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Chapter 1: Math 1571 Spring 2003

Section 1.1: Quiz 8

Quiz 1
01–14–03

(1) 1. $\tan\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

(1) 2. $\displaystyle\lim_{x\to 3}\frac{x^2-2x-3}{x-3} = \lim_{x\to 3}\frac{(x-3)(x+1)}{x-3} = \lim_{x\to 3}(x+1) = 4$

(1) 3.

(1) 4.

Quiz 2
01–17–03

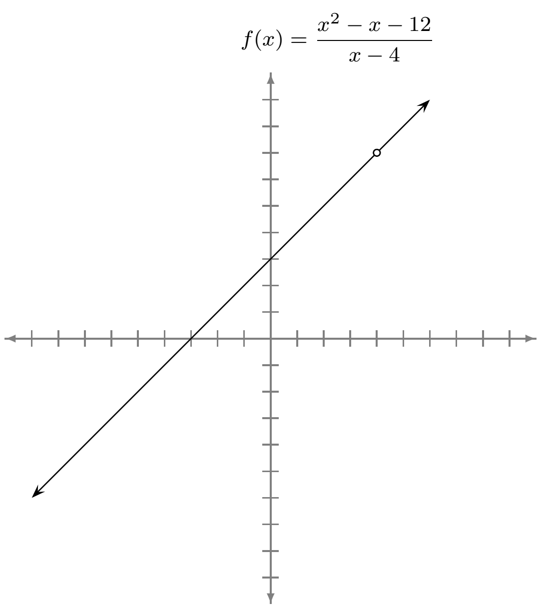
5. For each of the following, give the domain and range. Use your answers to sketch the graph.

(1) a. $f(x) = \dfrac{x^2-x-12}{x-4}$

D: $(-\infty,4)\cup(4,\infty)$

$$f(x) = \frac{x^2-x-12}{x-4} = \frac{(x-4)(x+3)}{x-4}$$

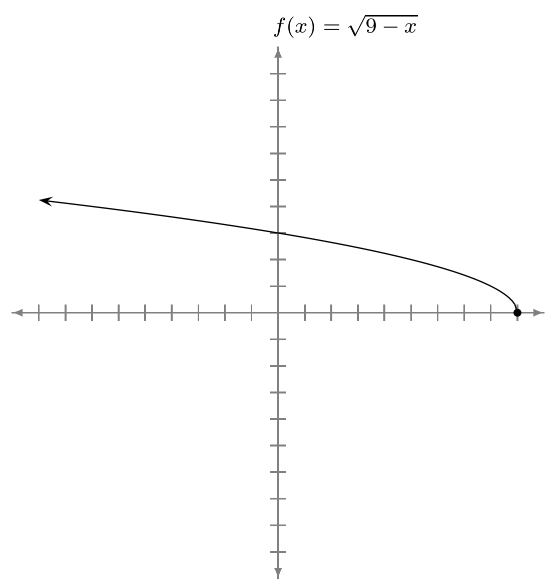
R: $(-\infty,7)\cup(7,\infty)$



(1) b. $f(x) = \sqrt{9-x}$

D: $(-\infty,9]$

R: $[0,\infty)$



Quiz 3

01–22–03

(2) 6. Prove that the product of an even function and an odd function is an odd function.

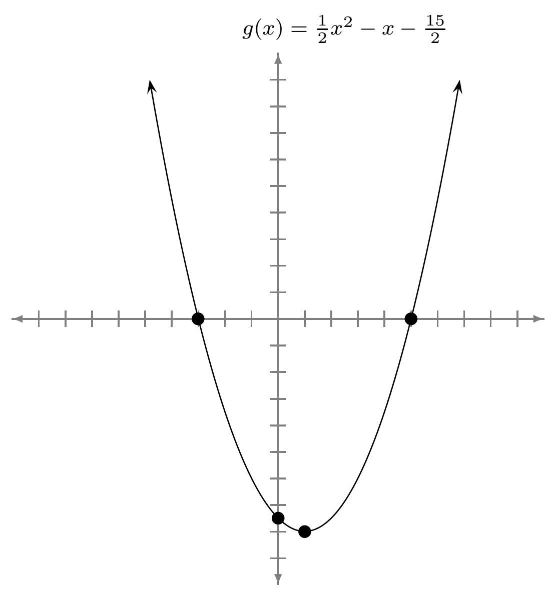
***Proof*:**

Let f be an even function and g be an odd function. Then $(f \cdot g)(\text{-}x) = f(\text{-}x) \cdot g(\text{-}x) = f(x) \cdot \text{-}g(x) = \text{-}f(x) \cdot g(x)$ $= \text{-}(f \cdot g)(x)$. ■

Quiz 4
01–24–03

For each of the following, find the vertex, y-intercepts, and x-intercept(s) (if any). Use your answers to sketch the graph.

(2) 7. $g(x) = \frac{1}{2}x^2 - x - \frac{15}{2}$



Vertex: (1,-8)

y-intercept: $\left(0,\text{-}\frac{15}{2}\right)$

$\frac{1}{2}x^2 - x - \frac{15}{2} = 0$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

$x =$ -3,5

x-intercepts: (-3,0) and (5,0)

Quiz 5
01–29–03

(2) 8. Give the domain and range of the function $h(x) = (\lfloor x \rfloor)^2$.

Domain: $(\text{-}\infty,\infty)$

Range: $\{0, 1, 4, 9, 16, 25, \ldots\}$

Quiz 6
01–31–03

9. (1) a. Simplify completely.

$(\sqrt{x+1}-3)(\sqrt{x+1}+3)$

$(\sqrt{x+1}-3)(\sqrt{x+1}+3) = x-8$

(3) b. Use the $\varepsilon-\delta$ definition of limit to prove that $\lim_{x\to 8}\sqrt{x+1}=3$. Hint: Choose $\delta=\varepsilon$ and use your previous answer.

***Proof*:** Let $\varepsilon>0$ be given. Choose $\delta=\varepsilon$. If $|x-8|<\delta$, then

$|x-8|<\varepsilon$

$|(\sqrt{x+1}-3)(\sqrt{x+1}+3)|<\varepsilon$

$|(\sqrt{x+1}-3)||(\sqrt{x+1}+3)|<\varepsilon$

$3|(\sqrt{x+1}-3)|<\varepsilon$

$|(\sqrt{x+1}-3)|<\varepsilon.$ ■

Quiz 7
02–12–03

(2) 10. Is the following function continuous at 1?

$$f(x)=\begin{cases}\dfrac{x^2-1}{x-1} & \text{if } x<1\\[2ex] x-1 & \text{if } x\geq 1\end{cases}$$

$f(1)=0$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\frac{x^2-1}{x-1}=\lim_{x\to 1^-}\frac{(x+1)(x-1)}{x-1}=\lim_{x\to 1^-}(x+1)=2$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(x-1)=0$$

Since $\lim_{x\to 1^-} f(x)\neq\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1} f(x)$ does not exist. Therefore, f is not continuous at 1.

Quiz 8
02–14–03

(2) 11. Does the equation $x^{14}+x^2-4x+1=0$ have a solution between 0 and 1?

Yes.

Proof: Let $f(x) = x^{14} + x^2 - 4x + 1 = 0$ and note that f is continuous on [0,1] since f is a polynomial. Since $f(0) = 1$ and $f(1) =$ -1, the intermediate value theorem implies that there is a number $c \in$ [0,1] such that $f(c) = 0$. ■

Quiz 9
02–18–03

(2) 12. Does the equation $x^{14} + x^2 + 4x + 1 = 0$ have a solution between 0 and 1?

No.

Note that for $x \geq 0$, $x^{14} + x^2 + 4x + 1 \geq 1$.

Quiz 10
02–19–03

(4) 13. Find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

$f(x) = \dfrac{x^2 + x + 1}{x}$

HA: none

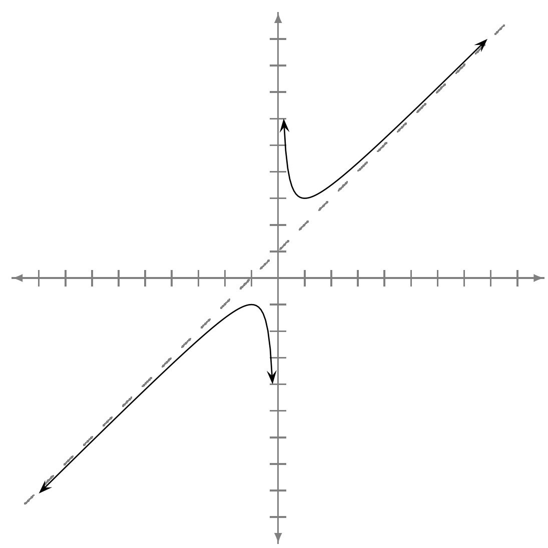
$\lim\limits_{x \to 0^-} \dfrac{x^2 + x + 1}{x} = -\infty$

$\lim\limits_{x \to 0^+} \dfrac{x^2 + x + 1}{x} = \infty$

VA: $x = 0$

$f(x) = \dfrac{x^2 + x + 1}{x} = x + 1 + \frac{1}{x}$

OA: $y = x + 1$



Quiz 11
02–25–03

(2) 14. Differentiate the following.

$f(x) = x^3 + x$

$\lim\limits_{h \to 0} \left[f(x+h) - f(x) \right] \frac{1}{h}$

$$= \lim_{h\to 0}\left[(x+h)^3 + (x+h) - (x^3+x)\right]\tfrac{1}{h}$$

$$= \lim_{h\to 0}\left[x^3 + 3x^2h + 3xh^2 + h^3 + x + h - x^3 - x\right]\tfrac{1}{h}$$

$$= \lim_{h\to 0}\left[3x^2h + 3xh^2 + h^3 + h\right]\tfrac{1}{h}$$

$$= \lim_{h\to 0}\left[3x^2 + 3xh + h^2 + 1\right]$$

$$= 3x^2 + 1$$

Quiz 12
02–26–03

Differentiate the following.

(2) 15. $f(x) = \sqrt{x+5}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{x+h+5}-\sqrt{x+5}}{h}$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{x+h+5}-\sqrt{x+5}}{h}\cdot\frac{\sqrt{x+h+5}+\sqrt{x+5}}{\sqrt{x+h+5}+\sqrt{x+5}}\right)$$

$$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h+5}+\sqrt{x+5})}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h+5}+\sqrt{x+5}}$$

$$= \frac{1}{2\sqrt{x+5}}$$

Quiz 13
02–28–03

(2) 16. Compute the derivative of the given function and find the equation of the line that is tangent to its graph at the specified point.

$g(x) = 5\sqrt{x}$; $x = 25$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5\sqrt{x+h} - 5\sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{5\sqrt{x+h} - 5\sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{5(x+h) - 5x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{5x + 5h - 5x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{5h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{5}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{5}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{5}{2\sqrt{x}}$$

$g(25) = 25$

$$g'(25) = \frac{5}{2\sqrt{25}} = \frac{1}{2}$$

$y - 25 = \frac{1}{2}(x - 25)$

Quiz 14
03–18–03

17. For a six second period of time, a particle moves according to a law of motion $s = \sin t + \cos t$ where t is measured in seconds and s in meters.

(1) a. Find the velocity at time t.

(1) b. When is the particle at rest?

(1) c. When is the particle moving in a positive direction? When is the particle moving in a negative direction?

(1) d. Find the total distance traveled during the six second time interval.

See exam 2.

Quiz 15
03–21–03

(2) 18. Find the equation of the line that is tangent to the given curve at the specified point.

$x^2y^3 + y + \cos x = 2$; (0,1)

$x^2y^3 + y + \cos x = 2$;

$2xy^3 + 3x^2y^2y' + y' - \sin x = 0$

$$y' = \frac{\text{-}2xy^3 + \sin x}{3x^2y^2 + 1}$$

$x = 0$

$y = 1$

$y' = 0$

$y - 1 = 0(x - 0)$

$y = 1$

Quiz 16
03–25–03

(4) 19. Find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, and local extrema. Use these answers to sketch the graph.

$$f(x) = \frac{2x^2 + 5x - 3}{x^2 + 4x + 3}$$

HA: $y = 2$

$$f(x) = \frac{(2x-1)(x+3)}{(x+3)(x+1)}$$

$$\lim_{x \to \text{-}3} \frac{(2x-1)(x+3)}{(x+3)(x+1)} = \lim_{x \to \text{-}3} \frac{2x-1}{x+1} = \frac{7}{2}$$

VA: $x = \text{-}1$

OA: none

If $x \neq \text{-}3$, $f(x) = \dfrac{2x-1}{x+1}$.

$$f'(x) = \frac{2(x+1) - (2x-1)}{(x+1)^2} = \frac{3}{(x+1)^2}$$

Dec: never

Inc: $(\text{-}\infty,\text{-}3) \cup (\text{-}3,\text{-}1) \cup (\text{-}1,\text{-}\infty)$

