty)$$

$$\text{b. } f(x) = x^2 + 1 \text{ and } g(x) = \frac{x}{x-1}$$

$$(g \circ f)(x)$$

$$(f \circ g)(x)$$

$$= g(f(x))$$

$$= f(g(x))$$

$$= g(x^2 + 1)$$

$$= f\left(\frac{x}{x-1}\right)$$

$$= \frac{x^2 + 1}{x^2 + 1 - 1}$$

$$= \left(\frac{x}{x-1}\right)^2 + 1$$

$$= \frac{x^2 + 1}{x^2}$$

$$= \frac{x^2}{(x-1)^2} + 1$$

$$\text{Dom: } (-\infty, 0) \cup (0, \infty)$$

5. Find the intercepts of the following functions. Also, determine whether the graphs of the functions are symmetric with respect to the y -axis or the origin.

$$\text{a. } f(x) = x^4 + x^3 + x^2$$

$$y\text{-intercept: none}$$

$$h(0) = 2$$

$$f(0) = 0$$

$$\frac{1}{x^3 - 3x} = 0$$

$$y\text{-intercept: } (0, 2)$$

$$y\text{-intercept: } (0, 0)$$

$$\text{No solution.}$$

$$2 - |x| = 0$$

$$x^4 + x^3 + x^2 = 0$$

$$x\text{-intercepts: none}$$

$$|x| = 2$$

$$x^2(x^2 + x + 1) = 0$$

$$g(-x)$$

$$x = \pm 2$$

$$x\text{-intercept: } (0, 0)$$

$$= \frac{1}{(-x^3) - 3(-x)}$$

$$x\text{-intercepts: } (-2, 0), (2, 0)$$

$$f(1) = 3$$

$$= \frac{1}{-x^3 + 3x}$$

$$h(-x)$$

$$f(-1) = 1$$

$$= -g(x)$$

$$= 2 - |-x|$$

Neither even nor odd.

So g is odd.

$$= 2 - |x|$$

Symmetry: none

$$= h(x)$$

$$\text{b. } g(x) = \frac{1}{x^3 - 3x}$$

Symmetry: origin

So h is even.

$$g(0) \text{ undefined}$$

$$\text{c. } h(x) = 2 - |x|$$

Symmetry: y -axis

6. For each of the following, find $\lim_{x \rightarrow c} f(x)$.

$$\text{a. } f(x) = \begin{cases} 4x - 2, & \text{if } x < 1 \\ 7, & \text{if } x = 1; \quad c = 1 \\ 2x - x^2, & \text{if } x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (4x - 2) = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2x - x^2) = 1$$

So $\lim_{x \rightarrow 1} f(x)$ does not exist.

$$\text{b. } f(x) = \begin{cases} x + 5, & \text{if } x \geq -2 \\ 2x^2 - x - 7, & \text{if } x < -2; \quad c = -2 \end{cases}$$

$$\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} (2x^2 - x - 7) = 3$$

$$\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^+} (x + 5) = 3$$

$$\lim_{x \rightarrow -2} f(x) = 3$$

7. Calculate the following limits.

$$\text{a. } \lim_{x \rightarrow 2} (\sqrt{x-1} - \sqrt{3x-2})$$

$$= \sqrt{2-1} - \sqrt{3 \cdot 2 - 2}$$

$$= -1$$

$$\text{b. } \lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 - 4x - 5}$$

$$= \lim_{x \rightarrow -1} \frac{(x+1)(x^2 - x + 1)}{(x+1)(x-5)}$$

$$= \lim_{x \rightarrow -1} \frac{x^2 - x + 1}{x - 5}$$

$$= \frac{(-1)^2 - (-1) + 1}{-1 - 5}$$

$$= -\frac{1}{2}$$

$$\text{c. } \lim_{x \rightarrow \infty} \frac{4x^3 + x - 5}{x^3 - 2}$$

Divide the leading coefficients.

$$\lim_{x \rightarrow \infty} \frac{4x^3 + x - 5}{x^3 - 2} = 4$$

$$\text{d. } \lim_{x \rightarrow 7^-} \frac{x^2 + 49}{x - 7}$$

If $x < 7$, then $x - 7 < 0$.

$$\lim_{x \rightarrow 7^-} \frac{x^2 + 49}{x - 7} = -\infty$$

$$\text{e. } \lim_{x \rightarrow 7^+} \frac{x^2 + 49}{x - 7}$$

If $x > 7$, then $x - 7 > 0$.

$$\lim_{x \rightarrow 7^+} \frac{x^2 + 49}{x - 7} = \infty$$

$$\text{f. } \lim_{x \rightarrow 7} \frac{x^2 + 49}{x - 7}$$

$$\lim_{x \rightarrow 7^-} \frac{x^2 + 49}{x - 7} = -\infty$$

$$\lim_{x \rightarrow 7^+} \frac{x^2 + 49}{x - 7} = \infty$$

$$\lim_{x \rightarrow 7} \frac{x^2 + 49}{x - 7} \text{ DNE}$$

$$\text{g. } \lim_{x \rightarrow \frac{1}{2}^+} \frac{|2x - 1|}{2x - 1}$$

If $x > \frac{1}{2}$, then $2x - 1 > 0$.

$$\lim_{x \rightarrow \frac{1}{2}^+} \frac{|2x - 1|}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}^+} \frac{2x - 1}{2x - 1} = 1$$

$$\text{h. } \lim_{x \rightarrow \frac{1}{2}^-} \frac{|2x - 1|}{2x - 1}$$

If $x < \frac{1}{2}$, then $2x - 1 < 0$.

$$\lim_{x \rightarrow \frac{1}{2}^-} \frac{|2x - 1|}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}^-} \frac{-(2x - 1)}{2x - 1} = -1$$

$$\text{i. } \lim_{x \rightarrow \frac{1}{2}} \frac{|2x - 1|}{2x - 1}$$

$$\lim_{x \rightarrow \frac{1}{2}^+} \frac{|2x - 1|}{2x - 1} = 1$$

$$\lim_{x \rightarrow \frac{1}{2}^-} \frac{|2x - 1|}{2x - 1} = -1$$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{|2x - 1|}{2x - 1} \text{ DNE}$$

$$\text{j. } \lim_{x \rightarrow -\infty} \frac{x^2 + 4x - 1}{x^4}$$

$$= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 + \frac{4}{x} - \frac{1}{x^2}\right)}{x^4}$$

$$= \lim_{x \rightarrow -\infty} \frac{1 + \frac{4}{x} - \frac{1}{x^2}}{x^2}$$

$$= 0$$

$$\text{k. } \lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x^3 + 2x^2 + 1}}{3x - 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x^3 \left(1 + \frac{2}{x} + \frac{1}{x^3}\right)}}{x \left(3 - \frac{5}{x}\right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \sqrt[3]{1 + \frac{2}{x} + \frac{1}{x^3}}}{x \left(3 - \frac{5}{x}\right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt[3]{1 + \frac{2}{x} + \frac{1}{x^3}}}{3 - \frac{5}{x}}$$

$$= \frac{1}{3}$$

$$\text{k. } \lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x^3 + 2x^2 + 1}}{3x - 5}$$

If $x \leq -3$, then

$$\frac{\sqrt[3]{x^3}}{3x - 5}$$

$$\leq \frac{\sqrt[3]{x^3 + 2x^2 + 1}}{3x - 5}$$

$$\leq \frac{\sqrt[3]{x^3 + 3x^2 + 3x + 1}}{3x - 5}.$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x^3}}{3x - 5} = \lim_{x \rightarrow -\infty} \frac{x}{3x - 5} = \frac{1}{3}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x^3 + 3x^2 + 3x + 1}}{3x - 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt[3]{(x + 1)^3}}{3x - 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{x + 1}{3x - 5}$$

$$= \frac{1}{3}$$

By the Squeeze Theorem,

$$\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x^3 + 2x^2 + 1}}{3x - 5} = \frac{1}{3}.$$

$$\text{l. } \lim_{x \rightarrow -3^+} \sqrt{x^2 - 9}$$

If $-3 < x < -2$, then $x^2 - 9 < 0$.

$$\lim_{x \rightarrow -3^+} \sqrt{x^2 - 9} \text{ DNE}$$

$$\text{m. } \lim_{x \rightarrow \infty} \frac{\sin \frac{1}{x}}{x}$$

$$\text{If } x > 0, \text{ then } -\frac{1}{x} \leq \frac{\sin \frac{1}{x}}{x} \leq \frac{1}{x}.$$

$$\lim_{x \rightarrow \infty} -\frac{1}{x} = 0$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\text{By the Squeeze Theorem, } \lim_{x \rightarrow \infty} \frac{\sin \frac{1}{x}}{x} = 0.$$

$$\text{n. } \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sec x - 1}{\tan x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{1}{\cos x} - 1}{\frac{\sin x}{\cos x}}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \cos x}{\sin x}$$

$$= \frac{1 - \cos \frac{\pi}{2}}{\sin \frac{\pi}{2}}$$

$$= \frac{1 - 0}{1}$$

$$= 1$$

$$\text{o. } \lim_{x \rightarrow 1} \frac{\sqrt{8+x^2} - 3}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{8+x^2} - 3}{x - 1} \cdot \frac{\sqrt{8+x^2} + 3}{\sqrt{8+x^2} + 3}$$

$$= \lim_{x \rightarrow 1} \frac{x^2 - 1}{(x - 1)(\sqrt{8+x^2} + 3)}$$

$$= \lim_{x \rightarrow 1} \frac{(x + 1)(x - 1)}{(x - 1)(\sqrt{8+x^2} + 3)}$$

$$= \lim_{x \rightarrow 1} \frac{x + 1}{\sqrt{8+x^2} + 3}$$

$$= \frac{1}{3}$$

$$\text{o. } \lim_{x \rightarrow 1} \frac{\sqrt{8+x^2} - 3}{x - 1}$$

$$\text{Let } f(x) = \sqrt{8+x^2} - 3.$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{8+x^2} - 3}{x - 1} = f'(1)$$