Local max: none

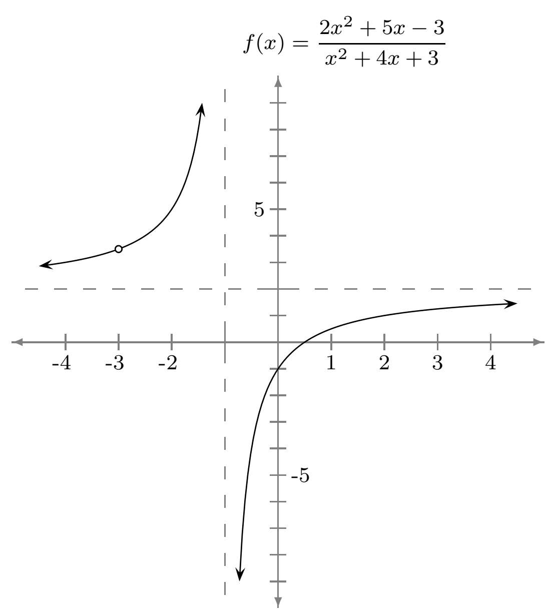
Local min: none

$$f''(x) = \frac{\text{-}6}{(x+1)^3}$$

CD: $(\text{-}1,\text{-}\infty)$

CU: $(\text{-}\infty,\text{-}3) \cup (\text{-}3,\text{-}1)$

IP: none



Quiz 17
03–28–03

(4) 20. Find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

$$g(x) = \frac{x^2 - 7x + 20}{x - 3}$$

$$\lim_{x\to\infty} \frac{x^2 - 7x + 20}{x - 3} = \infty$$

$$\lim_{x\to-\infty} \frac{x^2 - 7x + 20}{x - 3} = -\infty$$

HA: none

$$\lim_{x\to 3^-} \frac{x^2 - 7x + 20}{x - 3} = -\infty$$

$$\lim_{x\to 3^+} \frac{x^2 - 7x + 20}{x - 3} = \infty$$

VA: $x = 3$

$$g(x) = \frac{x^2 - 7x + 20}{x - 3} = x - 4 + \frac{8}{x - 3}$$

OA: $y = x - 4$

$$g(x) = \frac{x^2 - 7x + 20}{x - 3}$$

$$= x - 4 + \frac{8}{x - 3}$$

$$= x - 4 + 8(x - 3)^{-1}$$

$$g'(x) = 1 - 8(x - 3)^{-2}$$

$$= 1 - \frac{8}{(x - 3)^2}$$

$$= \frac{(x - 3)^2 - 8}{(x - 3)^2}$$

$$= \frac{x^2 - 6x + 1}{(x - 3)^2}$$

Inc: $(-\infty, 3 - 2\sqrt{2}] \cup [3 + 2\sqrt{2}, \infty)$

Dec: $[3 - 2\sqrt{2}, 3 + 2\sqrt{2}]$

Local max: $-1 - 4\sqrt{2}$ at $3 - 2\sqrt{2}$

Local min: $-1 + 4\sqrt{2}$ at $3 + 2\sqrt{2}$

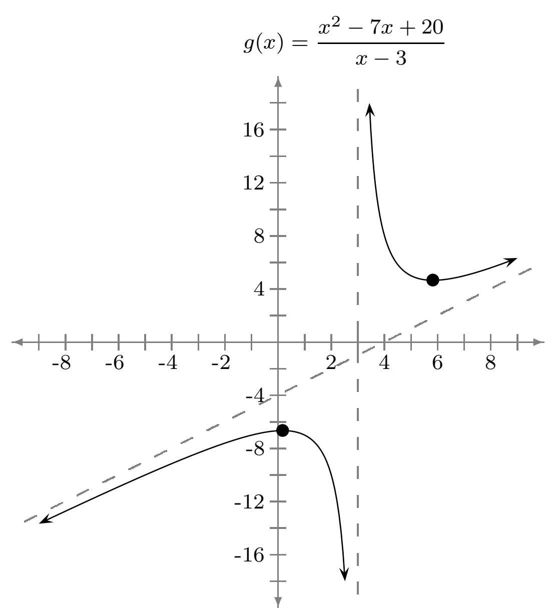
$$g'(x) = 1 - 8(x - 3)^{-2}$$

$$g''(x) = 16(x - 3)^{-3} = \frac{16}{(x - 3)^3}$$

CU: $(3, \infty)$

CD: $(-\infty, 3)$

IP: none



Quiz 18
04–01–03

21. A ball is thrown upward with an initial velocity of of 48 ft/sec from a height of 50 ft.

(1) a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 48t + 50$

(2) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 48$

$\text{-}32t + 48 = 0$

$\text{-}16(2t - 3) = 0$

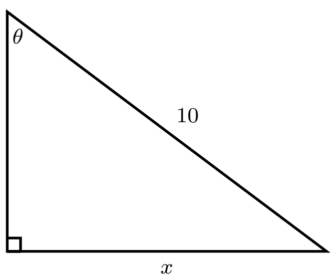
$t = \frac{3}{2}$

$h\left(\frac{3}{2}\right) = \text{-}12 + 72 + 50 = 86$

86 feet

Quiz 19
04–04–03

(3) 22. A 10-foot ladder is being pulled away from a wall in such a way that the top of the ladder is sliding down the wall at a rate of 4 ft/sec. At what rate is the angle between the ladder and the wall changing when the top of the ladder is 6 feet above the floor?



$\cos\theta = \frac{y}{10}$

$\text{-}\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$

$\text{-}\frac{x}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$

$\text{-}\frac{8}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10}(\text{-}4)$

$\frac{d\theta}{dt} = \frac{1}{2}$

$\frac{1}{2}$ radians per second

Quiz 20
04–16–03

(1) 23. $\sum\limits_{n=1}^{5} n^3 = \left(\frac{5\cdot 6}{2}\right)^2 = 225$

(1) 24. Given that $\sum\limits_{n=1}^{10} a_i = 8$ and $\sum\limits_{n=1}^{10} b_i = 10$, calculate $\sum\limits_{n=1}^{10} (2a_i + 7b_i)$.

$$\sum_{n=1}^{10} (2a_i + 7b_i) = \sum_{n=1}^{10} 2a_i + \sum_{n=1}^{10} 7b_i = 2\sum_{n=1}^{10} a_i + 7\sum_{n=1}^{10} b_i = 2\cdot 8 + 7\cdot 10 = 86$$

Quiz 21
04–18–03

(3) 25. Integrate.

$$\int_{-1}^{3}\left(2x^2+x+1\right)dx$$

$$\int_{-1}^{3}\left(2x^2+x+1\right)dx$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\tfrac{4i}{n}\right)\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[2\left(-1+\tfrac{4i}{n}\right)^2+\left(-1+\tfrac{4i}{n}\right)+1\right]\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[2\left(\tfrac{16i^2}{n^2}-\tfrac{8i}{n}+1\right)+\left(-1+\tfrac{4i}{n}\right)+1\right]\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\tfrac{32i^2}{n^2}-\tfrac{16i}{n}+2-1+\tfrac{4i}{n}+1\right)\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\tfrac{32i^2}{n^2}-\tfrac{12i}{n}+2\right)\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\tfrac{128i^2}{n^3}-\tfrac{48i}{n^2}+\tfrac{8}{n}\right)$$

$$= \lim_{n\to\infty}\left(\sum_{i=1}^{n}\tfrac{128i^2}{n^3}-\sum_{i=1}^{n}\tfrac{48i}{n^2}+\sum_{i=1}^{n}\tfrac{8}{n}\right)$$

$$= \lim_{n\to\infty}\left(\tfrac{128}{n^3}\sum_{i=1}^{n}i^2-\tfrac{48}{n^2}\sum_{i=1}^{n}i+\tfrac{8}{n}\sum_{i=1}^{n}1\right)$$

$$= \lim_{n\to\infty}\left(\tfrac{128}{n^3}\cdot\tfrac{n(n+1)(2n+1)}{6}-\tfrac{48}{n^2}\cdot\tfrac{n(n+1)}{2}+\tfrac{8}{n}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\tfrac{128}{n^3}\cdot\tfrac{2n^3+3n^2+n}{6}-\tfrac{48}{n^2}\cdot\tfrac{(n^2+n)}{2}+\tfrac{8}{n}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\tfrac{256n^3+384n^2+128n}{6n^3}-\tfrac{48n^2+48n}{2n^2}+8\right)$$

$$= \tfrac{256}{6}-\tfrac{48}{2}+8$$

$$= 26\tfrac{2}{3}$$

Quiz 22
04–22-03

(3) 26. Find the area of the region enclosed by the curves $y = x^2-1$, $y = 0$, $x = 0$, and $x = 2$.

Note that $x^2-1 \le 0$ for $0 \le x \le 1$ and $x^2-1 \ge 0$ for $1 \le x \le 2$. So the area of the region is

$$-\int_0^1 (x^2-1)\,dx + \int_1^2 (x^2-1)\,dx = -\left(\tfrac{1}{3}x^3 - x\right)\Big|_0^1 + \left(\tfrac{1}{3}x^3 - x\right)\Big|_1^2 = -\left(\tfrac{1}{3}-1\right) + \tfrac{8}{3} - 2 - \left(\tfrac{1}{3}-1\right) = 2.$$

Quiz 23
04–25-03

(3) 27. $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$

The above integral is the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$, and $y = \sqrt{1-x^2}$. This region is one-fourth of the interior of the circle centered at (0,0) with radius 1. Therefore, the area of the region is $\frac{\pi}{4}$.

Quiz 24
04–29-03

(4) 28. Find the area of the region enclosed by the curves $x = 0$, $x = \frac{\pi}{2}$, $y = \sin x \cos x$, and $y = \sin^2 x \cos x$.

Note that for $0 \le x \le \frac{\pi}{2}$, $y = \sin^2 x \cos x \le \sin x \cos x$.

$$\int_0^{\frac{\pi}{2}} \left(\sin x \cos x - \sin^2 x \cos x\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\sin x - \sin^2 x\right) \cos x\, dx$$

Let $u = \sin x$ and $du = \cos x\, dx$.

$$= \int_0^{\frac{\pi}{2}} \left(\sin x - \sin^2 x\right) \cos x\, dx$$

$$= \int_0^1 (u - u^2)\,du$$

$$= \left(\tfrac{1}{2}u^2 - \tfrac{1}{3}u^3\right)\Big|_0^1$$

$$= \tfrac{1}{6}$$

Section 1.2: Quiz 11

Quiz 1
01–14–03

(1) 29. $\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\ \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

(1) 30. $\displaystyle\lim_{x\to -1} \frac{x^2-2x-3}{x+1} = \lim_{x\to -1} \frac{(x-3)(x+1)}{x+1} = \lim_{x\to -1} (x-3) = -4$

(1) 31.

(1) 32.

Quiz 2
01–14–03

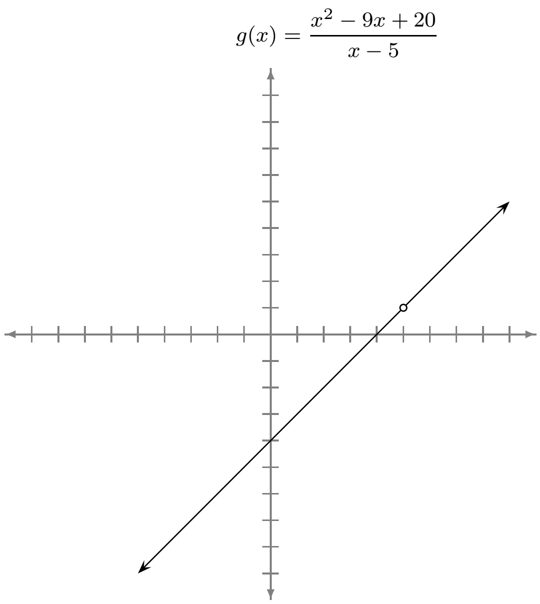
33. For each of the following, give the domain and range. Use your answers to sketch the graph.

(1) a. $g(x) = \dfrac{x^2 - 9x + 20}{x - 5}$

D: (-∞,5) ∪ (5,∞)

$g(x) = \dfrac{(x-5)(x-4)}{x-5}$

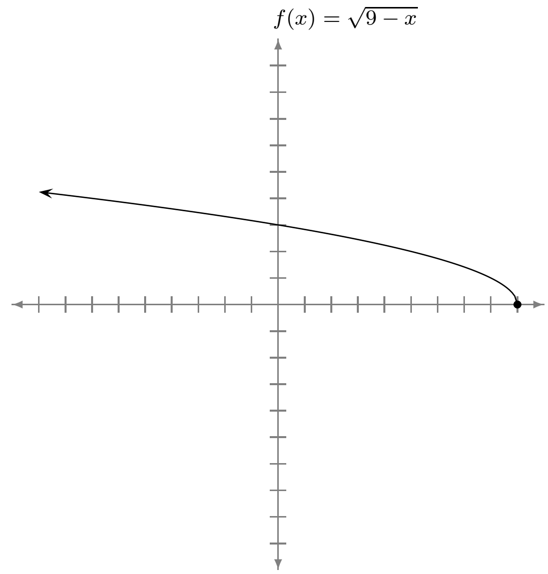
R: (-∞,1) ∪ (1,∞)



(1) b. $f(x) = \sqrt{9 - x}$

D: (-∞,9]

R: [0,∞)



Quiz 3
01–22–03

(2) 34. Prove that the composition of two odd functions is an odd function.

***Proof*:**

Let f and g be odd functions. Then $(f \circ g)(\text{-}x) = f(g(\text{-}x)) = f(\text{-}g(x)) = \text{-}f(g(x)) = \text{-}(f \circ g)(x)$. ■

Quiz 4
01–24–03

For each of the following, find the vertex, y-intercepts, and x-intercept(s) (if any). Use your answers to sketch the graph.

(2) 35. $g(x) = \frac{1}{2}x^2 - x - \frac{15}{2}$

Vertex: (1,-8)

y-intercept: $\left(0,\text{-}\frac{15}{2}\right)$

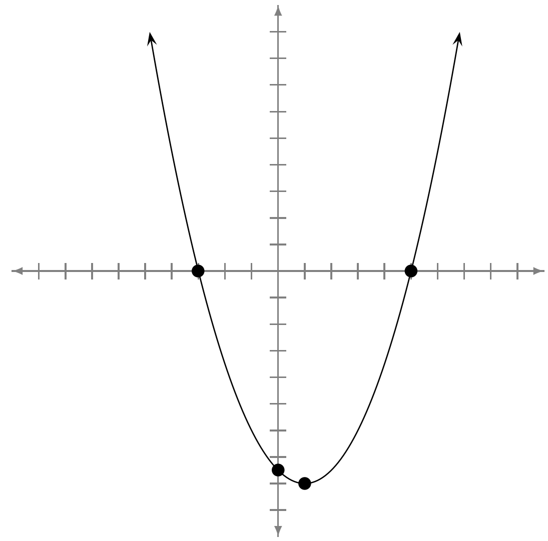
$\frac{1}{2}x^2 - x - \frac{15}{2} = 0$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x =$ -3,5

x-intercepts: (-3,0) and (5,0)



$g(x) = \frac{1}{2}x^2 - x - \frac{15}{2}$

Quiz 5
01–29–03

(2) 36. Give the domain and range of the function $h(x) = \sqrt{\lfloor x \rfloor}$.

Domain: $[0,\infty)$

Range: $\{0, 1, 2, 3, 4, 5, \ldots\}$

Quiz 6
01–31–03

37. (1) a. Simplify completely.

$(\sqrt{x+1} - 2)(\sqrt{x+1} + 2)$

$(\sqrt{x+1} - 2)(\sqrt{x+1} + 2) = x - 3$

(3) b. Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \to 3} \sqrt{x+1} = 2$. Hint: Choose $\delta = \varepsilon$ and use your previous answer.

***Proof*:** Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. If $|x-3| < \delta$, then

$|x-3| < \varepsilon$

$|(\sqrt{x+1} - 2)(\sqrt{x+1} + 2)| < \varepsilon$

$|(\sqrt{x+1} - 2)||(\sqrt{x+1} + 2)| < \varepsilon$

$2|(\sqrt{x+1} - 2)| < \varepsilon$

$|(\sqrt{x+1}-2)| < \varepsilon$. ■

Quiz 7
02–11–03

(2) 38. Is the following function continuous at 1?

$$g(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ \\ 2, & \text{if } x = 1 \end{cases}$$

$g(1) = 2$

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1}\big(x+1\big) = 2$$

The function g is continuous at 1 since $\lim\limits_{x\to 1} g(x) = g(1)$.

Quiz 8
02–14–03

(2) 39. Does the equation $x^{14} + x^2 + 4x + 1 = 0$ have a solution between 0 and 1?

No.

Note that for $x \geq 0$, $x^{14} + x^2 + 4x + 1 \geq 1$.

Quiz 9
02–18–03

(2) 40. Does the equation $x^{14} + x^2 - 4x + 1 = 0$ have a solution between 0 and 1?

Yes.

***Proof*:** Let $f(x) = x^{14} + x^2 - 4x + 1 = 0$ and note that f is continuous on [0,1] since f is a polynomial. Since $f(0) = 1$ and $f(1) =$ -1, the intermediate value theorem implies that there is a number $c \in$ [0,1] such that $f(c) = 0$. ■

Quiz 10
02–19–03

(4) 41. Find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

$f(x) = \dfrac{x^2+x+1}{x}$

HA: none

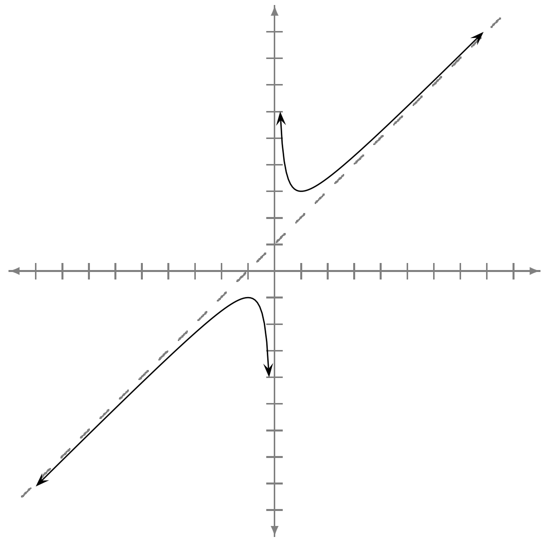
$\lim\limits_{x\to 0^-} \dfrac{x^2+x+1}{x} = -\infty$

$\lim\limits_{x\to 0^+} \dfrac{x^2+x+1}{x} = \infty$

VA: $x = 0$

$f(x) = \dfrac{x^2+x+1}{x} = x + 1 + \frac{1}{x}$

OA: $y = x + 1$



Quiz 11
02–25–03

(2) 42. Differentiate the following.

43. $f(x) = x^3 + x$

$\lim\limits_{h\to 0}\big[f(x+h) - f(x)\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[(x+h)^3 + (x+h) - (x^3+x)\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[x^3 + 3x^2h + 3xh^2 + h^3 + x + h - x^3 - x\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[3x^2h + 3xh^2 + h^3 + h\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[3x^2 + 3xh + h^2 + 1\big]$

$= 3x^2 + 1$

Quiz 12
02–26–03

(2) 44. Differentiate the following.

$g(x) = \sqrt{x-5}$

$\lim\limits_{h\to 0} \dfrac{g(x+h) - g(x)}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{(x+h)-5} - \sqrt{x-5}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \cdot \frac{\sqrt{x+h-5} + \sqrt{x-5}}{\sqrt{x+h-5} + \sqrt{x-5}} \right)$$

$$= \lim_{h\to 0} \frac{x+h-5-(x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \lim_{h\to 0} \frac{x+h-5-(x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \lim_{h\to 0} \frac{x+h-5-x+5}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h-5} + \sqrt{x-5}}$$

$$= \frac{1}{\sqrt{x-5} + \sqrt{x-5}}$$

$$= \frac{1}{2\sqrt{x-5}}$$

Quiz 13
02–28–03

(2) 45. Compute the derivative of the given function and find the equation of the line that is tangent to its graph at the specified point.

$g(x) = 5\sqrt{x}$; $x = 25$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{5\sqrt{x+h} - 5\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{5\sqrt{x+h} - 5\sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h\to 0} \frac{5(x+h) - 5x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{5x + 5h - 5x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{5h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{5}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{5}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{5}{2\sqrt{x}}$$

$g(25) = 25$

$$g'(25) = \frac{5}{2\sqrt{25}} = \frac{1}{2}$$

$y - 25 = \frac{1}{2}(x - 25)$

Quiz 14
03–18–03

46. For a six second period of time, a particle moves according to a law of motion $s = \sin t + \cos t$ where t is measured in seconds and s in meters.

(1) a. Find the velocity at time t.

(1) b. When is the particle at rest?

(1) c. When is the particle moving in a positive direction? When is the particle moving in a negative direction?

(1) d. Find the total distance traveled during the six second time interval.

See exam 2.

Quiz 15
03–21–03

(2) 47. Find the equation of the line that is tangent to the given curve at the specified point.

$x^2y^3 + y + \cos x = 2$; (0,1)

$x^2y^3 + y + \cos x = 2;$

$2xy^3 + 3x^2y^2y' + y' - \sin x = 0$

$$y' = \frac{\text{-}2xy^3 + \sin x}{3x^2y^2 + 1}$$

$x = 0$

$y = 1$

$y' = 0$

$y - 1 = 0(x - 0)$

$y = 1$

Quiz 16
03–25–03

(4) 48. Find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, and local extrema. Use these answers to sketch the graph.

$$f(x) = \frac{2x^2 + 5x - 3}{x^2 + 4x + 3}$$

HA: $y = 2$

$$f(x) = \frac{(2x-1)(x+3)}{(x+3)(x+1)}$$

$$\lim_{x \to \text{-}3} \frac{(2x-1)(x+3)}{(x+3)(x+1)} = \lim_{x \to \text{-}3} \frac{2x-1}{x+1} = \frac{7}{2}$$

VA: $x = $ -1

OA: none

If $x \neq$ -3, $f(x) = \dfrac{2x-1}{x+1}$.

$$f'(x) = \frac{2(x+1) - (2x-1)}{(x+1)^2} = \frac{3}{(x+1)^2}$$

Dec: never

Inc: (-∞,-3) ∪ (-3,-1) ∪ (-1,-∞)

Local max: none

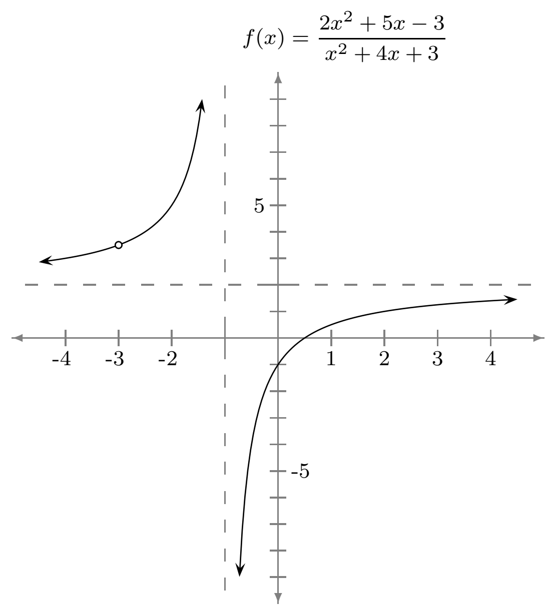
Local min: none

$$f''(x) = \frac{\text{-}6}{(x+1)^3}$$

CD: (-1,-∞)

CU: (-∞,-3) ∪ (-3,-1)

IP: none



Quiz 17
03–28–03

(4) 49. Find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

$f(x) = \frac{4}{27}x^3 - 4x - 1$

HA: none

VA: none

OA: none

$f'(x)$

$= \frac{4}{9}x^2 - 4$

$= \frac{4}{9}(x^2 - 9)$

$= \frac{4}{9}(x+3)(x-3)$

Dec: [-3,3]

Inc: (-∞,-3] ∪ [3,∞)

Local max: 7 at -3

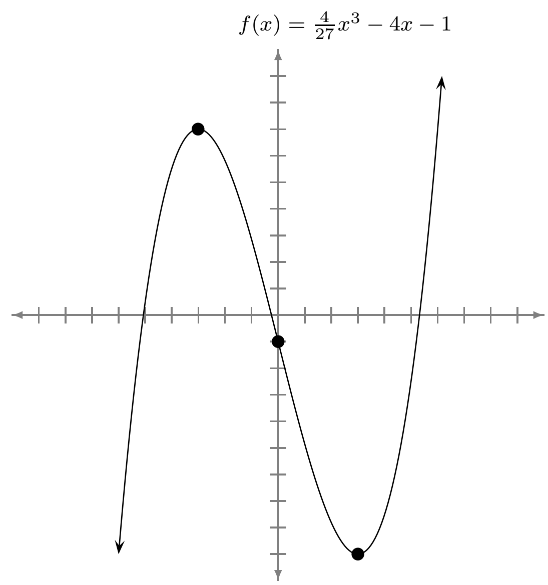
Local min: -9 at 3

$f''(x) = \frac{8}{9}x$

CD: (-∞,0)

CU: (0,∞)

IP: (0,-1)



Quiz 18
04–01–03

50. A ball is thrown upward with an initial velocity of of 48 ft/sec from a height of 50 ft.

(1) a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 48t + 50$

(2) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 48$

$\text{-}32t + 48 = 0$

$\text{-}16(2t - 3) = 0$

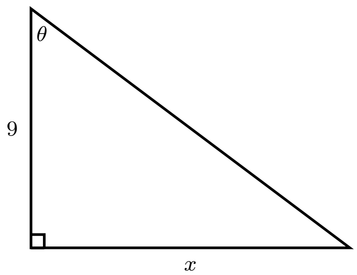
$t = \frac{3}{2}$

$h\left(\frac{3}{2}\right) = \text{-}12 + 72 + 50 = 86$

86 feet

Quiz 19
04–04–03

(3) 51. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is walking away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?



$\tan\theta = \frac{x}{9}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{1}{250}$

$\frac{1}{250}$ radians per second

Quiz 20
04–16–03

(1) 52. $\sum\limits_{n=1}^{5} n^3 = \left(\frac{5\cdot 6}{2}\right)^2 = 225$

(1) 53. Given that $\sum\limits_{n=1}^{10} a_i = 8$ and $\sum\limits_{n=1}^{10} b_i = 10$, calculate $\sum\limits_{n=1}^{10} (2a_i + 7b_i)$.

$$\sum_{n=1}^{10} (2a_i + 7b_i) = \sum_{n=1}^{10} 2a_i + \sum_{n=1}^{10} 7b_i = 2\sum_{n=1}^{10} a_i + 7\sum_{n=1}^{10} b_i = 2\cdot 8 + 7\cdot 10 = 86$$

Quiz 21
04–18–03

(3) 54. Integrate.

$$\int_{-1}^{3} \left(2x^2 + x + 1\right) dx$$

$$\int_{-1}^{3} \left(2x^2 + x + 1\right) dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1 + \frac{4i}{n}\right) \cdot \frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(-1 + \frac{4i}{n}\right)^2 + \left(-1 + \frac{4i}{n}\right) + 1\right] \cdot \frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(\frac{16i^2}{n^2} - \frac{8i}{n} + 1\right) + \left(-1 + \frac{4i}{n}\right) + 1\right] \cdot \frac{4}{n}$$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{32i^2}{n^2} - \frac{16i}{n} + 2 - 1 + \frac{4i}{n} + 1 \right) \cdot \frac{4}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{32i^2}{n^2} - \frac{12i}{n} + 2 \right) \cdot \frac{4}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{128i^2}{n^3} - \frac{48i}{n^2} + \frac{8}{n} \right)$

$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{128i^2}{n^3} - \sum_{i=1}^{n} \frac{48i}{n^2} + \sum_{i=1}^{n} \frac{8}{n} \right)$

$= \lim_{n\to\infty} \left(\frac{128}{n^3} \sum_{i=1}^{n} i^2 - \frac{48}{n^2} \sum_{i=1}^{n} i + \frac{8}{n} \sum_{i=1}^{n} 1 \right)$

$= \lim_{n\to\infty} \left(\frac{128}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{48}{n^2} \cdot \frac{n(n+1)}{2} + \frac{8}{n} \cdot n \right)$

$= \lim_{n\to\infty} \left(\frac{128}{n^3} \cdot \frac{2n^3+3n^2+n}{6} - \frac{48}{n^2} \cdot \frac{(n^2+n)}{2} + \frac{8}{n} \cdot n \right)$

$= \lim_{n\to\infty} \left(\frac{256n^3+384n^2+128n}{6n^3} - \frac{48n^2+48n}{2n^2} + 8 \right)$

$= \frac{256}{6} - \frac{48}{2} + 8$

$= 26\frac{2}{3}$

Quiz 22
04–22-03

(3) 55. Find the area of the region enclosed by the curves $y = x^2 - 1$, $y = 0$, $x = 0$, and $x = 2$.

Note that $x^2 - 1 \leq 0$ for $0 \leq x \leq 1$ and $x^2 - 1 \geq 0$ for $1 \leq x \leq 2$. So the area of the region is

$$-\int_0^1 (x^2 - 1)dx + \int_1^2 (x^2 - 1)dx = -\left(\tfrac{1}{3}x^3 - x\right)\Big|_0^1 + \left(\tfrac{1}{3}x^3 - x\right)\Big|_1^2 = -\left(\tfrac{1}{3} - 1\right) + \tfrac{8}{3} - 2 - \left(\tfrac{1}{3} - 1\right) = 2.$$

Quiz 23
04–25-03

(3) 56. $\int_0^1 \sqrt{1 - x^2}\, dx$

The above integral is the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$, and $y = \sqrt{1 - x^2}$. This region is one-fourth of the interior of the circle centered at (0,0) with radius 1. Therefore, the area of the region is $\frac{\pi}{4}$.

Quiz 24
04–29-03

(4) 57. Find the area of the region enclosed by the curves $x = 0$, $x = \frac{\pi}{2}$, $y = \sin x \cos x$, and $y = \sin^2 x \cos x$.

Note that for $0 \le x \le \frac{\pi}{2}$, $y = \sin^2 x \cos x \le \sin x \cos x$.

$$\int_0^{\frac{\pi}{2}} \left(\sin x \cos x - \sin^2 x \cos x\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\sin x - \sin^2 x\right) \cos x \, dx$$

Let $u = \sin x$ and $du = \cos x \, dx$.

$$= \int_0^{\frac{\pi}{2}} \left(\sin x - \sin^2 x\right) \cos x \, dx$$

$$= \int_0^1 \left(u - u^2\right) du$$

$$= \left(\tfrac{1}{2}u^2 - \tfrac{1}{3}u^3\right)\Big|_0^1$$

$$= \tfrac{1}{6}$$

Section 1.3: Exam 1

Exam 1 Math 1571 Spring 2003

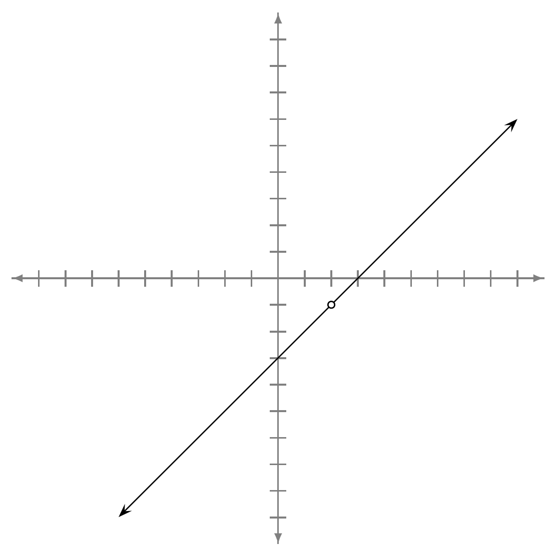
58. For each of the following, find the domain and range and sketch the graph.