$$f'(x) = \frac{1}{2}(8+x^2)^{-\frac{1}{2}}(2x)$$

$$f'(1) = \frac{1}{3}$$

$$\text{p. } \lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{3} + h\right) - \tan \frac{\pi}{3}}{h}$$

$$\text{Let } f(x) = \tan x.$$

$$\lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{3} + h\right) - \tan \frac{\pi}{3}}{h} = f'\left(\frac{\pi}{3}\right)$$

$$f'(x) = \sec^2 x$$

$$f'\left(\frac{\pi}{3}\right) = \sec^2\left(\frac{\pi}{3}\right) = 4$$

$$\text{q. } \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \sin \frac{\pi}{6}}{h}$$

$$\text{Let } f(x) = \sin x.$$

$$\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \sin \frac{\pi}{6}}{h} = f'\left(\frac{\pi}{6}\right)$$

$$f'(x) = \cos x$$

$$f'\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\text{r. } \lim_{h \rightarrow 0} \frac{\cos(x + h) - \cos x}{h}$$

$$\text{Let } f(x) = \cos x.$$

$$\lim_{h \rightarrow 0} \frac{\cos(x + h) - \cos x}{h} = f'(x)$$

$$f'(x) = -\sin x$$

8. For each of the following, define P such that the given function is continuous at 3.

$$\text{a. } f(x) = \begin{cases} \frac{3(x^4 - 81)}{x^2 - 9}, & \text{if } x \neq 3 \\ Px + 9, & \text{if } x = 3 \end{cases}$$

$$\lim_{x \rightarrow 3} f(x)$$

$$= \lim_{x \rightarrow 3} \frac{3(x^4 - 81)}{x^2 - 9}$$

$$= \lim_{x \rightarrow 3} \frac{3(x^2 - 9)(x^2 + 9)}{x^2 - 9}$$

$$= \lim_{x \rightarrow 3} 3(x^2 + 9)$$

$$= 54$$

$$f(3) = 54$$

$$3P + 9 = 54$$

$$P = 15$$

$$\text{b. } g(x) = \begin{cases} \frac{x^3 - 27}{x^2 - 9}, & \text{if } x \neq 3 \\ P, & \text{if } x = 3 \end{cases}$$

$$\lim_{x \rightarrow 3} g(x)$$

$$= \lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$$

$$= \lim_{x \rightarrow 3} \frac{(x - 3)(x^2 + 3x + 9)}{(x + 3)(x - 3)}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 + 3x + 9}{x + 3}$$

$$= \frac{9}{2}$$

$$P = \frac{9}{2}$$

9. Determine the intervals on which the functions defined below are continuous.

$$\text{a. } f(x) = \begin{cases} 8 - 7x, & \text{if } x \leq 4 \\ -x - 16, & \text{if } x > 4 \end{cases}$$

$$f(4) = -20$$

$$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} (8 - 7x) = -20$$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} (-x - 16) = -20$$

$$\lim_{x \rightarrow 4} f(x) = -20$$

$$\lim_{x \rightarrow 4} f(x) = f(4)$$

So f is continuous at 4.

Hence, f is continuous on $(-\infty, \infty)$.

$$\text{b. } g(x) = \begin{cases} x^2 + 1, & \text{if } x \leq -3 \\ 5 - x, & \text{if } x > -3 \end{cases}$$

$$g(-3) = 10$$

$$\lim_{x \rightarrow -3^-} g(x) = \lim_{x \rightarrow -3^-} (x^2 + 1) = 10$$

$$\lim_{x \rightarrow -3^+} g(x) = \lim_{x \rightarrow -3^+} (5 - x) = 8$$

$$\lim_{x \rightarrow -3} g(x) \text{ DNE}$$

So g is not continuous at -3.

Note that g is left continuous at -3.

Hence, g is continuous on $(-\infty, -3]$ and $\cup(-3, \infty)$.

10. Identify all asymptotes of the following.

a. $y = \frac{x-2}{x-1}$

$$\lim_{x \rightarrow \infty} \frac{x-2}{x-1} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x-2}{x-1} = 1$$

HA: $y = 1$

$$\lim_{x \rightarrow 1^-} \frac{x-2}{x-1} = \infty$$

$$\lim_{x \rightarrow 1^+} \frac{x-2}{x-1} = -\infty$$

VA: $x = 1$

OA: none

b. $y = \frac{x^2 - 3x + 2}{x^2 - 4}$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 2}{x^2 - 4} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 3x + 2}{x^2 - 4} = 1$$

HA: $y = 1$

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x-1)}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{x-1}{x+2}$$

$$= \frac{1}{4}$$

$$\lim_{x \rightarrow -2^-} \frac{x^2 - 3x + 2}{x^2 - 4}$$

$$= \lim_{x \rightarrow -2^-} \frac{(x-2)(x-1)}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow -2^-} \frac{x-1}{x+2}$$

$$= \infty$$

$$= \lim_{x \rightarrow -2^+} \frac{x^2 - 3x + 2}{x^2 - 4}$$

$$= \lim_{x \rightarrow -2^+} \frac{(x-2)(x-1)}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow -2^+} \frac{x-1}{x+2}$$

$$= -\infty$$

VA: $x = -2$

OA: none

c. $y = \frac{\sqrt{x^2 + 4}}{x}$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 4}}{x}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 \left(1 + \frac{4}{x^2}\right)}}{x}$$

$$= \lim_{x \rightarrow \infty} \frac{x \sqrt{1 + \frac{4}{x^2}}}{x}$$

$$= \lim_{x \rightarrow \infty} \sqrt{1 + \frac{4}{x^2}}$$

$$= 1$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 4}}{x}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 \left(1 + \frac{4}{x^2}\right)}}{x}$$

$$= \lim_{x \rightarrow -\infty} \frac{|x| \sqrt{1 + \frac{4}{x^2}}}{x}$$

$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} \frac{-x\sqrt{1 + \frac{4}{x^2}}}{x} \\
&= \lim_{x \rightarrow -\infty} -\sqrt{1 + \frac{4}{x^2}} \\
&= -1
\end{aligned}$$

HA: $y = -1$, $y = 1$

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{x^2 + 4}}{x} = -\infty$$

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x^2 + 4}}{x} = \infty$$

VA: $x = 0$

OA: none

d. $y = \frac{x^2 - 9}{x^2 - 5x + 6}$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 9}{x^2 - 5x + 6} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 9}{x^2 - 5x + 6} = 1$$

HA: $y = 1$

$$\lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 - 5x + 6}$$

$$= \lim_{x \rightarrow -3} \frac{(x - 3)(x + 3)}{(x - 3)(x - 2)}$$

$$= \lim_{x \rightarrow -3} \frac{x + 3}{x - 2}$$

$$= 0$$

$$\lim_{x \rightarrow 2^-} \frac{x^2 - 9}{x^2 - 5x + 6}$$

$$= \lim_{x \rightarrow 2^-} \frac{(x - 3)(x + 3)}{(x - 3)(x - 2)}$$

$$= \lim_{x \rightarrow 2^-} \frac{x + 3}{x - 2}$$

$$= -\infty$$

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 9}{x^2 - 5x + 6}$$

$$= \lim_{x \rightarrow 2^+} \frac{(x - 3)(x + 3)}{(x - 3)(x - 2)}$$

$$= \lim_{x \rightarrow 2^+} \frac{x + 3}{x - 2}$$

$$= \infty$$

VA: $x = 2$

OA: none

11. Give a specific example to show that it is possible for $\lim_{x \rightarrow a} f(x)$ to exist if $f(a)$ is undefined.

$$f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$$

$f(3)$ DNE

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 5x + 6}$$

$$= \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{(x - 3)(x - 2)}$$

$$= \lim_{x \rightarrow 3} \frac{x + 3}{x - 2}$$

$$= 6$$

There are infinitely many correct answers.

12. Determine y' when $x = 2$ if $y = \frac{7}{\sqrt{x^4 - 15}}$.

$$y = 7(x^4 - 15)^{-\frac{1}{2}}$$

$$y' = -\frac{7}{2}(x^4 - 15)^{-\frac{3}{2}}(4x^3)$$

$$x = 2$$

$$y' = -112$$

$$y' = -\frac{7}{2}(2^4 - 15)^{-\frac{3}{2}}(4 \cdot 2^3)$$

13. Determine $f'(5)$ if $f(x) = \sqrt[3]{(2x+17)^2}$.

$$f(x) = (2x+17)^{\frac{2}{3}}$$

$$f'(5) = \frac{2}{3}(2 \cdot 5 + 17)^{-\frac{1}{3}}(2)$$

$$f'(x) = \frac{2}{3}(2x+17)^{-\frac{1}{3}}(2)$$

$$f'(5) = \frac{4}{9}$$

14. Determine y' when $x = 1$ if $y = \frac{8x}{3} - \frac{3}{8x}$.

$$y = \frac{8}{3}x - \frac{3}{8}x^{-1}$$

$$y' = \frac{8}{3} + \frac{3}{8}$$

$$y' = \frac{8}{3} + \frac{3}{8}x^{-2}$$

$$y' = \frac{73}{24}$$

$$x = 1$$

15. Determine y' when $x = 1$ if $y = (2-x)\sqrt{x^2+8}$.

$$y = (2-x)(x^2+8)^{\frac{1}{2}}$$

$$x = 1$$

$$y' = (-1)(x^2+8)^{\frac{1}{2}} + (2-x) \cdot \frac{1}{2}(x^2+8)^{-\frac{1}{2}}(2x)$$