(6) a.

$$r(x) = \frac{x^2 - 5x + 6}{x - 2}$$

Domain: $(-\infty,2) \cup (2,\infty)$

Range: $(-\infty,-1) \cup (-1,\infty)$



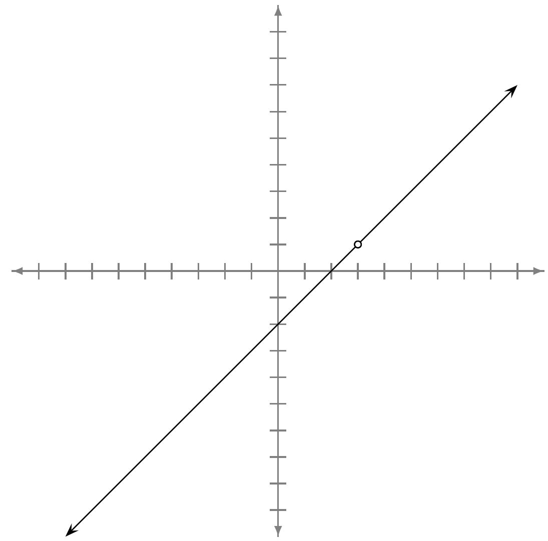
$$r(x) = \frac{x^2 - 5x + 6}{x - 3}$$

Domain: $(-\infty,3) \cup (3,\infty)$

Range: $(-\infty,1) \cup (1,\infty)$

$$r(x) = \frac{x^2 - 5x + 6}{x - 2}$$

$$r(x) = \frac{x^2 - 5x + 6}{x - 3}$$



59. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$.

(3) a. Give the domain and range of $f \circ g$.

$(f \circ g)(x) = x - 1$

Domain: $[0,\infty)$

Range: $[-1,\infty)$

(3) b. Give the domain and range of $g \circ f$.

$(g \circ f)(x) = \sqrt{x^2 - 1}$

Domain: $(-\infty,-1] \cup [1,\infty)$

Range: $[0,\infty)$

(3) 60. Is the following function even, odd, or neither?

$f(x) = x^3 - x + 2$

$f(2) = 8$

$f(\text{-}2) = \text{-}4$

neither

$f(x) = \text{-}x^3 + x + 2$

$f(2) = \text{-}4$

$f(\text{-}2) = 8$

neither

(3) 61. Give the equation of the line perpendicular to the line $y = 4x - 7$ and passing through the point (2,3).

$m = \text{-}\frac{1}{4}$

$y - 3 = \text{-}\frac{1}{4}(x - 2)$

Give the equation of the line perpendicular to the line $y = 7x - 4$ and passing through the point (3,2).

$m = \text{-}\frac{1}{7}$

$y - 2 = \text{-}\frac{1}{7}(x - 3)$

(2) 62. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(3) 63. Find θ such that $0 \leq \theta \leq \pi$ and $\tan\theta = \text{-}1$.

$\theta = \frac{3\pi}{4}$

Find θ such that $0 \leq \theta \leq \pi$ and $\cot\theta = \text{-}1$.

$\theta = \frac{3\pi}{4}$

(3) 64.

$\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{\pi}{5}\right) - \sin\left(\frac{2\pi}{15}\right)\sin\left(\frac{\pi}{5}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\sin\left(\frac{3\pi}{10}\right)\cos\left(\frac{2\pi}{10}\right) + \cos\left(\frac{3\pi}{10}\right)\sin\left(\frac{2\pi}{10}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

(5) 65. Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x\to 7}\big(4x-1\big) = 27$.

Proof:

Let $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta < \frac{\varepsilon}{4}$. If $|x-7| < \delta$, then

$|x-7| < \frac{\varepsilon}{4}$

$4|x-7| < \varepsilon$

$|4x-28| < \varepsilon$

$|(4x-1)-27| < \varepsilon$. ■

66. For each of the following, calculate the limit (if it exists).

(2) a.

$\lim_{x\to 3}\big(x^3-4x^2+x-2\big) =$ -8

$\lim_{x\to 2}\big(x^3-4x^2+x-2\big) =$ -8

(4) b.

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-5x+6} = \lim_{x\to 2}\frac{(x+3)(x-2)}{(x-3)(x-2)} = \lim_{x\to 2}\frac{x+3}{x-3} = \text{-5}$$

$$\lim_{x\to 3}\frac{x^2-x-6}{x^2-5x+6} = \lim_{x\to 3}\frac{(x-3)(x+2)}{(x-3)(x-2)} = \lim_{x\to 3}\frac{x+2}{x-2} = 5$$

(4) c.

$$\lim_{x\to\sqrt{5}}\frac{x-\sqrt{5}}{x^2-5} = \lim_{x\to\sqrt{5}}\frac{x-\sqrt{5}}{(x-\sqrt{5})(x+\sqrt{5})} = \lim_{x\to\sqrt{5}}\frac{1}{(x+\sqrt{5})} = \frac{1}{2\sqrt{5}}$$

$$\lim_{x\to\sqrt{7}}\frac{x-\sqrt{7}}{x^2-7} = \lim_{x\to\sqrt{7}}\frac{x-\sqrt{7}}{(x-\sqrt{7})(x+\sqrt{7})} = \lim_{x\to\sqrt{7}}\frac{1}{(x+\sqrt{7})} = \frac{1}{2\sqrt{7}}$$

(4) d. $\displaystyle\lim_{\theta\to 0}\frac{\cos^2\theta-\cos\theta}{\theta} = \lim_{\theta\to 0}\frac{\cos\theta(\cos\theta-1)}{\theta} = \lim_{\theta\to 0}\left(\cos\theta\cdot\frac{\cos\theta-1}{\theta}\right) = 1\cdot 0 = 0$

(5) e. $\lim_{x\to 0} x\cos\left(\left\lfloor\frac{1}{x}\right\rfloor\right)$

Since -$|x| \le x\cos\left(\left\lfloor\frac{1}{x}\right\rfloor\right) \le |x|$ and $\lim_{x\to 0}$ -$|x| = \lim_{x\to 0}|x| = 0$, $\lim_{x\to 0} x\cos\left(\left\lfloor\frac{1}{x}\right\rfloor\right) = 0$ by the squeeze theorem.

Section 1.4: Exam 2

Exam 2 Math 1571 Spring 2003

(2) 67. Is the following function continuous at 5?

$$f(x) = \frac{x+2}{x+5}$$

Since rational functions are continuous where they are defined, f is continuous at 5.

(3) 68. Is the following function continuous at 1?

$$g(x) = \begin{cases} \dfrac{x^2+3x-4}{x^2-3x+2}, & \text{if } x < 1 \\ 5, & \text{if } x = 1 \\ x^2-6, & \text{if } x > 1 \end{cases}$$

$g(1) = 5$

$$\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} \frac{x^2+3x-4}{x^2-3x+2} = \lim_{x \to 1^-} \frac{(x+4)(x-1)}{(x-2)(x-1)} = \lim_{x \to 1^-} \frac{x+4}{x-2} = \text{-}5$$

$$\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} \left(x^2-6\right) = \text{-}5$$

$$\lim_{x \to 1} g(x) = \text{-}5$$

Since $g(1) = 5$ and $\lim_{x \to 1} g(x) = \text{-}5$, g is not continuous at 1.

69. (3) a. State the intermediate value theorem.

(2) b. Consider the equation $x^3 + 3x^2 - 3 = 0$. Is there a solution between -1 and 1?

Yes. Let $f(x) = x^3 + 3x^2 - 3$ and note that f is continuous since f is a polynomial. Since $f(\text{-}1) = \text{-}1$ and $f(1) = 1$, the intermediate value theorem implies that there is a solution between -1 and 1.

70. For each of the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

(8) a. $f(x) = \dfrac{2x^2 - 4x - 6}{x^2 + 5x + 4}$

HA: $y = 2$

$$f(x) = \frac{2x^2 - 4x - 6}{x^2 + 5x + 4} = \frac{2(x+1)(x-3)}{(x+1)(x+4)}$$

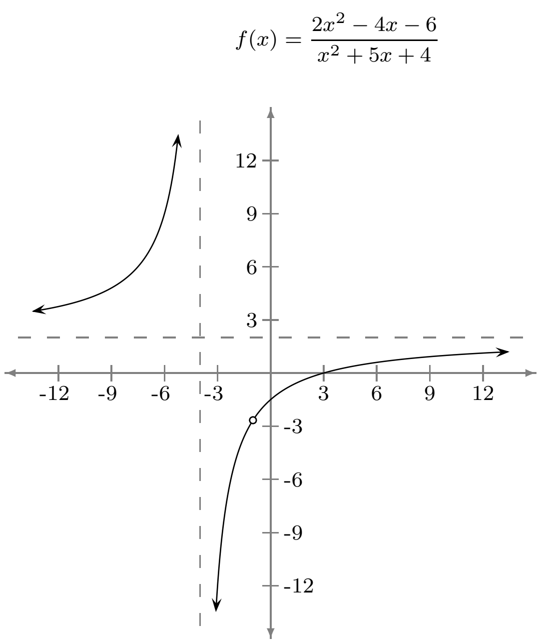
$$\lim_{x \to -1} \frac{2(x+1)(x-3)}{(x+1)(x+4)}$$

$$= \lim_{x \to -1} \frac{2(x-3)}{(x+4)}$$

$$= -\tfrac{8}{3}$$

VA: $x = -4$

OA: none



(8) b. $g(x) = \dfrac{x^3 + 6x^2 + 8x}{x^2 + 3x + 2}$

HA: none

$$g(x) = \frac{x^3 + 6x^2 + 8x}{x^2 + 3x + 2} = \frac{x(x+2)(x+4)}{(x+2)(x+1)}$$

$$\lim_{x \to -2} \frac{x(x+2)(x+4)}{(x+2)(x+1)}$$

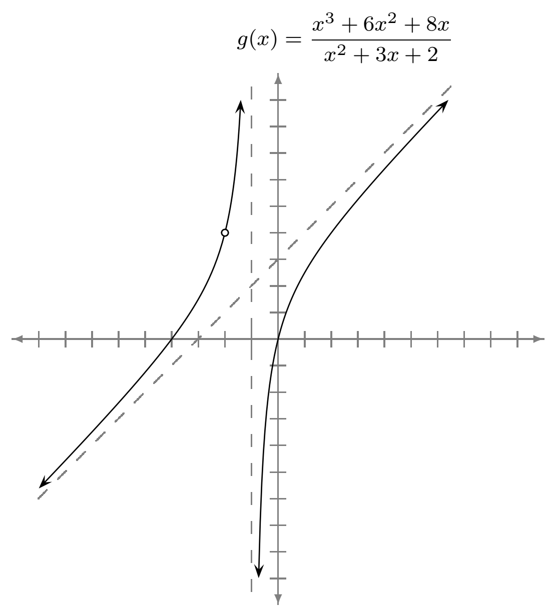
$$= \lim_{x \to -2} \frac{x(x+4)}{(x+1)}$$

$$= 4$$

VA: $x = -1$

$g(x) = x + 3 + \frac{-3x+6}{x^2+3x+2}$

OA: $y = x + 3$



(3) 71. Use the definition of derivative to differentiate $k(x) = x^2 - x + 1$.

$k'(x)$

$$= \lim_{h \to 0} \frac{k(x+h) - k(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - (x+h) + 1 - (x^2 - x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h + 1 - x^2 + x - 1}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 1)}{h}$$

$$= \lim_{h \to 0} \big(2x + h - 1\big)$$

$$= 2x - 1$$

(3) 72. Calculate the following limit.

$$\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$$

$f(x) = \tan x$

$$f'\left(\tfrac{\pi}{4}\right) = \lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$$

$f'(x) = \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right) = 2$

73. Differentiate.

(3) a. $f(x) = (x^2 - 6)(x^3 + 2x - 3)$

$f'(x) = (2x)(x^3 + 2x - 3) + (x^2 - 6)(3x^2 + 2)$

(3) b. $g(x) = \dfrac{x^3 + x^2 + 1}{\cos x}$

$$g'(x) = \frac{(3x^2 + 2x)(\cos x) - (x^3 + x^2 + 1)(\text{-}\sin x)}{\cos^2 x}$$

(3) c. $h(x) = \sqrt{x^2 + 1}$

$h(x) = (x^2 + 1)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(2x)$

$$h'(x) = \frac{x}{\sqrt{x^2 + 1}}$$

74. For a six second period of time, a particle moves according to a law of motion $s = \sin t + \cos t$ where t is measured in seconds and s in meters.

(2) a. Find the velocity at time t.

$v(t) = \frac{ds}{dt} = s = \cos t - \sin t$

(3) b. When is the particle at rest?

$\cos t - \sin t = 0$

$\cos t = \sin t$

$t = \frac{\pi}{4}, \frac{5\pi}{4}$

(3) c. When is the particle moving in a positive direction? When is the particle moving in a negative direction?

Neg: $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$

Pos: $\left[0,\frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4},6\right]$

(3) d. Find the total distance traveled during the six second time interval.

$\left|s\left(\frac{\pi}{4}\right) - s\left(0\right)\right| + \left|s\left(\frac{5\pi}{4}\right) - s\left(\frac{\pi}{4}\right)\right| + \left|s\left(6\right) - s\left(\frac{5\pi}{4}\right)\right|$

$= \left|\sqrt{2} - 1\right| + \left|\text{-}\sqrt{2} - \sqrt{2}\right| + \left|\sin 6 + \cos 6 - \text{-}\sqrt{2}\right|$

$= \sqrt{2} - 1 + 2\sqrt{2} + \sin 6 + \cos 6 + \sqrt{2}$

$= 4\sqrt{2} + \sin 6 + \cos 6 - 1$

Section 1.5: Exam 3

Exam 3 Math 1571 Spring 2003

(4) 75. Find the equation of the line that is tangent to the given curve at the specified point.