$$y' = -3 + \frac{1}{3}$$

$$= -\sqrt{x^2+8} + \frac{2-x}{\sqrt{x^2+8}}$$

$$y' = -\frac{8}{3}$$

16. Find the equation of the tangent line to the curve defined by $y = 2x^2 - 5x + 8$ when $x = 2$.

$$y = 2x^2 - 5x + 8$$

$$y = 6$$

$$y = 3x$$

$$y' = 4x - 5$$

$$y' = 3$$

$$x = 2$$

$$y - 6 = 3(x - 2)$$

17. At what point (x, y) is the tangent line to the curve $y = 2x^2 - 5x + 8$ parallel to the line $y = 3x - 7$?

$$y = 2x^2 - 5x + 8$$

$$4x = 8$$

$$(2, 6)$$

$$y' = 4x - 5$$

$$x = 2$$

$$4x - 5 = 3$$

$$y = 6$$

18. Find the equation of the line tangent to the graph of $x^2 + 2xy^2 + 3y = 31$ at the point $(2, -3)$.

$$x^2 + 2xy^2 + 3y = 31$$

$$y = -3$$

$$2x + 2y^2 + 4xyy' + 3y' = 0$$

$$y' = \frac{-2 \cdot 2 - 2(-3)^2}{4 \cdot 2(-3) + 3}$$

$$4xyy' + 3y' = -2x - 2y^2$$

$$y' = \frac{22}{21}$$

$$y'(4xy + 3) = -2x - 2y^2$$

$$y + 3 = \frac{22}{21}(x - 2)$$

$$y' = \frac{-2x - 2y^2}{4xy + 3}$$

$$y = \frac{22}{21}x - \frac{107}{21}$$

$$x = 2$$

19. Let $y = \sqrt{x^2 - 1}$. Find y'' when $x = 2$.

$$y = (x^2 - 1)^{\frac{1}{2}}$$

$$y'' = \frac{-1}{\sqrt{(x^2 - 1)^3}}$$

$$y' = \frac{1}{2}(x^2 - 1)^{-\frac{1}{2}}(2x)$$

$$x = 2$$

$$y' = x(x^2 - 1)^{-\frac{1}{2}}$$

$$y'' = \frac{-1}{\sqrt{27}}$$

$$y'' = (1)(x^2 - 1)^{-\frac{1}{2}} + x \cdot -\frac{1}{2}(x^2 - 1)^{-\frac{3}{2}}(2x)$$

$$y'' = \frac{-1}{3\sqrt{3}}$$

20. If $f(x) = \sqrt{x + \sqrt{x}}$, find $f'(1)$.

$$f(x) = \left(x + x^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$f'(x) = \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x + \sqrt{x}}}$$

$$f'(x) = \frac{1}{2} \left(x + x^{\frac{1}{2}}\right)^{-\frac{1}{2}} \left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$f'(1) = \frac{3}{4\sqrt{2}}$$

21. Determine $f'(1)$ if $f(x) = \sqrt[3]{\frac{x}{x^3 + 1}}$.

$$f(x) = \left(\frac{x}{x^3 + 1}\right)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} \left(\frac{x}{x^3 + 1}\right)^{-\frac{2}{3}} \left(\frac{(1)(x^3 + 1) - x(3x^2)}{(x^3 + 1)^2}\right)$$

$$f'(1) = -\frac{\sqrt[3]{4}}{12}$$

22. Determine $f'(3)$ if $f(x) = \frac{1}{x - \sqrt{x^2 - 5}}$.

$$f(x) = \left(x - (x^2 - 5)^{\frac{1}{2}} \right)^{-1}$$

$$f'(x) = - \left(x - (x^2 - 5)^{\frac{1}{2}} \right)^{-2} \left(1 - \frac{1}{2} (x^2 - 5)^{-\frac{1}{2}} (2x) \right)$$

$$f'(3) = \frac{1}{2}$$

23. Determine y'' at $x = 1$ if $y = 3\sqrt[3]{x^4} - \frac{1}{3x^3}$.

$$y = 3x^{\frac{4}{3}} - \frac{1}{3}x^{-3}$$

$$y'' = \frac{4}{3}x^{-\frac{2}{3}} - 4x^{-5}$$

$$y' = 4x^{\frac{1}{3}} + x^{-4}$$

$$x = 1$$

$$y'' = -\frac{8}{3}$$

24. Let $y = [\cos(2x - \pi)]^3$. Find y' at $x = \frac{\pi}{6}$.

$$y = \cos^3(2x - \pi).$$

$$x = \frac{\pi}{6}$$

$$y' = 3 \cos^2(2x - \pi)(-\sin(2x - \pi))(2)$$

$$y' = \frac{3\sqrt{3}}{4}$$

25. Determine $f' \left(\frac{\pi}{4} \right)$ if $f(x) = \frac{\tan x}{1 + \cos x}$.

$$f(x) = \frac{\tan x}{1 + \cos x}$$

$$= \frac{2 + \frac{3\sqrt{2}}{2}}{\frac{3}{2} + \sqrt{2}}$$

$$f'(x) = \frac{(\sec^2 x)(1 + \cos x) - (\tan x)(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\frac{4+3\sqrt{2}}{2}}{\frac{3+2\sqrt{2}}{2}}$$

$$f' \left(\frac{\pi}{4} \right)$$

$$= \frac{2 \left(1 + \frac{\sqrt{2}}{2} \right) - 1 \left(-\frac{\sqrt{2}}{2} \right)}{\left(1 + \frac{\sqrt{2}}{2} \right)^2}$$

$$= \frac{4 + 3\sqrt{2}}{3 + 2\sqrt{2}}$$

$$= \frac{4 + 3\sqrt{2}}{3 + 2\sqrt{2}} \cdot \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$$= \frac{2 + \sqrt{2} + \frac{\sqrt{2}}{2}}{1 + \sqrt{2} + \frac{1}{2}}$$

$$= \sqrt{2}$$

26. If $f(x) = \sin 3x \cos 2x$, find $f' \left(\frac{\pi}{6} \right)$.

$$f(x) = \sin 3x \cos 2x$$

$$f' \left(\frac{\pi}{6} \right) = 3 \cdot 0 \cdot \frac{1}{2} - 2 \cdot 1 \cdot \frac{\sqrt{3}}{2}$$

$$f'(x) = 3(\cos 3x)(\cos 2x) + (\sin 3x)(-2 \sin 2x)$$

$$f' \left(\frac{\pi}{6} \right) = -\sqrt{3}$$

27. If $y^4 + x^4 - 2x^2y + 9x = 9$. Find y' at $(1, 1)$.

$$y^4 + x^4 - 2x^2y + 9x = 9$$

$$y' = \frac{-4x^3 + 4xy - 9}{4y^3 - 2x^2}$$

$$4y^3y' + 4x^3 - 4xy - 2x^2y' + 9 = 0$$

$$x = 1$$

$$4y^3y' - 2x^2y' = -4x^3 + 4xy - 9$$

$$y = 1$$

$$y'(4y^3 - 2x^2) = -4x^3 + 4xy - 9$$

$$y' = -\frac{9}{2}$$

28. Let $f'(x) = x^2(4x - 3)$ and $f''(x) = 6x(2x - 1)$. Determine the intervals on which f is increasing.

$$f'(x) = x^2(4x - 3)$$

$$\text{Inc: } \left[\frac{3}{4}, \infty\right)$$

29. Determine the intervals on which f is decreasing if $f'(x) = \frac{x-1}{2x+3}$ and $f''(x) = \frac{5}{(2x+3)^2}$.

$$f'(x) = \frac{x-1}{2x+3}$$

$$\text{Dec: } \left(-\frac{3}{2}, 1\right]$$

30. Determine the intervals on which f is concave upward if $f(x) = \frac{1}{10}x^5 + \frac{1}{6}x^4 - 4x^3 + 87x + 69$.

$$f(x) = \frac{1}{10}x^5 + \frac{1}{6}x^4 - 4x^3 + 87x + 69$$

$$f''(x) = 2x(x^2 + x - 12)$$

$$f'(x) = \frac{1}{2}x^4 + \frac{2}{3}x^3 - 12x^2 + 87$$

$$f''(x) = 2x(x+4)(x-3)$$

$$f''(x) = 2x^3 + 2x^2 - 24x$$

$$\text{CU: } (-4, 0), (3, \infty)$$

31. Determine the intervals on which f is concave downward if $f'(x) = \frac{-3}{2(x^2-4)^2}$ and

$$f''(x) = \frac{6x}{(x^2-4)^3}.$$

$$f''(x) = \frac{6x}{(x^2-4)^3}$$

$$f''(x) = \frac{6x}{[(x+2)(x-2)]^3}$$

$$\text{CD: } (-\infty, -2), (0, 2)$$

32. Determine all points of inflection for $f(x) = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 6x^2$.

$$f(x) = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 6x^2$$

$$f''(x) = (x+4)(x-3)$$

$$f'(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x$$

$$\text{CD: } (-4, 3)$$

$$f''(x) = x^2 + x - 12$$

$$\text{CU: } (-\infty, -4), (3, \infty)$$

$$\text{IP: } \left(-4, -\frac{256}{3}\right), \left(3, -\frac{171}{4}\right)$$

33. Determine all points of inflection for $f(x) = \frac{1}{12}x^4 - \frac{7}{3}x^3 + \frac{49}{2}x^2 + 88$.

$$f(x) = \frac{1}{12}x^4 - \frac{7}{3}x^3 + \frac{49}{2}x^2 + 88$$

$$f''(x) = (x - 7)^2$$

$$f'(x) = \frac{1}{3}x^3 - 7x^2 + 49x$$

CD: never

$$f''(x) = x^2 - 14x + 49$$

CU: $(-\infty, \infty)$

IP: none

34. Let $f(x) = \frac{1}{3}x^3 + 3x^2 + x - 2$. Find all local extrema for f .

$$f(x) = \frac{1}{3}x^3 + 3x^2 + x - 2$$

$$x = -3 \pm 2\sqrt{2}$$

$$f'(x) = x^2 + 6x + 1$$

$$\text{Dec: } [-3 - 2\sqrt{2}, -3 + 2\sqrt{2}]$$

$$x^2 + 6x + 1 = 0$$

$$\text{Inc: } (-\infty, -3 - 2\sqrt{2}], [-3 + 2\sqrt{2}, \infty)$$

$$x = \frac{-6 \pm \sqrt{36 - 4}}{2}$$

$$\text{Local max: } 13 + \frac{32}{3}\sqrt{2} \text{ at } -3 - 2\sqrt{2}$$

$$x = \frac{-6 \pm 4\sqrt{2}}{2}$$

$$\text{Local min: } 13 - \frac{32}{3}\sqrt{2} \text{ at } -3 + 2\sqrt{2}$$

35. Let $f(x) = \frac{1}{4}x^4 - \frac{5}{2}x^2 + 3$. Find all local extrema for f .

$$f(x) = \frac{1}{4}x^4 - \frac{5}{2}x^2 + 3$$

$$\text{Dec: } (-\infty, -\sqrt{5}], [0, \sqrt{5}]$$

$$f'(x) = x^3 - 5x$$

$$\text{Inc: } [-\sqrt{5}, 0], [\sqrt{5}, \infty)$$

$$f'(x) = x(x^2 - 5)$$

Local max: 3 at 0

$$f'(x) = x(x + \sqrt{5})(x - \sqrt{5})$$

$$\text{Local min: } -\frac{13}{4} \text{ at } -\sqrt{5}, -\frac{13}{4} \text{ at } \sqrt{5}$$