$x^3y^2 + x + y = \text{-}3$; (-1,2)

$\frac{d}{dx}\left(x^3y^2 + x + y\right) = \frac{d}{dx}\text{-}3$

$3x^2y^2 + 2x^3yy' + 1 + y' = 0$

$$y' = \frac{\text{-}3x^2y^2 - 1}{2x^3y + 1}$$

$x = \text{-}1$

$y = 2$

$y' = \text{-}\frac{13}{3}$

$y - 2 = \frac{13}{3}(x + 1)$

76. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(10) a. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 1}$

$f(x) = \dfrac{(x+1)(x-2)}{(x+1)(x-1)}$

HA: $y = 1$

VA: $x = 1$

OA: none

For $x \neq 1$, $f(x) = \dfrac{x-2}{x-1}$.

$f'(x)$

$= \dfrac{(1)(x-1) - (x-2)(1)}{(x-1)^2}$

$= \dfrac{1}{(x-1)^2}$

Dec: never

Inc: (-∞,-1) ∪ (-1,1) ∪ (1,∞)

Local max: none

Local min: none

$f'(x) = (x-1)^{-2}$

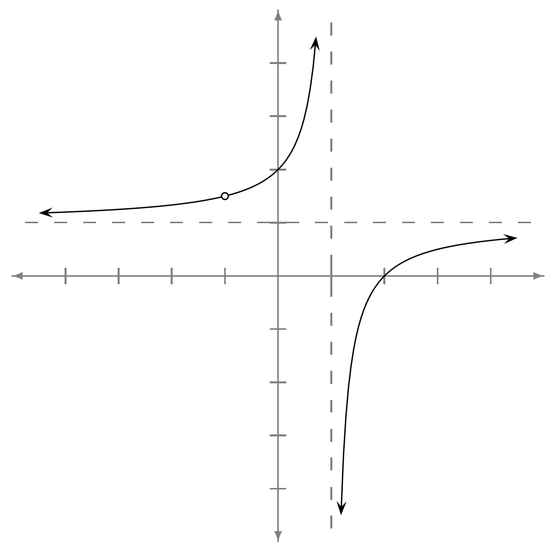
$f''(x) = -2(x-1)^{-3} = \dfrac{-2}{(x-1)^3}$

CD: (1,∞)

CU: (-∞,-1) ∪ (-1,1)

IP: none

$f(x) = \dfrac{x^2 - x - 2}{x^2 - 1}$



(6) b. $g(x) = x^3 + 3x^2 - 3$

HA: none

VA: none

OA: none

$g'(x) = 3x^2 + 6x = 3x(x+2)$

Dec: [-2,0]

Inc: (-∞,-2] ∪ [0,∞)

Local max: 1 at -2

Local min: -3 at 0

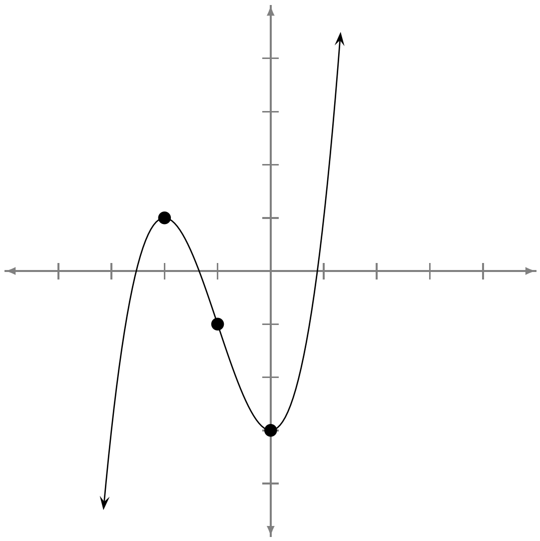
$g''(x) = 6x + 6 = 6(x+1)$

CD: (-∞,-1)

CU: (-1,∞)

IP: (-1,-1)

$g(x) = x^3 + 3x^2 - 3$



77. A ball is thrown upward with an initial velocity of of 64 ft/sec from a height of 80 ft.

(2) a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 64t + 80$

(3) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 64$

$\text{-}32t + 64 = 0$

$\text{-}32(t - 2) = 0$

$t = 2$

$h(2) = \text{-}64 + 128 + 80 = 144$

144 feet

(4) 78. A stone is thrown into a lake creating a circular ripple. The radius of the ripple is increasing at a rate of 2 cm per second. At what rate is the area of the ripple increasing after 3 seconds?

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

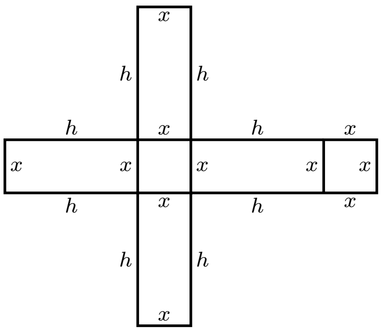
$\frac{dA}{dt} = 2\pi \cdot 6 \cdot 2$

$\frac{dA}{dt} = 24\pi$

24π cm^2

(6) 79. A manufacturing company needs to construct a box with a square base that will have a volume of 54 cm^3. The material to construct the top and bottom costs \$1 per square cm. The material to construct the sides costs \$1 per square cm. Find the dimensions that will minimize the cost of making the box.

Let x be the length of one side of the base. Then the height of the box is $h = \frac{54}{x^2}$.



$C(x) = 1 \cdot 2x^2 + 1 \cdot 4xh$

$C(x) = 2x^2 + \frac{216}{x}$

$C'(x) = 4x - \frac{216}{x^2}$

$$C'(x) = \frac{4x^3 - 216}{x^2}$$

$$C'(x) = \frac{4(x^3 - 54)}{x^2}$$

Dec: $\left(0,3\sqrt[3]{2}\right]$

Inc: $\left[3\sqrt[3]{2},\infty\right)$

Absolute min: $54\sqrt[3]{4}$ at $3\sqrt[3]{2}$

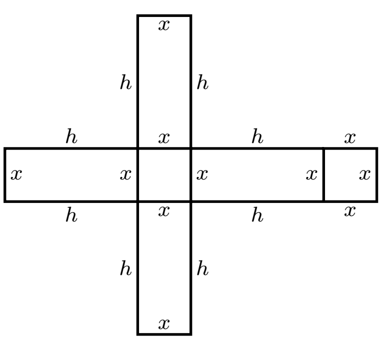
Length of base side: $3\sqrt[3]{2}$

Height: $3\sqrt[3]{2}$

Dimensions: $3\sqrt[3]{2} \times 3\sqrt[3]{2} \times 3\sqrt[3]{2}$

80. A manufacturing company needs to construct a box with a square base that will have a volume of 54 cm^3. The material to construct the top and bottom costs \$2 per square cm. The material to construct the sides costs \$1 per square cm. Find the dimensions that will minimize the cost of making the box.

Let x be the length of one side of the base. Then the height of the box is $h = \frac{54}{x^2}$.



$C(x) = 2 \cdot 2x^2 + 1 \cdot 4xh$

$C(x) = 4x^2 + \frac{216}{x}$

$C'(x) = 8x - \frac{216}{x^2}$

$$C'(x) = \frac{8x^3 - 216}{x^2}$$

$$C'(x) = \frac{8(x^3 - 27)}{x^2}$$

Dec: $(0,3]$

Inc: $[3,\infty)$

Absolute min: 108 at 3

Length of base side: 3

Height: 6

Dimensions: $2 \times 2 \times 6$

Page 267: 23.

(6) 81. A kite is carried horizontally at the rate of 1.5 m/sec and is rising at 2.0 m/sec. How fast is the string released to maintain this flight when 100 m of string have been released and the kite is at an altitude of 80 m?

82. Let $f(x) = \sqrt[3]{x}$.

(3) a. Find the linearization of f at 125.

$f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f(125) = 5$

$f'(125) = \frac{1}{75}$

$L(x) = \frac{1}{75}(x - 125) + 5$

(2) b. Use your answer from the previous part to approximate $\sqrt[3]{126}$.

$f(126) \approx L(126) = 5\frac{1}{75}$

83. Let $f(x) = x^3 + 2x + 1$.

(3) a. Use the intermediate value theorem to prove that the equation $f(x) = 0$ has a solution.

Proof: Note that $f(0) = 1$ and $f(\text{-}1) = \text{-}2$. Since f is a polynomial, f is continuous on [-1,0]. So the intermediate value theorem implies that there is a number $c \in [\text{-}1,0]$ such that $f(c) = 0$. ■

(4) b. Use the mean value theorem to prove that the equation $f(x) = 0$ has only one solution.

Proof: Suppose that there exist real numbers a and b such that $a < b$ and $f(a) = f(b) = 0$. By the mean value theorem, there exists a number c such that $a < c < b$ and $f'(c) = \frac{f(b)-f(a)}{b-a} = 0$. This is not possible since $f'(x) = 3x^2 + 2 \geq 2$ for all x. Therefore, $f(x) = 0$ has only one solution. ■

Chapter 2: Math 1571 Fall 2004

Section 2.1: Exam 2

Exam 2 Math 1571 Fall 2004

84. Calculate the following limits.

(3) a. $\lim\limits_{x\to 2}\left(x^3 - 3x^2 + 6\right) = 2$

(4) b. $\lim\limits_{x\to\text{-}3} \dfrac{x^2 + 6x + 9}{x + 3}$

$= \lim\limits_{x\to\text{-}3} \dfrac{(x+3)(x+3)}{x+3}$

$= \lim\limits_{x\to\text{-}3}\left(x + 3\right)$

$= 6$

(4) c. $\lim\limits_{x\to 1} \dfrac{\sqrt{x} - 1}{x - 1}$

$$= \lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)$$

$$= \lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1}\frac{1}{\sqrt{x}+1}$$

$$= \frac{1}{2}$$

(3) d. $\lim\limits_{\theta\to 0}\dfrac{\sin 3\theta}{2\theta}$

$$= \lim_{\theta\to 0}\left(\frac{3}{2}\cdot\frac{\sin 3\theta}{3\theta}\right)$$

$$= \tfrac{3}{2}\lim_{\theta\to 0}\left(\frac{\sin 3\theta}{3\theta}\right)$$

$$= \tfrac{3}{2}\cdot 1$$

$$= \tfrac{3}{2}$$

(3) e. $\lim\limits_{x\to 0}|x|\cos\left(\frac{1}{x}\right)=0$

Since $0\le |x|\cos\left(\frac{1}{x}\right)\le |x|$ and $\lim\limits_{x\to 0}0=\lim\limits_{x\to 0}|x|=0$, $\lim\limits_{x\to 0}|x|\cos\left(\frac{1}{x}\right)=0$ by the Squeeze Theorem.

(8) 85. For the following function, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

$$f(x)=\frac{x^2+1}{x}$$

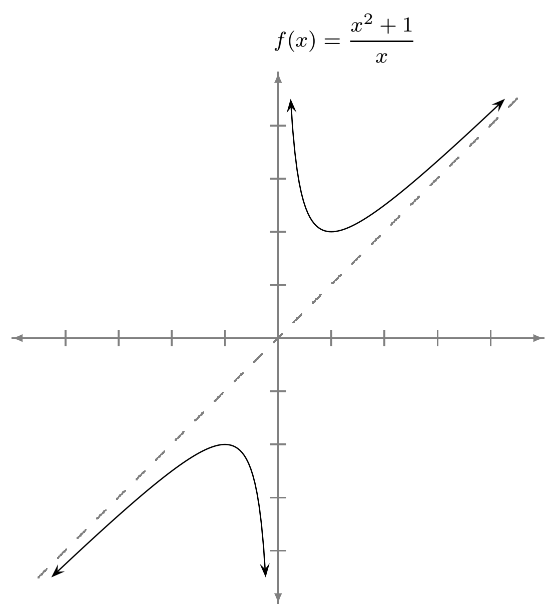
$$f(x)=\frac{x^2+1}{x}=x+\tfrac{1}{x}$$

HA: none

VA: $x=0$

$f(x)=x+\frac{1}{x}$

OA: $y=x$



86. **(2) a.** State the Intermediate Value Theorem.

(3) b. Does the following equation have a solution?

$x^2\sqrt{x+1} = 15$

Yes.

Proof: Let $g(x) = x^2\sqrt{x+1}$ and note that g is continuous on its domain. Also, note that $g(0) = 0$ and $g(3) = 18$. Since $0 < 15 < 18$, the Intermediate Value Theorem implies that there is a number $c \in [0,3]$ such that $f(c) = 15$. ∎

87. For each of the following, express the given derivative as a limit. Do not calculate the limit.

(2) a. $f'\left(\frac{\pi}{4}\right)$ where $f(x) = \tan x$

$$\lim_{x\to\frac{\pi}{4}} \frac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$$

(2) b. $f'(x)$ where $f(x) = \sqrt{x+1}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

(2) 88. Calculate the following limit.

$$\lim_{x\to 10} \frac{x^3 - 1000}{x - 10} = 300$$

$f(x) = x^3$

$f'(x) = 3x^2$

$$\lim_{x\to 10} \frac{x^3 - 1000}{x - 10} = f'(10) = 300$$

89. Differentiate.

(3) a. $f(x) = x^4 + x^2 + 3x - 11$

$f'(x) = 4x^3 + 2x + 3$

(3) b. $g(x) = \dfrac{x^3 + 2x + 1}{x^2 + 2}$

$$g'(x) = \frac{(3x^2+2)(x^2+2) - (x^3+2x+1)(2x)}{(x^2+2)^2}$$

(3) c. $h(x) = \sqrt{x^2+1}$

$h(x) = (x^2+1)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)$

$$h'(x) = \frac{x}{\sqrt{x^2+1}}$$

90. Let $f(x) = x^2 - 3x + 1$.

(3) a. Find the equation of the line that is tangent to the graph of f at the point where $x = 3$.

$f'(x) = 2x - 3$

$x = 3$

$y = f(3) = 1$

$m = f'(3) = 3$

$y - 1 = 3(x - 3)$

(2) b. Find the rate of change of f with respect to x when $x = 4$.

$f(x) = x^2 - 3x + 1$

$f'(x) = 2x - 3$

$f'(4) = 5$

(2) 91. BONUS: Given that $\frac{d}{dx} e^x = e^x$, find $\frac{d}{dx} e^{3x}$.

$\frac{d}{dx} e^{3x} = 3e^{3x}$

Total Points: 341

Section 2.2: Exam 3

Exam 3 Math 1571 Fall 2004

92. For each of the following, find the equation of the line that is tangent to the graph of the given function at the indicated point.

(3) a. $f(t) = \sin^2 t$; $t = \frac{\pi}{6}$

$f'(t) = 2 \sin t \cos t$

$t = \frac{\pi}{6}$

$f\left(\frac{\pi}{6}\right) = \frac{1}{4}$

$f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$y - \frac{1}{4} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

(3) b. $4 \cos x \sin x = 1$

See the student solutions manual.

93. A ball is thrown upward with an initial velocity of of 64 ft/sec from a height of 80 ft.

(2) a. Give the height of the ball as a function of time.

$h(t) = -16t^2 + 64t + 80$

(2) b. Find the maximum height of the ball.

$v(t) = h'(t) = -32t + 64$

$-32t + 64 = 0$

$-32(t - 2) = 0$

$t = 2$

$h(2) = -64 + 128 + 80 = 144$

144 feet

(2) c. What is the velocity of the ball when the ball hits the ground?

$-16t^2 + 64t + 80 = 0$

$-16(t^2 - 4t - 5) = 0$

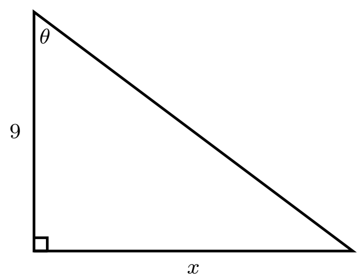
$-16(t-5)(t+1) = 0$

$t = 5$

$v(5) = -96$

-96 ft/sec

94. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is walking away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?



$\tan\theta = \frac{x}{9}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{1}{250}$

$\frac{1}{250}$ radians per second

Exam 3 Math 1571 Fall 2004

Name:__________________________

E-mail (optional):__________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

95. For each of the following, find the intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(5) a. $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$

(5) b. $g(x) = \sqrt{x^2+1}$

(5) 96. Find the equation of the line that is tangent to the graph of the given curve at the indicated point.

$f(t) = \sin^2 t;\ t = \frac{\pi}{6}$

(5) 97. Find $\frac{dy}{dx}$.

$4 \cos x \sin y = 1$

98. A ball is thrown upward with an initial velocity of of 64 ft/sec from a height of 80 ft.

(2) a. Give the height of the ball as a function of time.

(2) b. Find the maximum height of the ball.

(2) c. What is the velocity of the ball when the ball hits the ground?

(5) 99. A spherical snowball is melting in such a way that its volume is decreasing at a rate of 4 in^3 per hour. At what rate is the radius of the snowball decreasing when the radius is 1 in? Recall: $V = \frac{4}{3}\pi r^3$.

(5) 100. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is walking away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?

(5) 101. Find the maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x$ on the interval [0,3].

(5) 102. A farmer has 400 yards of fence in which to enclose a rectangular field. Find the dimensions that maximize the area.

(5) 103. Find the maximum area of a rectangle that can be inscribed in the unit circle.

Chapter 3: Math 1571 Spring 2007

Section 3.1: Exam 1

Exam 1 Math 1571 Spring 2007

(5) 104. Find the equation of the line passing through the points (3,-1) and (4,6).

$m = 7$

$y + 1 = 7(x - 3)$

105. For each of the following, find the domain and range and sketch the graph.

(5) a. $f(x)$

$= \dfrac{x^2 + x - 2}{x + 2}$

$= \dfrac{(x+2)(x-1)}{x+2}$

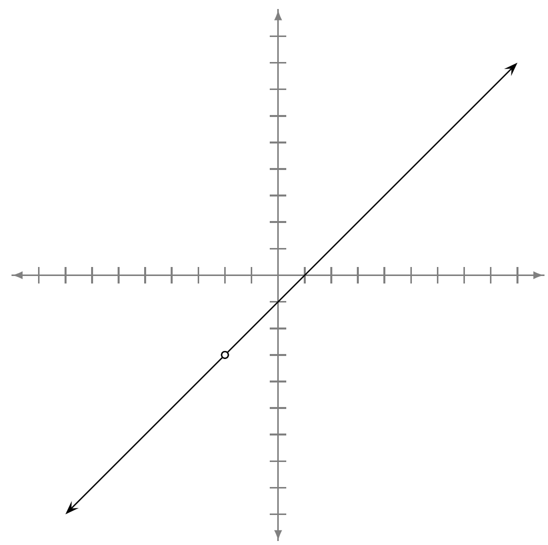
$\neq x - 1$

D: (-∞,-2) ∪ (-2,∞)

R: (-∞,-3) ∪ (-3,∞)

HA: none

VA: none



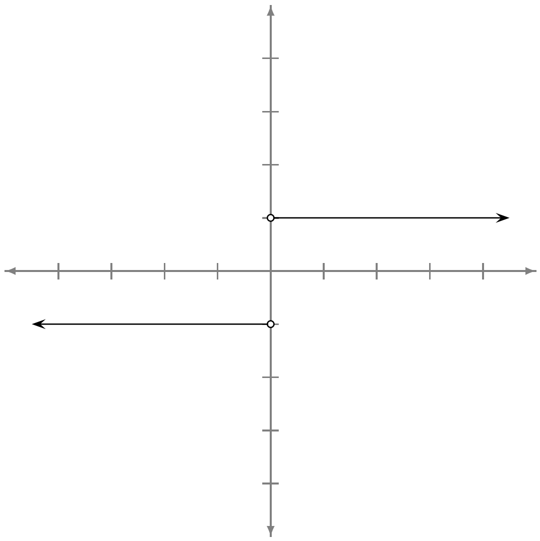
(5) b. $f(x) = \dfrac{|x|}{x}$

D: (-∞,0) ∪ (0,∞)

R: {-1,1}

$f(x) = \dfrac{x^2 + x - 2}{x + 2}$

$f(x) = \dfrac{|x|}{x}$



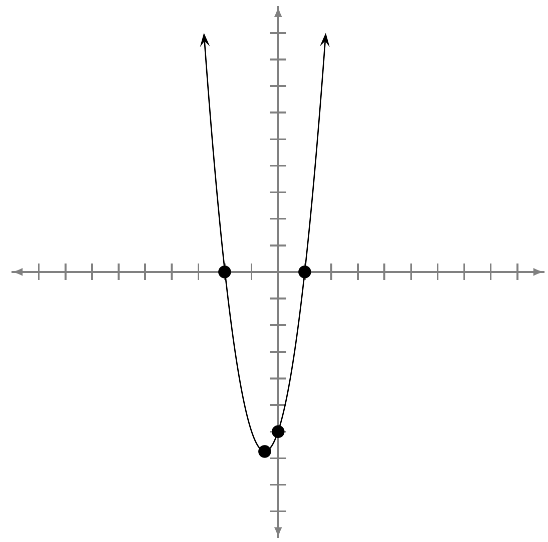
(Page 23: 40) **(5) c.** $h(x) = \sqrt{4 - x^2}$

(5) 106.

For the following, find the vertex, y-intercept, and x-intercept(s) (if any). Use your answers to sketch the graph.

$f(x) = 3x^2 + 3x - 6$

$f(x) = 3x^2 + 3x - 6$



Vertex: $\left(-\frac{1}{2}, -6\frac{3}{4}\right)$

$3x^2 + 3x - 6 = 0$

$3(x^2 + x - 2) = 0$

$x^2 + x - 2 = 0$

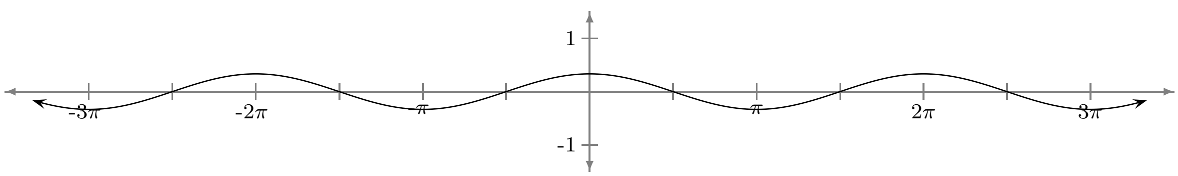
$(x + 2)(x - 1) = 0$

$x = -2, 1$

x-intercepts: (-2,0) and (1,0)

y-intercept: (0,-6)

(5) 107. Sketch the graph of $y = \frac{1}{3}\sin\left(x + \frac{\pi}{2}\right)$



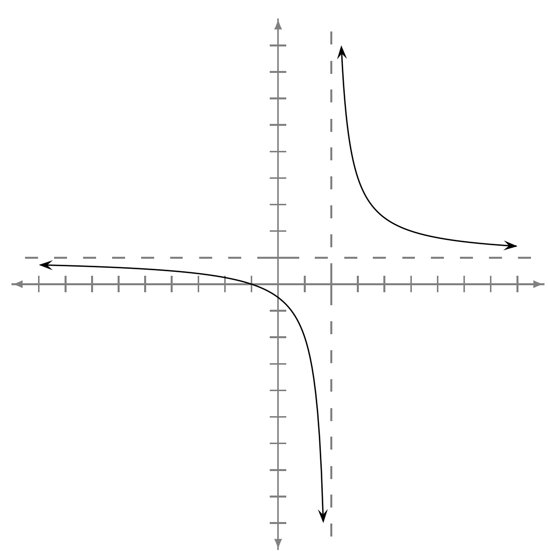
(5) 108.

List the transformations needed to obtain the graph of $y = \frac{3}{x-2} + 1$ from the graph of $y = \frac{1}{x}$. Sketch the graph of $y = \frac{3}{x-2} + 1$.

Shift right 2 units.

Vertical stretch by a factor of 3.

Shift up 1 unit.



$f(x) = \frac{3}{x-2} + 1$

(Page A32: 53) **(5) 109.** Prove the following identity.

$\sin x \sin 2x + \cos x \cos 2x = \cos x$

(5) 110. Use the $\varepsilon - \delta$ definition of limit to prove that $\lim\limits_{x \to 4} \text{-}3x + 2 = \text{-}10$.

Proof: Let $\varepsilon > 0$ and let $\delta < \frac{\varepsilon}{3}$. If $|x - 4| < \delta$, then

$|x - 4| < \delta$

$|x - 4| < \frac{\varepsilon}{3}$

$3|x - 4| < \varepsilon$

$|3x - 12| < \varepsilon$

$|3x - 2 - 10| < \varepsilon$

$|\text{-}3x + 2 + 10| < \varepsilon$

$|(\text{-}3x + 2) - \text{-}10| < \varepsilon$.

111. Calculate the following limits.

(5) a. $\lim\limits_{x \to 1} \dfrac{x^3 + 2x^2 - 3x}{x^2 + 3x - 4}$

$= \lim\limits_{x \to 1} \dfrac{x(x^2 + 2x - 3)}{x^2 + 3x - 4}$

$= \lim\limits_{x \to 1} \dfrac{x(x + 3)(x - 1)}{(x + 4)(x - 1)}$

$= \lim\limits_{x \to 1} \dfrac{x(x + 3)}{x + 4}$

$= \dfrac{4}{5}$

(5) b. $\lim\limits_{h \to 0} \dfrac{\sqrt{9 + h} - 3}{h}$

$= \lim\limits_{h \to 0} \dfrac{\sqrt{9 + h} - 3}{h} \cdot \dfrac{\sqrt{9 + h} + 3}{\sqrt{9 + h} + 3}$

$= \lim\limits_{h \to 0} \dfrac{9 + h - 9}{h(\sqrt{9 + h} + 3)}$

$= \lim\limits_{h \to 0} \dfrac{h}{h(\sqrt{9 + h} + 3)}$

$= \lim\limits_{h \to 0} \dfrac{1}{\sqrt{9 + h} + 3}$

$= \dfrac{1}{6}$

Section 3.2: Exam 2

Exam 2 Math 1571 Spring 2007

112. For each of the following, decide if the given function is continuous at the specified value of x.

(4) a. $f(x) = 3x^3 - 5x + 6$; $x = 2$

Polynomials are continuous everywhere.

(4) b. $g(x) = \begin{cases} 4x + 1, & \text{if } x \leq \text{-}1 \\ x^2 - 4, & \text{if } x > \text{-}1 \end{cases}$; $x = \text{-}1$

$g(\text{-}1) = \text{-}3$

$\lim\limits_{x \to \text{-}1^-} g(x) = \lim\limits_{x \to \text{-}1^-} \big(4x + 1\big) = \text{-}3$

$\lim\limits_{x \to \text{-}1^+} g(x) = \lim\limits_{x \to \text{-}1^+} \big(x^2 - 4\big) = \text{-}3$

$\lim\limits_{x \to \text{-}1} g(x) = \text{-}3$

The function is continuous at -1.

113. Determine whether or not each of the following equations has a solution.

(4) a. $x^4 - 4x^2 + 1 = 0$

Let $f(x) = x^4 - 4x^2 + 1$. Note that $f(0) = 1$ and $f(1) =$ -2. Since f is continuous on [0,1], the Intermediate Value Theorem implies that there exists a number $c \in$ [0,1] such that $f(c) = 0$.

(4) b. $x^4 + 4x^2 + 1 = 0$

Since $x^4 + 4x^2 + 1 \geq 1$ for all $x \in \mathbb{R}$, the equation $x^4 + 4x^2 + 1 = 0$ has no solution.