36. For each of the following, find the maximum and minimum values of the given function on the indicated interval.

a. $f(x) = x^3 - 12x; [0, 3]$

$$f(2) = -16 \text{ (min)}$$

$$f'(x) = 3x^2 - 12$$

Note that $-2 \notin [0, 3]$.

$$f'(x) = 3(x^2 - 4)$$

b. $f(x) = x^3 - 12x; [-3, 0]$

$$f'(x) = 3(x + 2)(x - 2)$$

$$f'(x) = 3x^2 - 12$$

$$f(0) = 0 \text{ (max)}$$

$$f'(x) = 3(x^2 - 4)$$

$$f(3) = -9$$

$$f'(x) = 3(x + 2)(x - 2)$$

$$f(-3) = 9$$

$$f(-2) = 16 \text{ (max)}$$

$$f(0) = 0 \text{ (min)}$$

Note that $2 \notin [-3, 0]$.

37. A rock thrown from the top of a cliff is $s(t) = 192 + 64t - 16t^2$ feet above the ground t seconds after being thrown.

a. Determine the height of the cliff.

$$s(0) = 192$$

$$192 \text{ ft}$$

b. Determine the time it takes the rock to reach the ground.

$$-16t^2 + 64t + 192 = 0$$

$$t = 6$$

$$t^2 - 4t - 12 = 0$$

$$6 \text{ sec}$$

$$(t + 2)(t - 6) = 0$$

c. Find the velocity of the rock when it strikes the ground.

$$s(t) = -16t^2 + 64t + 192$$

$$v(6) = -128$$

$$v(t) = -32t + 64$$

$$-128 \text{ ft/sec}$$

38. A rock is thrown vertically upward from the roof of a house 32 feet high with an initial velocity of 128 ft/sec.

a. What is the speed of the rock at the end of 2 seconds?

$$s(t) = -16t^2 + 128t + 32$$

$$v(2) = 64$$

$$v(t) = -32t + 128$$

$$64 \text{ ft/sec}$$

b. What is the maximum height the rock will reach?

$$v(t) = -32t + 128$$

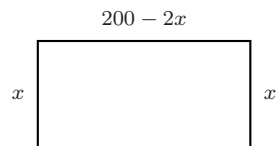
$$s(4) = 288$$

$$-32t + 128 = 0$$

$$288 \text{ ft}$$

$$t = 4$$

39. What is the maximum area which can be enclosed by 200 ft of fencing if the enclosure is in the shape of a rectangle and one side of the rectangle requires no fencing?



$$A(x) = x(200 - 2x)$$

$$A(x) = -2x^2 + 200x$$

$$A'(x) = -4x + 200$$

$$A'(x) = -4(x - 50)$$

$$A(0) = 0$$

$$A(100) = 0$$

$$A(50) = 5000 \text{ (max)}$$

$$5000 \text{ ft}^2$$

40. A woman throws a ball vertically upward from the ground. The equation of its motion is given by $s(t) = -16t^2 + ct$, where c is the initial velocity of the ball. If she wants the ball to reach a maximum height of 100 ft, find c .

$$s(t) = -16t^2 + ct$$

$$v(t) = -32t + c$$

$$-32t + c = 0$$

$$t = \frac{c}{32}$$

$$s\left(\frac{c}{32}\right) = 100$$

$$-16\left(\frac{c}{32}\right)^2 + c\left(\frac{c}{32}\right) = 100$$

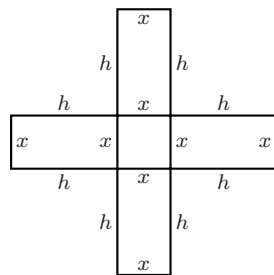
$$-\frac{c^2}{64} + \frac{c^2}{32} = 100$$

$$\frac{c^2}{64} = 100$$

$$c^2 = 6400$$

$$c = 80$$

41. A rectangular open tank is to have a square base, and its volume is to be 125 yd³. The cost per square yard for the base is \$8 and for the sides is \$4. Find the dimensions of the tank in order to minimize the cost of the material.



$$x^2h = 125$$

$$h = \frac{125}{x^2}$$

$$C(x) = 8 \cdot x^2 + 4 \cdot 4xh$$

$$C(x) = 8x^2 + 16x\left(\frac{125}{x^2}\right)$$

$$C(x) = 8x^2 + \frac{2000}{x}$$

$$C'(x) = 16x - \frac{2000}{x^2}$$

$$C'(x) = \frac{16x^3 - 2000}{x^2}$$

$$C'(x) = \frac{16(x^3 - 125)}{x^2}$$

$$\text{Dec: } (0, 5]$$

$$\text{Inc: } [5, \infty)$$

$$\text{Absolute min: } 600 \text{ at } 5$$

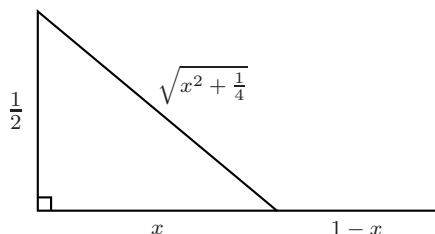
$$\text{Length of base side: } 5$$

$$\text{Height: } 5$$

$$\text{Dimensions: } 5 \times 5 \times 5$$

42. A power station is on one side of a river which is $\frac{1}{2}$ mile wide, and a factory is 1 mile downstream on the other side of the river. It costs \$300 per foot to run power lines overland and \$500 per foot to run them underwater. Find the most economical way to run the power lines from the power station to the factory.

Recall: 1 mile = 5280 feet



$$C(x) = 500\sqrt{x^2 + \frac{1}{4}} \cdot 5280 + 300(1-x) \cdot 5280$$

$$C(x) = 528000 \left(5\sqrt{x^2 + \frac{1}{4}} + 3(1-x) \right)$$

$$C(x) = 528000 \left(5\sqrt{x^2 + \frac{1}{4}} - 3x + 3 \right)$$

$$C(x) = 528000 \left(5 \left(x^2 + \frac{1}{4} \right)^{\frac{1}{2}} - 3x + 3 \right)$$

$$C'(x) = 528000 \left(\frac{5}{2} \left(x^2 + \frac{1}{4} \right)^{-\frac{1}{2}} (2x) - 3 \right)$$

$$C'(x) = 528000 \left(\frac{5x}{\sqrt{x^2 + \frac{1}{4}}} - 3 \right)$$

Solve $C'(x) = 0$.

$$528000 \left(\frac{5x}{\sqrt{x^2 + \frac{1}{4}}} - 3 \right) = 0$$

$$\frac{5x}{\sqrt{x^2 + \frac{1}{4}}} - 3 = 0$$

$$\frac{5x}{\sqrt{x^2 + \frac{1}{4}}} = 3$$

$$5x = 3\sqrt{x^2 + \frac{1}{4}}$$

$$25x^2 = 9 \left(x^2 + \frac{1}{4} \right)$$

$$25x^2 = 9x^2 + \frac{9}{4}$$

$$16x^2 = \frac{9}{4}$$

$$x^2 = \frac{9}{64}$$

$$x = \frac{3}{8}$$

$$C(0) = 2904000$$

$$C(1) = 2951609.73$$

$$C\left(\frac{3}{8}\right) = 2640000 \text{ (min)}$$

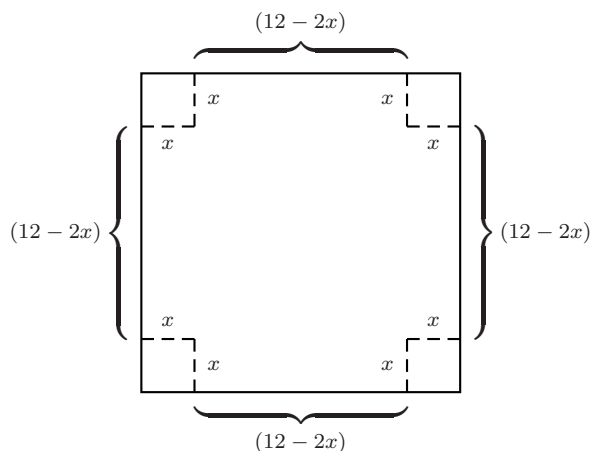
$$1 - \frac{3}{8} = \frac{5}{8}$$

$$\sqrt{\left(\frac{3}{8}\right)^2 + \frac{1}{4}} = \frac{5}{8}$$

Overland $\frac{5}{8}$ mile

Underwater $\frac{3}{8}$ mile

43. A cardboard box manufacturer wishes to make open boxes from pieces of cardboard 12 in square by cutting equal squares from the four corners and turning up the sides. Find the length of the side of the square to be cut out in order to obtain a box of the largest possible volume. What is the largest possible volume?



$$V(x) = x(12 - 2x)(12 - 2x)$$

$$V(x) = 4x^3 - 48x^2 + 144x$$

$$V'(x) = 12x^2 - 96x + 144$$

$$V'(x) = 12(x^2 - 8x + 12)$$

$$V'(x) = 12(x - 2)(x - 6)$$

$$V(0) = 0$$

$$V(6) = 0$$

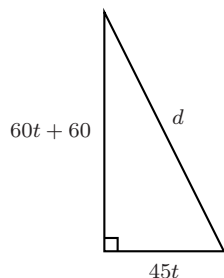
$$V(2) = 128 \text{ (max)}$$

$$2 \times 2 \text{ square}$$

$$\text{Volume of } 128 \text{ in}^3$$

44. A train leaves a station traveling north at the rate of 60 mph. One hour later, a second train leaves the same station traveling east at the rate of 45 mph. Find the rate at which the trains are separating 2 hours after the second train leaves the station.

Let t be the amount of time the second train has traveled.



$$d = \sqrt{(60t + 60)^2 + (45t)^2}$$

$$d = \sqrt{3600t^2 + 7200t + 3600 + 2025t^2}$$

$$d = \sqrt{5625t^2 + 7200t + 3600}$$

$$d = 15\sqrt{25t^2 + 32t + 16}$$

$$d = 15(25t^2 + 32t + 16)^{\frac{1}{2}}$$

$$d' = \frac{15}{2}(25t^2 + 32t + 16)^{-\frac{1}{2}}(50t + 32)$$

$$d' = \frac{15(50t + 32)}{2\sqrt{25t^2 + 32t + 16}}$$

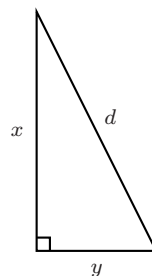
$$t = 2$$

$$d' = \frac{165}{\sqrt{5}}$$

$$d' = 33\sqrt{5}$$

$$33\sqrt{5} \text{ mph}$$

Alternatively,



$$x^2 + y^2 = d^2$$

$$\sqrt{5}d' = 165$$

$$2xx' + 2yy' = 2dd'$$

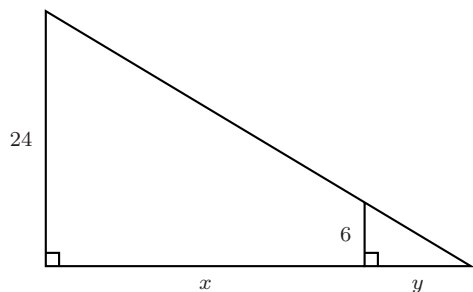
$$d' = 33\sqrt{5}$$

$$2 \cdot 180 \cdot 60 + 2 \cdot 90 \cdot 45 = 2 \cdot 90\sqrt{5}d'$$

$$33\sqrt{5} \text{ mph}$$

$$29700 = 180\sqrt{5}d'$$

45. A street light hangs 24 ft above the sidewalk. A man 6 ft tall walks away from the light at the rate of 3 ft/sec. At what rate is the length of his shadow increasing?



$$\frac{y}{6} = \frac{x+y}{24}$$

$$y' = \frac{1}{3}x'$$

$$4y = x + y$$

$$x' = 3$$

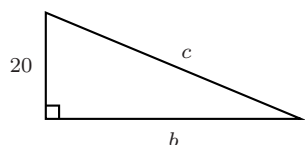
$$3y = x$$

$$y' = 1$$

$$y = \frac{1}{3}x$$

$$1 \text{ ft/sec}$$

46. A barge is pulled toward a dock by means of a taut cable. If the barge is 20 ft below the level of the dock, and the cable is pulled in at the rate of 36 ft/min, find the speed of the barge when the cable is 52 ft long.



$$b^2 + 20^2 = c^2$$

$$b' = 39$$

$$2bb' = 2cc'$$

$$39 \text{ ft/min}$$

$$2 \cdot 48 \cdot b' = 2 \cdot 52 \cdot 36$$

47. Find the values of Δy and dy if $y = 2x^2 - x$, $x = 2$, and $dx = \Delta x = .01$.

$$y = 2x^2 - x$$

$$dy = (4x - 1)dx$$

$$dy = .07$$

$$\Delta y = 6.0702 - 6 = .0702$$

$$x = 2$$

$$y' = 4x - 1$$

$$dx = .01$$

48. Use differentials to approximate the maximum possible error that can be produced when calculating the volume of a cube if the length of an edge is known to be $2 \pm .005$ ft.

$$V = x^3$$

$$x = 2$$

$$\pm .06 \text{ ft}^3$$

$$V' = 3x^2$$

$$dx = .005$$

$$dV = 3x^2 dx$$

$$dV = .06$$

49. Approximate $\sqrt{50}$ using each of the following.

a. Differentials	$\sqrt{50}$	$f(49) = 7$
$y = \sqrt{x}$	$\approx \sqrt{49} + \frac{1}{14}$	$f'(49) = \frac{1}{14}$
$y' = \frac{1}{2\sqrt{x}}$	$= 7 + \frac{1}{14}$	$L(x) = \frac{1}{14}(x - 49) + 7$
$dy = \frac{1}{2\sqrt{x}}dx$	b. A linearization	$f(50)$
$x = 49$	$f(x) = \sqrt{x}$	$\approx L(50)$
$dx = 1$	$f'(x) = \frac{1}{2\sqrt{x}}$	$= 7 + \frac{1}{14}$
$dy = \frac{1}{14}$	$x = 49$	

50. The moment of inertia of an annular cylinder is $I = .5M(R_2^2 - R_1^2)$, where M is the mass of the cylinder, R_2 is its outer radius, and R_1 is its inner radius. If $R_1 = 2$, and R_2 changes from 4 to 4.01, use differentials to estimate the resulting change in the moment of inertia.

$I = .5M(R_2^2 - 2^2)$	$dI = MR_2 dR_2$	$dI = M \times 4 \times .01$
$I = .5MR_2^2 - 2M$	$R_2 = 4$	$dI = .04M$
$I' = MR_2$	$dR_2 = .01$	

51. The range of a shell shot from a certain ship is $R = 300 \sin(2\theta)$ meters, where θ is the angle above horizontal of the gun when it is shot. If the gun is intended to be fired at an angle of $\frac{\pi}{6}$ radians to hit its target, but due to waves it actually shot .05 radians too low, use differentials to estimate how far short of its target the shell will fall.

$R = 300 \sin(2\theta)$	$\theta = \frac{\pi}{6}$	$dR = 600 \times \frac{1}{2} \times -.05$
$R' = 300 \cos(2\theta)(2)$	$d\theta = -.05$	$dR = -15$
$R' = 600 \cos(2\theta)$	$dR = 600 \cos\left(\frac{\pi}{3}\right)(-.05)$	15 meters
$dR = 600 \cos(2\theta)d\theta$		

52. For each of the following, determine whether the Intermediate Value Theorem guarantees the equation has a solution in the specified interval.

a. $x^5 + 2x^2 - 10x + 5 = 0$; $[1, 2]$	$f(1) = -2 < 0$
$f(x) = x^5 + 2x^2 - 10x + 5$	$f(2) = 25 > 0$

Yes.

$$\mathbf{b.} \quad x - \sqrt{x} = 5; [4, 9]$$

$$f(x) = x - \sqrt{x}$$

$$f(4) = 2 < 5$$

$$f(9) = 6 > 5$$

Yes.

$$\mathbf{c.} \quad x = \cos x; [0, \pi]$$

$$f(x) = \cos x - x$$

$$f(0) = 1 > 0$$

$$f(\pi) = -1 - \pi < 0$$

Yes.

$$\mathbf{d.} \quad x = \tan x; \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

$$f(x) = \tan x$$

No.

Note that f is not continuous on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

53. Find $f(x)$ if $f''(x) = 4x + 3$, $f(1) = 2$, and $f'(1) = -3$.

$$f''(x) = 4x + 3$$

$$f'(x) = 2x^2 + 3x + C$$

$$f'(1) = -3$$

$$5 + C = -3$$

$$C = -8$$

$$f'(x) = 2x^2 + 3x - 8$$

$$f(x) = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 8x + C$$

$$f(1) = 2$$

$$\frac{2}{3} + \frac{3}{2} - 8 + C = 2$$

$$C = \frac{47}{6}$$

$$f(x) = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 8x + \frac{47}{6}$$

54. Find $f(x)$ if $f'(x) = x^2 + 3x + 2$ and $f(-3) = -\frac{3}{2}$.

$$f'(x) = x^2 + 3x + 2$$

$$f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + C$$

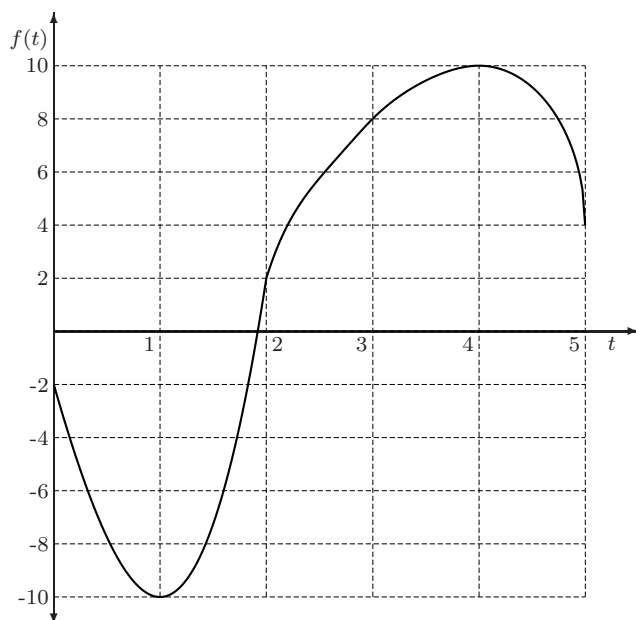
$$f(-3) = -\frac{3}{2}$$

$$-9 + \frac{27}{2} - 6 + C = -\frac{3}{2}$$

$$C = 0$$

$$f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x$$

For problems 55 and 56, let $f(t)$ be the function defined by the graph shown.