114. For each of the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

(4) a. $g(x)$

$$= \frac{x^2 + x - 2}{x^2 + 3x + 2}$$

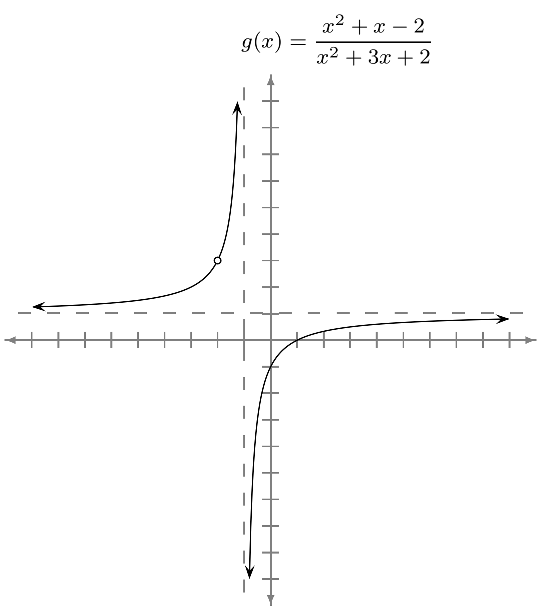
$$= \frac{(x+2)(x-1)}{(x+2)(x+1)}$$

$$\neq \frac{x-1}{x+1}$$

HA: $y = 1$

VA: $x =$ -1

OA: none



(4) b. $g(x)$

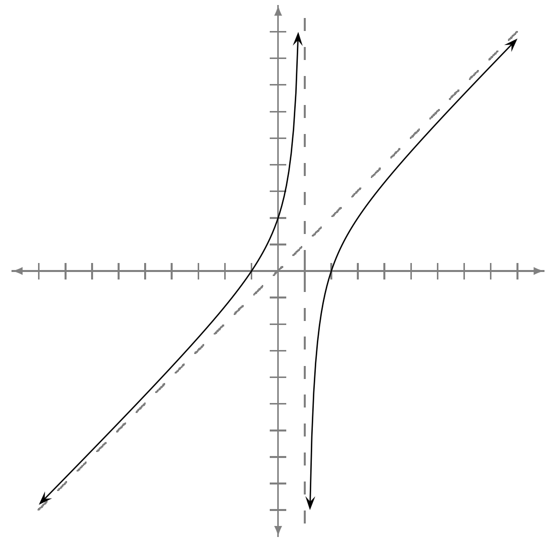
$= \dfrac{x^2 - x - 2}{x - 1}$

$= x - \frac{2}{x-1}$

HA: none

VA: $x = 1$

OA: $y = x$



115. For each of the following, use the definition of derivative to differentiate.

(4) a. $f(x) = x^2 + 2x + 1$

$\lim\limits_{h \to 0} \frac{1}{h}[f(x+h) - f(x)]$

$= \lim\limits_{h \to 0} \frac{1}{h}[(x+h)^2 + (x+h) + 1 - (x^2 + x + 1)]$

$= \lim\limits_{h \to 0} \frac{1}{h}(x^2 + 2xh + h^2 + x + h + 1 - x^2 - x - 1)$

$= \lim\limits_{h \to 0} \frac{1}{h}(2xh + h^2 + h)$

$= \lim\limits_{h \to 0}(2x + h + 1)$

$= 2x + 1$

116. Differentiate.

(4) a. $f(x) = x^5 - 4x^3 + 3x^2 + 17$

$f'(x) = 5x^4 - 12x^2 + 6x$

(4) b. $g(x) = \dfrac{x^2 + 1}{3x - 4}$

$g'(x)$

$= \dfrac{2x(3x-4) - (x^2+1)3}{(3x-4)^2}$

$= \dfrac{6x^2 - 8x - 3x^2 - 3}{(3x-4)^2}$

$= \dfrac{3x^2 - 8x - 3}{(3x-4)^2}$

(4) c. $h(x) = \sqrt[3]{x^4 - 11x}$

$h(x) = (x^4 - 11x)^{\frac{1}{3}}$

$h'(x) = \frac{1}{3}(x^4 - 11x)^{-\frac{2}{3}}(4x^3 - 11)$

$h'(x) = \dfrac{4x^3 - 11}{3\sqrt[3]{(x^4 - 11x)^2}}$

(4) 117. Find the equation of the line that is tangent to the graph of the function $f(x) = x^3 - x - 1$ at the point where $x = 2$.

$f'(x) = 3x^2 - 1;$

$x = 2$

$y = 5$

$m = 11$

$y - 5 = 11(x - 2)$

118. A particle moves according to a law of motion $s = 2t^3 - 15t^2 + 36t$ where s is measured in meters and t in seconds.

(4) a. Find the velocity at time t.

$v(t)$

$= s'(t)$

$= 6t^2 - 30t + 36$

(4) b. Find the acceleration of the particle at time $t = 2$.

$a(t)$

$= v'(t)$

$= 12t - 30$

$= 6(2t - 5)$

$a(2) =$ -6

Section 3.3: Exam 3

Exam 3 Math 1571 Spring 2007

119. For each of the following, determine whether the function is odd, even, or neither. Also, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(6) a. $f(x) = x^3 + 6x^2 + 9x + 1$

$f(1) = 17$

f(-1) = -3

Neither even nor odd.

HA: none

VA: none

OA: none

$f'(x) = 3x^2 + 12x + 9 = 3(x + 1)(x + 3)$

Dec: [-3,-1]

Inc: (-∞,-3], [-1,∞)

Local max: 1 at -3

Local min: -3 at -1

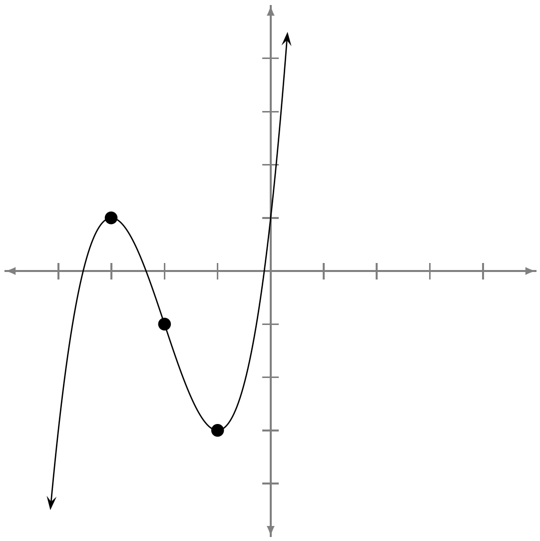
$f''(x) = 6x + 12 = 6(x + 2)$

CU: (-2,∞)

CD: (-∞,-2)

IP: (-2,-1)

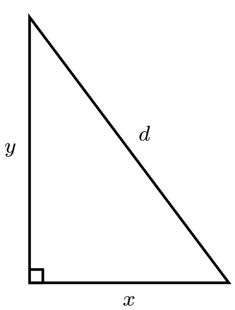
$f(x) = x^3 + 6x^2 + 9x + 1$



(6) b. $g(x) = \dfrac{x^2}{x^2+3}$

(Page 203: 15) **(6) 120.** The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100cm^2? Recall that the area of a triangle is $A = \frac{1}{2}bh$.

(6) 121. Two cars leave the same place at 2:00 P.M. The first car travels north at 60 mph while the other car travels east at 45 mph. At what rate is the distance between the two cars changing at 3:00 P.M.?



$x^2 + y^2 = d^2$

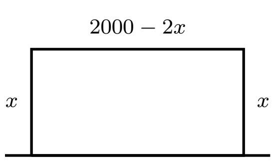
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

$2 \cdot 45 \cdot 45 + 2 \cdot 60 \cdot 60 = 2 \cdot 75 \cdot \frac{dd}{dt}$

$\frac{dd}{dt} = 75$

75 mph

(6) 122. A farmer has 2000 yards of fence in which to enclose a rectangular field next to an existing fence (i.e., fencing is only needed for three sides of the field). Find the dimensions that will maximize the area of the field.



$A(x) = x(2000 - 2x)$

$A(x) = -2x^2 + 2000x$

$A'(x) = -4x + 2000$

$A'(x) = -4(x - 500)$

$A(0) = 0$

$A(1000) = 0$

$A(500) = 500{,}000$

500×1000

(6) 123. For the following, find the absolute maximum and absolute minimum of the given function on the given interval.

$f(x) = x^3 - 3x + 1$; [0,2]

$f'(x) = 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3(x-1)(x+1)$

$f(0) = 1$

$f(2) = 3$ (max)

$f(1) =$ -1 (min)

(6) 124. Use a linear approximation to approximate $\sqrt{81.1}$.

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$x = 81$

$y = 9$

$m = \frac{1}{18}$

$L(x) = \frac{1}{18}(x - 81) + 9$

$L(81.1) = 9 + \frac{1}{180}$

(6) 125. Let $f(x) = x^{17} - x^{12} + x^6 + x^3 - x + 2$. Show that there is a number c in the interval [0,1] such that $f'(c) = 1$.

***Proof*:** By the Mean Value Theorem, there is a number $c \in$ [0,1] such that $f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{3-2}{1} = 1$. ■

(6) 126. Calculate the following limit.

$$\lim_{t\to 0} \frac{1-\cos t}{\sin t} \overset{\text{L'H}}{=} \lim_{t\to 0} \frac{\sin t}{\cos t} = 0$$

Chapter 4: Math 1571 Spring 2008

Section 4.1: Exam 1

Exam 1 Math 1571 Spring 2008

(3) 127. Find the equation of the line passing through the points (-1,4) and (2,3).

$m =$ -$\frac{1}{3}$

$y - 3 =$ -$\frac{1}{3}(x - 2)$

128. For each of the following, give the domain and range. Use your answers to sketch the graph.

(6) a. $f(x)$

$$= \frac{2x^2 - 4x - 6}{x+1}$$

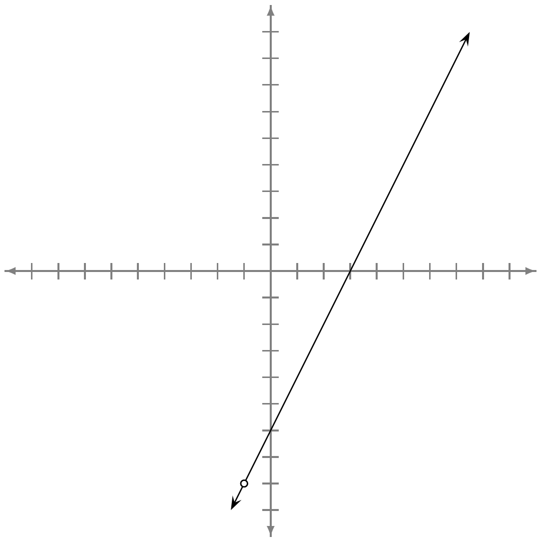
$$= \frac{2(x-3)(x+1)}{x+1}$$

$\neq 2(x-3)$

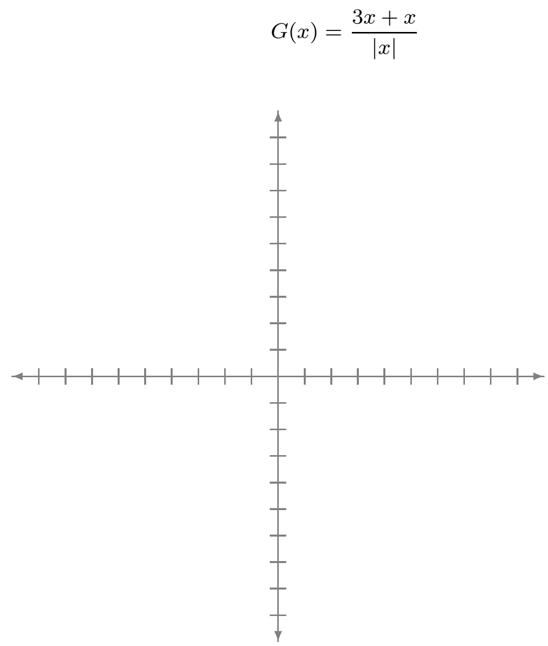
D: (-∞,-1) ∪ (-1,∞)

R: (-∞,-8) ∪ (-8,∞)

$$f(x) = \frac{2x^2 - 4x - 6}{x+1}$$



(Page 22: 39) **(6) b.** $G(x) = \dfrac{3x + x}{|x|}$



(5) 129. List the transformations (in a proper order) needed to transform the graph of $y = x^2$ into the graph of $y = \frac{1}{2}x^2 - 4x + 3$. Sketch the graph of $y = \frac{1}{2}x^2 - 4x + 3$.

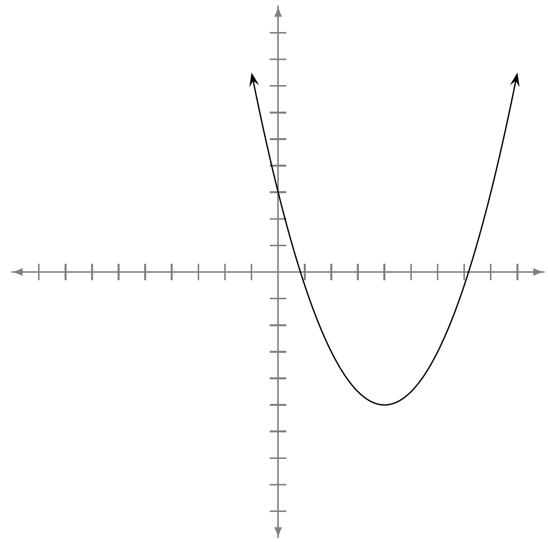
$y = \frac{1}{2}(x-4)^2 - 5$

Transformations:

Shift right 4 units.

Vertical compression by a factor of 2.

Shift down 5 units.



$y = \frac{1}{2}x^2 - 4x + 3$

(5) 130. Find the amplitude, period, phase shift, and average value of the following. Sketch the graph.

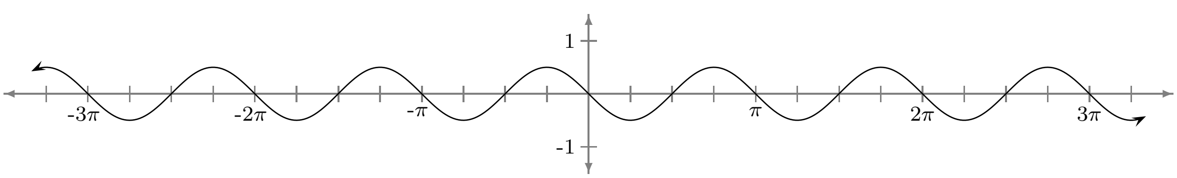
$y = \frac{1}{2}\cos\left(2x + \frac{\pi}{2}\right)$

Amplitude: $\frac{1}{2}$

Period: π

Phase Shift: $\frac{\pi}{4}$

Average Value: 0



(5) 131. Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$. Find $(f \circ g)(x)$. Give the domain and range of $(f \circ g)(x)$.

$(f \circ g)(x) = \sqrt{x^2 + 1}$

D: $\mathbb{R}$

R: $[1, \infty)$

(5) 132. Prove the following identity.

$\sin\theta + \cos\theta\cot\theta = \csc\theta$

Proof:

$\sin\theta + \cos\theta\cot\theta$

$= \sin\theta + \cos\theta\frac{\cos\theta}{\sin\theta}$

$= \sin\theta + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$= \csc\theta$

■

133. Note that $\lim\limits_{x\to 4}\big(2x+1\big) = 9$.

(5) a. Given $\varepsilon > 0$, find $\delta > 0$ such that $|(2x+1)-9| < \varepsilon$ whenever $|x-4| < \delta$.

$|(2x+1)-9| < \varepsilon$

$|2x-8| < \varepsilon$

$|2(x-4)| < \varepsilon$

$|x-4| < \frac{\varepsilon}{2}$

$\delta = \frac{\varepsilon}{2}$

(5) b. Use the $\varepsilon - \delta$ definition of limit to prove $\lim\limits_{x\to 4}\big(2x+1\big) = 9$.

Proof: Let $\varepsilon > 0$, be given and let $\delta = \frac{\varepsilon}{2}$. Suppose that $|x-4| < \delta$ and consider

$|x-4| < \delta$

$|x-4| < \frac{\varepsilon}{2}$

$2|x-4| < \varepsilon$

$|2x-8| < \varepsilon$

$|(2x+1)-9| < \varepsilon$

as desired. ■

(5) 134. Two cars leave the same place at 2:00 P.M. The first car travels north at 60 mph while the other car travels east at 45 mph. Express the distance between the cars as a function of time.