55. Estimate each of the following. Round numbers to the nearest integer.

a. The instantaneous rate of change of f at $t = 3$.

Note that the slope of the tangent line at the point $(3, 8)$ is approximately 4. So $f'(3) \approx 4$.

b. The average rate of change of f over the interval $[0, 4]$.

$$f(0) = -2$$

$$\frac{10 - (-2)}{4 - 0} = 3$$

$$f(4) = 10$$

c. The intervals where $f(t)$ is increasing and where it is decreasing.

Dec: $[0, 1]$, $[4, 5]$

Inc: $[1, 4]$

d. The inflection point or points of $f(t)$.

IP: $(2, 2)$

56. Find the intervals where $f'(t)$ is increasing and where it is decreasing. Round numbers to the nearest integer.

Recall that f' is increasing where f is concave upward and that f' is decreasing where f is concave downward.

The function f

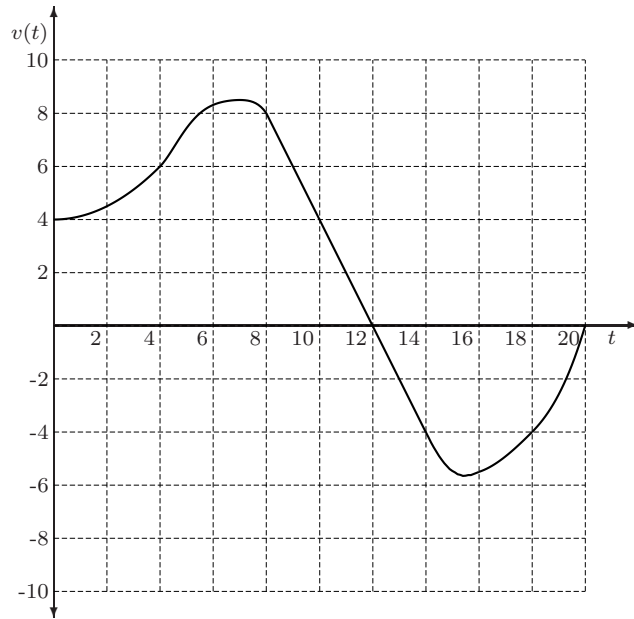
CU: $(0, 2)$

CD: $(2, 5)$

The function f'

Dec: $[2, 5]$ Inc: $[0, 2]$

57. Let $v(t)$, as shown in the graph below, be the velocity of a car in meters per second at time t in seconds, where positive velocity means the car is moving forward. Round your answers to the nearest integer.



a. When did the car stop?

$$v(12) = 0$$

12 seconds

b. Approximately how far did the car travel in the time interval 8 to 12 seconds?

$$\text{Let } s(t) \text{ be the position at time } t. \quad = \frac{1}{2} \cdot 4 \cdot 8 \quad (\text{area under the curve})$$

$$s(12) - s(8) \quad = 16$$

$$= \int_8^{12} v(t) dt \quad 16 \text{ meters}$$

c. Approximately how far did it travel in the time interval 12 to 14 seconds?

$$\text{Let } s(t) \text{ be the position at time } t. \quad = \frac{1}{2} \cdot 2 \cdot 4 \quad (\text{area above the curve})$$

$$s(12) - s(14) \quad = 4$$

$$= \int_{14}^{12} v(t) dt \quad 4 \text{ meters}$$

d. Approximately how far did it travel in the time interval 8 to 14 seconds?

From time $t = 8$ to time $t = 12$, the car travels forward 16 meters.

From time $t = 12$ to time $t = 14$, the car travels backward 4 meters.

The total distance that the car travels is 20 meters.

The displacement of the car is 12 meters.

e. At the time 2 seconds, is the car moving forward or backward? Is the driver's foot on the gas or the brake?

$$v(2) > 0$$

$$v'(2) > 0$$

Forward

Accelerator

f. At the time 16 seconds, is the car moving forward or backward? Is the driver's foot on the gas or the brake?

$$v(16) < 0$$

Accelerating

Backward

Brake

$$v'(16) > 0$$

The above answer may be counterintuitive. When you are driving a car in reverse, the velocity of the car is negative. When you apply the brake, the speed decreases but the velocity increases. This is the reason that the car is accelerating when your foot is on the brake.

58. Integrate.

a. $\int x(3x^2 + 4)^4 dx$

Let $u = 2x^4 + 4x$ and $du = (8x^3 + 4) dx$.

Let $u = 3x^2 + 4$ and $du = 6x dx$.

$$\int (2x^4 + 4x)^3 (2x^3 + 1) dx$$

$$\int x(3x^2 + 4)^4 dx$$

$$= \int \frac{1}{4} u^3 du$$

$$= \int \frac{1}{6} u^4 du$$

$$= \frac{1}{16} u^4 + C$$

$$= \frac{1}{30} u^5 + C$$

$$= \frac{1}{16} (2x^4 + 4x)^4 + C$$

$$= \frac{1}{30} (3x^2 + 4)^5 + C$$

c. $\int_1^2 \frac{2x^3 + 3x + 5}{x^5} dx$

b. $\int (2x^4 + 4x)^3 (2x^3 + 1) dx$

$$= \int_1^2 \left(\frac{2}{x^2} + \frac{3}{x^4} + \frac{5}{x^5} \right) dx$$

$$\begin{aligned}
&= \int_1^2 (2x^{-2} + 3x^{-4} + 5x^{-5}) dx \\
&= \left(-2x^{-1} - x^{-3} - \frac{5}{4}x^{-4} \right) \Big|_1^2 \\
&= \left(-\frac{2}{x} - \frac{1}{x^3} - \frac{5}{4x^4} \right) \Big|_1^2 \\
&= -1 - \frac{1}{8} - \frac{5}{64} - \left(-2 - 1 - \frac{5}{4} \right) \\
&= \frac{195}{64}
\end{aligned}$$

$$\begin{aligned}
\text{d. } &\int \frac{3x^2 + 6x + 2}{x^4} dx \\
&= \int \left(\frac{3}{x^2} + \frac{6}{x^3} + \frac{2}{x^4} \right) dx \\
&= \int (3x^{-2} + 6x^{-3} + 2x^{-4}) dx \\
&= -3x^{-1} - 3x^{-2} - \frac{2}{3}x^{-3} + C \\
&= -\frac{3}{x} - \frac{3}{x^2} - \frac{2}{3x^3} + C
\end{aligned}$$

$$\begin{aligned}
\text{e. } &\int_3^8 \sqrt{12-x} dx \\
&= \int_3^8 (12-x)^{\frac{1}{2}} dx \\
&= -\frac{2}{3}(12-x)^{\frac{3}{2}} \Big|_3^8 \\
&= -\frac{2}{3}\sqrt{(12-x)^3} \Big|_3^8 \\
&= -\frac{2}{3}(8-27) \\
&= \frac{38}{3}
\end{aligned}$$

$$\text{f. } \int (3x-1)\sqrt{3x^2-2x+1} dx$$

Let $u = 3x^2 - 2x + 1$ and $du = (6x - 2) dx$.

$$\begin{aligned}
&\int (3x-1)\sqrt{3x^2-2x+1} dx \\
&= \int \frac{1}{2}\sqrt{u} du \\
&= \int \frac{1}{2}u^{\frac{1}{2}} du \\
&= \frac{1}{3}u^{\frac{3}{2}} + C \\
&= \frac{1}{3}\sqrt{u^3} + C \\
&= \frac{1}{3}\sqrt{(3x^2-2x+1)^3} + C
\end{aligned}$$

$$\begin{aligned}
\text{g. } &\int \frac{1}{(x+5)^4} dx \\
&= \int (x+5)^{-4} dx \\
&= -\frac{1}{3}(x+5)^{-3} + C \\
&= \frac{-1}{3(x+5)^3} + C
\end{aligned}$$

$$\text{h. } \int \frac{5x + \frac{1}{2}}{(10x^2 + 2x + 40)^4} dx$$

Let $u = 10x^2 + 2x + 40$
and $du = (20x + 2) dx$.

$$\begin{aligned}
&\int \frac{5x + \frac{1}{2}}{(10x^2 + 2x + 40)^4} dx \\
&= \int \frac{\frac{1}{4}}{u^4} du \\
&= \int \frac{1}{4}u^{-4} du \\
&= -\frac{1}{12}u^{-3} + C \\
&= -\frac{1}{12u^3} + C
\end{aligned}$$

$$= \frac{-1}{12(10x^2 + 2x + 40)^3} + C$$

$$\text{i. } \int 3x^2 \sqrt{6x^3 + 1} dx$$

Let $u = 6x^3 + 1$ and $du = 18x^2 dx$.

$$\int 3x^2 \sqrt{6x^3 + 1} dx$$

$$= \int \frac{1}{6} \sqrt{u} du$$

$$= \int \frac{1}{6} u^{\frac{1}{2}} du$$

$$= \frac{1}{9} u^{\frac{3}{2}} + C$$

$$= \frac{1}{9} \sqrt{u^3} + C$$

$$= \frac{1}{9} \sqrt{(6x^3 + 1)^3} + C$$

$$\text{j. } \int x \sqrt{2 - x} dx$$

Let $u = 2 - x$ and $du = -dx$.

Note that $x = 2 - u$.

$$\int x \sqrt{2 - x} dx$$

$$= \int -(2 - u) \sqrt{u} du$$

$$= \int \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right) du$$

$$= \frac{2}{5} u^{\frac{5}{2}} - \frac{4}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{5} \sqrt{u^5} - \frac{4}{3} \sqrt{u^3} + C$$

$$= \frac{2}{5} \sqrt{(2 - x)^5} - \frac{4}{3} \sqrt{(2 - x)^3} + C$$

$$\text{k. } \int \frac{\sqrt[3]{3 + z^{-1}}}{z^2} dx$$

Let $u = 3 + z^{-1}$ and $du = -z^{-2} dx$.

$$\int \frac{\sqrt[3]{3 + z^{-1}}}{z^2} dx$$

$$= \int -\sqrt[3]{u} du$$

$$= \int -u^{\frac{1}{3}} du$$

$$= -\frac{3}{4} u^{\frac{4}{3}} + C$$

$$= -\frac{3}{4} \sqrt[3]{u^4} + C$$

$$= -\frac{3}{4} \sqrt[3]{(3 + z^{-1})^4} + C$$

$$\text{l. } \int \sec x \tan x \cos(\sec x) dx$$

Let $u = \sec x$ and $du = \sec x \tan x dx$.

$$\int \sec x \tan x \cos(\sec x) dx$$

$$= \int \cos u du$$

$$= \sin u + C$$

$$= \sin(\sec x) + C$$

$$\text{m. } \int \sin \frac{x}{3} \left(\cos \frac{x}{3} \right)^3 dx$$

Let $u = \cos \frac{x}{3}$ and $du = -\frac{1}{3} \sin \frac{x}{3} dx$.

$$\int \sin \frac{x}{3} \left(\cos \frac{x}{3} \right)^3 dx$$

$$= \int -3u^3 du$$

$$= -\frac{3}{4} u^4 + C$$

$$= -\frac{3}{4} \left(\cos \frac{x}{3} \right)^4 + C$$

$$\text{n. } \int \tan x (\sec x)^2 dx$$

Let $u = \tan x$ and $du = \sec^2 x dx$.

$$\int \tan x (\sec x)^2 dx$$

$$\begin{aligned}
 &= \int u \, du & &= \frac{1}{2}u^2 + C \\
 & & &= \frac{1}{2}\tan^2 x + C
 \end{aligned}$$

59. Differentiate.

Use the Fundamental Theorem of Calculus Part I.

$$\begin{aligned}
 \text{a. } f(x) &= \int_3^x (t^2 - 8) \, dt & \text{b. } g(x) &= \int_4^{x^2} (t^2 + 6)^{\frac{5}{2}} \, dt \\
 f'(x) &= x^2 - 8 & g'(x) &= (x^4 + 6)^{\frac{5}{2}} (2x)
 \end{aligned}$$

60. Find the area of the region bounded by $y = x^2$ and $y = x^4$.

$$\begin{aligned}
 x^4 &= x^2 & x^2(x-1)(x+1) &= 0 & &= \left(\frac{1}{3}x^3 - \frac{1}{5}x^5\right)\Big|_{-1}^1 \\
 x^4 - x^2 &= 0 & x &= 0, x = -1, x = 1 & &= \frac{1}{3} - \frac{1}{5} - \left(-\frac{1}{3} + \frac{1}{5}\right) \\
 x^2(x^2 - 1) &= 0 & \int_{-1}^1 (x^2 - x^4) \, dx & & &= \frac{4}{15}
 \end{aligned}$$

61. Find the area of the region bounded by $y = x^2$ and $y = 2 - x$.

$$\begin{aligned}
 x^2 &= 2 - x & x &= -2, x = 1 & &= \left(2x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_{-2}^1 \\
 x^2 + x - 2 &= 0 & \int_{-2}^1 (2 - x - x^2) \, dx & & &= 2 - \frac{1}{2} - \frac{1}{3} - \left(-4 - 2 + \frac{8}{3}\right) \\
 (x+2)(x-1) &= 0 & & & &= \frac{9}{2}
 \end{aligned}$$

62. Find the area of the region bounded by $y = x^4$ and $y = 8x$.

$$\begin{aligned}
 x^4 &= 8x & x &= 0, x = 2 & &= \left(4x^2 - \frac{1}{5}x^5\right)\Big|_0^2 \\
 x^4 - 8x &= 0 & \int_0^2 (8x - x^4) \, dx & & &= 16 - \frac{32}{5} \\
 x(x^3 - 8) &= 0 & & & &= \frac{48}{5}
 \end{aligned}$$

63. Find the volume of the solid generated by revolving the region bounded by $y = x^{\frac{3}{2}}$, the x -axis, and the line $x = 4$ about the x -axis.

Disk Method

$$\int_0^4 \pi \left(x^{\frac{3}{2}}\right)^2 \, dx = \int_0^4 \pi x^3 \, dx$$

$$= \frac{\pi}{4} x^4 \Big|_0^4$$

$$= 64\pi$$

Shell Method

$$y = x^{\frac{3}{2}}$$

$$x = y^{\frac{2}{3}}$$

$$\int_0^8 2\pi y \left(4 - y^{\frac{2}{3}}\right) dy$$

$$= 2\pi \int_0^8 \left(4y - y^{\frac{5}{3}}\right) dy$$

$$= 2\pi \left(2y^2 - \frac{3}{8}y^{\frac{8}{3}}\right) \Big|_0^8$$

$$= 2\pi (128 - 96)$$

$$= 64\pi$$

64. Find the volume of the solid generated by revolving the region bounded by $y^2 = 4x$ and $x^2 = 4y$ about the x -axis.

First, find the points of intersection.

$$x^2 = 4y$$

$$y = \frac{1}{4}x^2$$

$$y^2 = 4x$$

$$x = \frac{1}{4}y^2$$

$$x = \frac{1}{4} \left(\frac{1}{4}x^2\right)^2$$

$$x = \frac{1}{64}x^4$$

$$64x = x^4$$

$$x^4 - 64x = 0$$

$$x(x^3 - 64) = 0$$

$$x = 0, x = 4$$

Intersection Points:
(0, 0), (4, 4)

Find the volume using either the washer method or the shell method.

Washer Method

$$x^2 = 4y$$

$$y = \frac{1}{4}x^2$$

$$y^2 = 4x$$

$$y = 2\sqrt{x}$$

$$\int_0^4 \pi \left[(2\sqrt{x})^2 - \left(\frac{1}{4}x^2\right)^2 \right] dx$$

$$= \pi \int_0^4 \left(4x - \frac{1}{16}x^4\right) dx$$

$$= \pi \left(2x^2 - \frac{1}{80}x^5\right) \Big|_0^4$$

$$= \pi \left(32 - \frac{1024}{80}\right)$$

$$= \frac{96\pi}{5}$$

Shell Method

$$x^2 = 4y$$

$$x = 2\sqrt{y}$$

$$y^2 = 4x$$

$$x = \frac{1}{4}y^2$$

$$\int_0^4 2\pi y \left(2\sqrt{y} - \frac{1}{4}y^2\right) dy$$

$$= 2\pi \int_0^4 \left(2y^{\frac{3}{2}} - \frac{1}{4}y^3\right) dy$$

$$= 2\pi \left(\frac{4}{5}y^{\frac{5}{2}} - \frac{1}{16}y^4\right) \Big|_0^4$$

$$= 2\pi \left(\frac{128}{5} - 16\right)$$

$$= \frac{96\pi}{5}$$

65. Find the volume of the solid generated by revolving the region in the first quadrant bounded by $y = x^2$, $y = 4$, and the y -axis about each of the following.

a. the y -axis

$$x = \sqrt{y}$$

$$= \int_0^4 \pi y dy$$

Disk Method

$$y = x^2$$

$$\int_0^4 \pi (\sqrt{y})^2 dy$$

$$= \pi \frac{1}{2} y^2 \Big|_0^4$$

$$= 8\pi$$

Shell Method

$$y = x^2$$

$$\int_0^2 2\pi x(4 - x^2) dx$$

$$= \pi \int_0^2 (-2x^3 + 8x) dx$$

$$= \pi \left(-\frac{1}{2}x^4 + 4x^2 \right) \Big|_0^2$$

$$= \pi(-8 + 16)$$

$$= 8\pi$$

$$\mathbf{b.} \quad y = 4$$

Disk Method

$$\int_0^2 \pi (4 - x^2)^2 dx$$

$$= \pi \int_0^2 (x^4 - 8x^2 + 16) dx$$

$$= \pi \left(\frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x \right) \Big|_0^2$$

$$= \pi \left(\frac{32}{5} - \frac{64}{3} + 32 \right)$$

$$= \frac{256\pi}{15}$$

Shell Method

$$y = x^2$$

$$x = \sqrt{y}$$

$$\int_0^4 2\pi(4 - y)\sqrt{y} dy$$

$$= 2\pi \int_0^4 \left(4y^{\frac{1}{2}} - y^{\frac{3}{2}} \right) dy$$

$$= 2\pi \left(\frac{8}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}} \right) \Big|_0^4$$

$$= 2\pi \left(\frac{64}{3} - \frac{64}{5} \right)$$

$$= \frac{256\pi}{15}$$

66. Find the volume of the solid generated by revolving the region bounded by $y = x + \frac{4}{x}$, the x -axis, and the lines $x = 1$ and $x = 3$ about the y -axis.

Shell Method

$$\int_1^3 2\pi x \left(x + \frac{4}{x} \right) dx$$

$$= 2\pi \int_1^3 (x^2 + 4) dx$$

$$= 2\pi \left(\frac{1}{3}x^3 + 4x \right) \Big|_1^3$$

$$= 2\pi \left[9 + 12 - \left(\frac{1}{3} + 4 \right) \right]$$

$$= 2\pi \left(21 - \frac{13}{3} \right)$$

$$= \frac{100\pi}{3}$$

67. A solid has as its base the region in the first quadrant bounded by $x^2 + y^2 = 25$. Every plane section of the solid taken perpendicular to the x -axis is a square. Find the volume of the solid.

$$x^2 + y^2 = 25$$

$$y = \sqrt{25 - x^2}$$

$$A(x) = \left(\sqrt{25 - x^2} \right)^2$$

$$A(x) = 25 - x^2$$

$$\int_0^5 (-x^2 + 25) dx$$

$$= \left(-\frac{1}{3}x^3 + 25x \right) \Big|_0^5$$

$$= -\frac{125}{3} + 125$$

$$= \frac{250}{3}$$

68. A solid has as its base the region in the xy -plane bounded by the graphs of $y = x$ and $y^2 = x$. Find the volume of the solid if every cross section by a plane perpendicular to the x -axis is a semicircle with diameter in the xy -plane.

$$y^2 = x$$

$$y = \sqrt{x}$$

$$r = \frac{1}{2}(\sqrt{x} - x)$$

$$\begin{aligned}
A(x) &= \frac{1}{2}\pi \left(\frac{1}{2}(\sqrt{x} - x)\right)^2 & A(x) &= \frac{\pi}{8} \left(x^2 - 2x^{\frac{3}{2}} + x\right) & &= \frac{\pi}{8} \left(\frac{1}{3}x^3 - \frac{4}{5}x^{\frac{5}{2}} + \frac{1}{2}x^2\right) \Big|_0^1 \\
&\text{(semicircle)} & & & & \\
& & \int_0^1 \frac{\pi}{8} \left(x^2 - 2x^{\frac{3}{2}} + x\right) dx & &= \frac{\pi}{8} \left(\frac{1}{3} - \frac{4}{5} + \frac{1}{2}\right) \\
& & & &= \frac{\pi}{240}
\end{aligned}$$

69. Find the average value of the function $f(x) = x^2 + x + 1$ on the interval $[-1, 2]$.

$$\frac{1}{3} \int_{-1}^2 (x^2 + x + 1) dx = \frac{1}{3} \left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x \right) \Big|_{-1}^2 = \frac{1}{3} \left[\frac{8}{3} + 2 + 2 - \left(-\frac{1}{3} + \frac{1}{2} - 1 \right) \right] = \frac{1}{3} \left(\frac{20}{3} - \frac{5}{6} \right) = \frac{5}{2}$$

70. Find the average value of the function $f(x) = \sin x$ on the interval $[0, \pi]$.

$$\frac{1}{\pi} \int_0^{\pi} \sin x dx = \frac{1}{\pi} (-\cos x) \Big|_0^{\pi} = \frac{1}{\pi} (1 - (-1)) = \frac{2}{\pi}$$

71. Assume that the density of water is 62.5 lb/ft³. A cylindrical water tank with a circular base has radius 3 feet and height 10 feet. Find the work required to empty the tank by pumping the water out of the top for each of the following situations.

a. The tank is full.

28125 π foot-pounds

Partition the tank into n equal slices.

Alternatively,

The height of each slice is $\frac{10}{n}$ ft.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{5625\pi i}{n} \cdot \frac{10}{n}$$

The volume of each slice is $\frac{90\pi}{n}$ ft³.

The weight of each slice is $\frac{5625\pi}{n}$ lb.

$$= \lim_{n \rightarrow \infty} \frac{56250\pi}{n^2} \sum_{i=1}^n i$$

The work required to “lift” out the i^{th} slice is $\frac{5625\pi i}{n} \cdot \frac{10}{n}$ foot-pounds.

$$= \lim_{n \rightarrow \infty} \left(\frac{56250\pi}{n^2} \cdot \frac{n(n+1)}{2} \right)$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{5625\pi i}{n} \cdot \frac{10}{n}$$

$$= 28125\pi$$

28125 π foot-pounds

$$= 562.5\pi \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{10i}{n} \cdot \frac{10}{n}$$

b. The tank is half full.

Partition half of the tank into n equal slices.

$$= 562.5\pi \int_0^{10} x dx$$

The height of each slice is $\frac{5}{n}$ ft.

$$= 562.5\pi \left(\frac{1}{2}x^2 \right) \Big|_0^{10}$$

The volume of each slice is $\frac{45\pi}{n}$ ft³.

$$= 28125\pi$$

The weight of each slice is $\frac{2812.5\pi}{n}$ lb.

The work required to “lift” out the i^{th} slice is $\frac{2812.5\pi}{n} \left(\frac{5i}{n} + 5\right)$ foot-pounds.

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2812.5\pi}{n} \left(\frac{5i}{n} + 5\right) \\ &= 562.5\pi \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{5}{n} \left(\frac{5i}{n} + 5\right) \\ &= 562.5\pi \int_0^5 (x + 5) dx \\ &= 562.5\pi \left(\frac{1}{2}x^2 + 5x\right) \Big|_0^5 \\ &= 21093.75\pi \end{aligned}$$

21093.75 π foot-pounds

Alternatively,

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2812.5\pi}{n} \left(\frac{5i}{n} + 5\right) \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{14062.5\pi i}{n^2} + \frac{14062.5\pi}{n}\right) \\ &= \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \frac{14062.5\pi i}{n^2} + \sum_{i=1}^n \frac{14062.5\pi}{n}\right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{14062.5\pi}{n^2} \sum_{i=1}^n i + \sum_{i=1}^n \frac{14062.5\pi}{n}\right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{14062.5\pi}{n^2} \cdot \frac{n(n+1)}{2} + 14062.5\pi\right) \\ &= 7031.25\pi + 14062.5\pi \\ &= 21093.75\pi \end{aligned}$$

21093.75 π foot-pounds

72. A bucket with 24 lb of water is raised 30 feet from the bottom of a well. Find the work done in each of the following cases.

a. The weight of the empty bucket is 4 lb and the weight of the rope is negligible.

$$30(24 + 4) = 840$$

840 foot-pounds

b. The bucket weighs 4 lb and the rope weighs 4 oz/ft.

$$\begin{aligned} f(x) &= \frac{1}{4}(30 - x) + 24 + 4 &= \left(-\frac{1}{8}x^2 + \frac{71}{2}x\right) \Big|_0^{30} &= 952.5 \\ \int_0^{30} \left(-\frac{1}{4}x + \frac{71}{2}\right) dx & &= -\frac{900}{8} + \frac{2130}{2} &952.5 \text{ foot-pounds} \end{aligned}$$

c. The bucket weighs 4 lb, the rope weighs 4 oz/ft, and water is leaking out of the bucket at a constant rate so that only 18 lb of water remain in the bucket when it reaches the top.

$$\begin{aligned} f(x) &= \frac{1}{4}(30 - x) + (24 - \frac{1}{5}x) + 4 &= \left(-\frac{9}{40}x^2 + \frac{71}{2}x\right) \Big|_0^{30} &= 862.5 \\ \int_0^{30} \left(-\frac{9}{20}x + \frac{71}{2}\right) dx & &= -\frac{8100}{40} + \frac{2130}{2} &862.5 \text{ foot-pounds} \end{aligned}$$

73. Match each numbered item with a lettered item. (There are more lettered items than numbered items. Some lettered items do not match any numbered item.)

- | | |
|---|---|
| <p>1. Definition of $\lim_{x \rightarrow a} f(x) = L$.</p> <p>2. Definition of $\lim_{x \rightarrow a^+} f(x) = L$.</p> <p>3. Definition of $\lim_{x \rightarrow \infty} f(x) = L$.</p> <p>4. Definition of “f is continuous at a”.</p> <p>5. The Intermediate Value Theorem.</p> <p>6. Definition of the derivative of f at a.</p> <p>7. Definition of a function f being differentiable at a.</p> <p>8. Theorem relating differentiability and continuity.</p> <p>9. The power rule for differentiation.</p> <p>10. Definition of the differential.</p> <p>11. Definition of a function f having an absolute maximum at c.</p> <p>12. Definition of a function f having a local maximum at c.</p> <p>13. The Extreme Value Theorem.</p> <p>14. Definition of a function that is increasing on an interval I.</p> <p>15. The Mean Value Theorem.</p> <p>16. Definition of an antiderivative of f on an interval I.</p> | <p>a. $\lim_{x \rightarrow a} f(x) = f(a)$.</p> <p>b. For every $\varepsilon > 0$ there is a corresponding number N such that $f(x) - L < \varepsilon$ whenever $x > N$.</p> <p>c. If f is continuous on the closed interval $[a, b]$, and N is a number strictly between $f(a)$ and $f(b)$, then there exists a number c in (a, b) such that $f(c) = N$.</p> <p>d. The limit $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ exists.</p> <p>e. If f is differentiable at a, then f is continuous at a.</p> <p>f. If f is continuous at a, then f is differentiable at a.</p> <p>g. $\frac{d}{dx} x^n = nx^{n-1}$.</p> <p>h. The function g has the property $g'(x) = f(x)$ for all x in I.</p> <p>i. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$</p> <p>j. $f(c) \geq f(x)$ for all x in the domain of f.</p> <p>k. For every $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $f(x) - L < \varepsilon$ whenever $a < x < a + \delta$.</p> <p>l. For every $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $f(x) - L < \varepsilon$ whenever $0 < x - a < \delta$.</p> <p>m. There is an open interval I containing c such that $f(c) \geq f(x)$ for all x in I.</p> <p>n. $f'(x) > 0$ for all x in I.</p> <p>o. If f is continuous on $[a, b]$, then there are numbers c and d in $[a, b]$ such that $f(c)$ is an absolute maximum for f in $[a, b]$ and $f(d)$ is an absolute minimum for f in $[a, b]$.</p> <p>p. If f is differentiable, $dy = f'(x)dx$.</p> <p>q. $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ and x_1 and x_2 are in I.</p> <p>r. If f is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.</p> |
|---|---|

- | | | | | | | | |
|------|-------|-------|-------|-------|-------|-------|-------|
| 1. l | 2. k | 3. b | 4. a | 5. c | 6. i | 7. d | 8. e |
| 9. g | 10. p | 11. j | 12. m | 13. o | 14. q | 15. r | 16. h |