$d(t) = \sqrt{2025t^2 + 3600t^2} = 75t$

Exam 1 Math 1571 Spring 2008

Name:____________________________

E-mail (optional):____________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

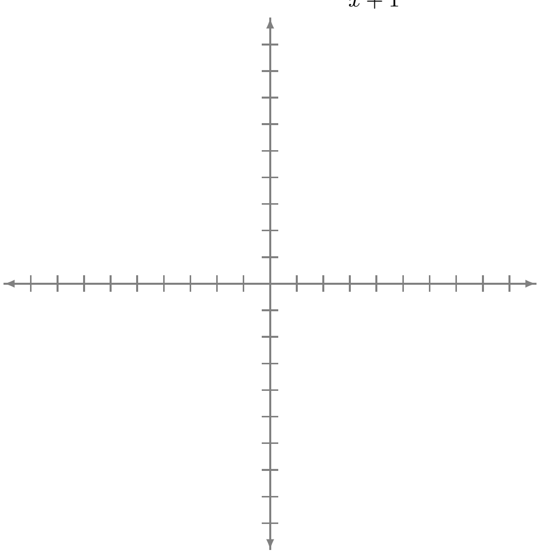
(3) 135. Find the equation of the line passing through the points (-1,4) and (2,3).

136. For each of the following, give the domain and range. Use your answers to sketch the graph.

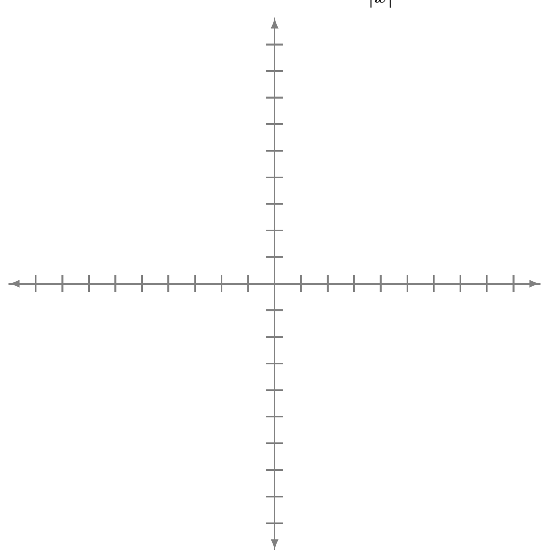
(6) a. $f(x) = f(x) = \dfrac{2x^2 - 4x - 6}{x + 1}$

(Page 22: 39) **(6) b.** $G(x) = \dfrac{3x + x}{|x|}$

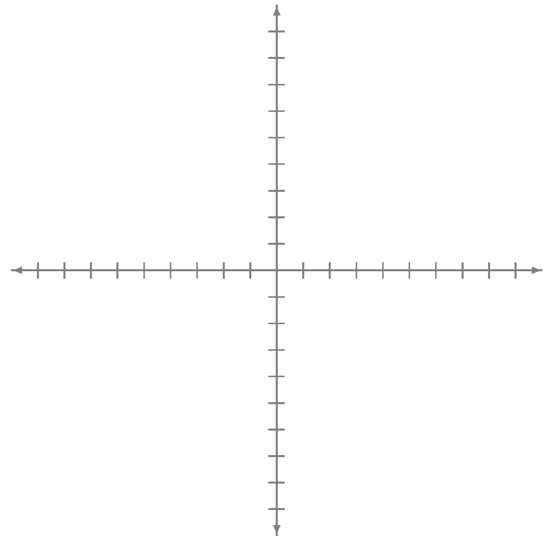
$f(x) = \dfrac{2x^2 - 4x - 6}{x + 1}$



$G(x) = \dfrac{3x + x}{|x|}$

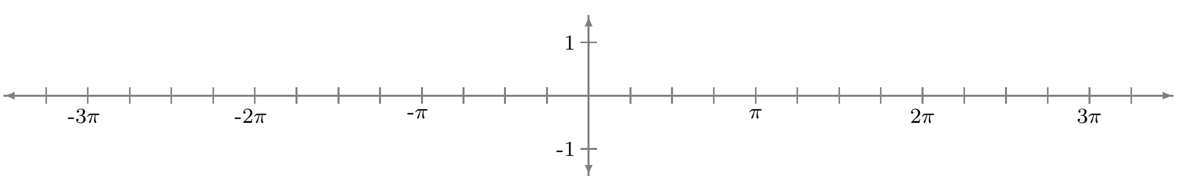


(5) 137. List the transformations (in a proper order) needed to transform the graph of $y = x^2$ into the graph of $y = \frac{1}{2}x^2 - 4x + 3$. Sketch the graph of $y = \frac{1}{2}x^2 - 4x + 3$.



(5) 138. Find the amplitude, period, phase shift, and average value of the following. Sketch the graph.

$y = \frac{1}{2}\cos\left(2x + \frac{\pi}{2}\right)$



(5) 139. Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$. Find $(f \circ g)(x)$. Give the domain and range of $(f \circ g)(x)$.

(5) 140. Prove the following identity.

$\sin\theta + \cos\theta\cot\theta = \csc\theta$

(5) 141. Two cars leave the same place at 2:00 P.M. The first car travels north at 60 mph while the other car travels east at 45 mph. Express the distance between the cars as a function of time.

142. Note that $\lim_{x\to 4}\big(2x+1\big) = 9$.

(5) a. Given $\varepsilon > 0$, find $\delta > 0$ such that $|(2x+1)-9| < \varepsilon$ whenever $|x-4| < \delta$.

(5) b. Use the $\varepsilon - \delta$ definition of limit to prove $\lim_{x\to 4}\big(2x+1\big) = 9$.

Section 4.2: Exam 2

Exam 2 Math 1571 Spring 2008

143. Calculate the following limits.

(5) a. $\lim_{x\to 3}\big(x^3 - 2x^2 + 1\big) = 10$

(5) b. $\lim_{x\to 1}\dfrac{x^3-1}{x-1}$

$= \lim_{x\to 1}\dfrac{(x-1)(x^2+x+1)}{x-1}$

$= \lim_{x\to 1}\big(x^2+x+1\big)$

$= 3$

(4) c. $\lim_{\theta\to 0}\dfrac{\sin 5\theta}{\theta}$

$= \lim_{\theta\to 0}\dfrac{5\sin 5\theta}{5\theta}$

$= 5\lim_{\theta\to 0}\dfrac{\sin 5\theta}{5\theta}$

$= 5\cdot 1$

$= 5$

(4) d. $\lim_{x\to 0^+} x^2\sqrt{|\sin\frac{1}{x}|}$

Since $0 \le x^2\sqrt{|\sin\frac{1}{x}|} \le x^2$, $\lim_{x\to 0^+} x^2\sqrt{|\sin\frac{1}{x}|} = 0$ by the Squeeze Theorem.

(5) 144. Is the following function continuous at 3?

$$f(x) = \begin{cases} x^2+1, & \text{if } x \le 3 \\ 2x+4, & \text{if } x > 3 \end{cases}$$

$f(3) = 10$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\big(x^2+1\big) = 10$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-}\big(2x+4\big) = 10$

$\lim_{x\to 3} f(x) = 10$

The function is continuous at 3.

145. Which of the following equations have solutions?

(4) a. $x^4 - 2x + 1 = 3$

Yes. Let $f(x) = x^4 - 2x + 1$ and note that f is continuous since it is a polynomial. Also, note that $f(0) = 1$ and $f(2) = 13$. Therefore, by the Intermediate Value Theorem, there is a number $c \in [0,2]$ such that $f(c) = 3$.

(4) b. $x^6 + |x| + 2 = 1$

Since $x^6 + |x| + 2 \ge 2$ for all x, there is no solution to the equation.

146. For each of the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

(5) a.

$$f(x) = \frac{2x^2+2x-12}{x^2-x-2}$$

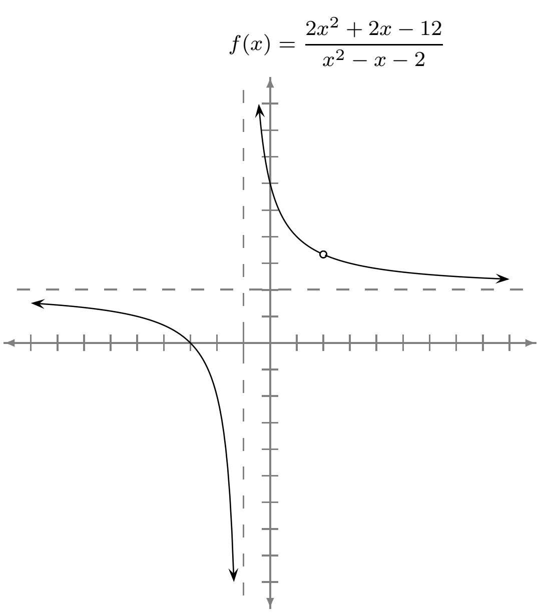
$$= \frac{2(x-2)(x+3)}{(x-2)(x+1)}$$

$$\neq \frac{2(x+3)}{(x+1)}$$

HA: $y = 2$

VA: $x =$ -1

OA: none



(5) b.

$f(x)$

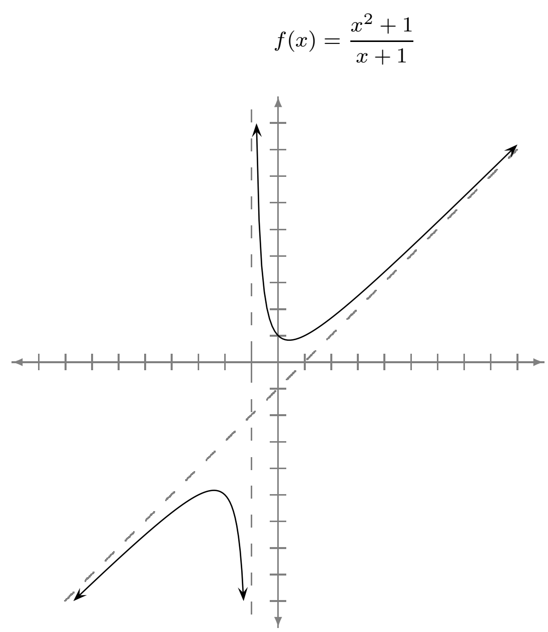
$$= \frac{x^2+1}{x+1}$$

$$= x - 1 + \frac{2}{x+1}$$

HA: none

VA: $x =$ -1

OA: $y = x - 1$



(4) 147. Use the definition of derivative to show that the derivative of $f(x) = x^2 + 2x + 3$ is $f'(x) = 2x + 2$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 3 - (x^2+2x+3)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 3 - x^2 - 2x - 3}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \big(2x + h + 2\big)$$

$$= 2x + 2$$

148. Differentiate.

(4) a. $f(x) = \dfrac{x^3+3x-1}{x^2+1}$

$$f'(x) = \frac{(3x^2+3)(x^2+1)-(x^3+3x-1)(2x)}{(x^2+1)^2}$$

(4) b. $g(x) = \sin^4 x$

$g'(x) = 4\sin^3 x \cdot \cos x$

Exam 2 Math 1571 Spring 2008

Name:__________________________

E-mail (optional):__________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

149. Calculate the following limits.

(5) a. $\lim_{x\to 3}\left(x^3 - 2x^2 + 1\right)$

(5) b. $\lim_{x\to 1}\frac{x^3-1}{x-1}$

(4) c. $\lim_{\theta\to 0}\frac{\sin 5\theta}{\theta}$

(4) d. $\lim_{x\to 0^+} x^2\sqrt{|\sin\frac{1}{x}|}$

(5) 150. Is the following function continuous at 3?

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x \leq 3 \\ 2x + 4, & \text{if } x > 3 \end{cases}$$

151. Which of the following equations have solutions?

(4) a. $x^4 - 2x + 1 = 3$

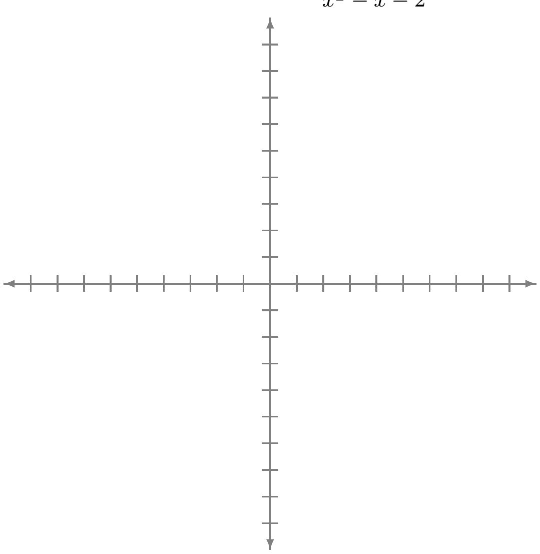
(4) b. $x^6 + |x| + 2 = 1$

152. For each of the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

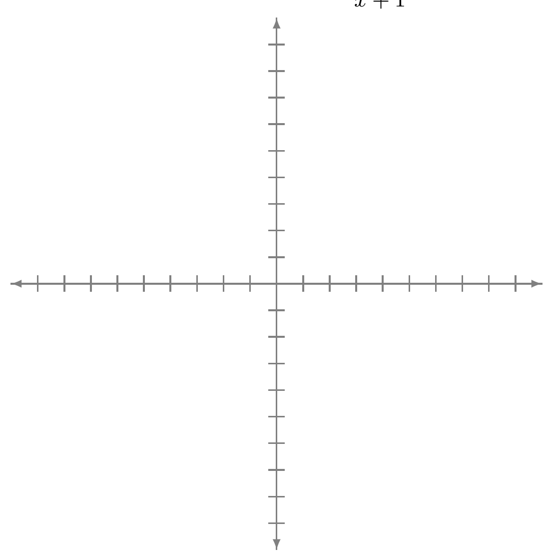
(5) a. $f(x) = \dfrac{2x^2 + 2x - 12}{x^2 - x - 2}$

(5) b. $f(x) = \dfrac{x^2 + 1}{x + 1}$

$f(x) = \dfrac{2x^2 + 2x - 12}{x^2 - x - 2}$



$f(x) = \dfrac{x^2 + 1}{x + 1}$



(4) 153. Use the definition of derivative to show that the derivative of $f(x) = x^2 + 2x + 3$ is $f'(x) = 2x + 2$.

154. Differentiate.

(4) a. $f(x) = \dfrac{x^3 + 3x - 1}{x^2 + 1}$

(4) b. $g(x) = \sin^4 x$

Section 4.3: Exam 3

Exam 3 Math 1571 Spring 2008

155. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(8) a. $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$

HA: none

VA: none

OA: none

$f'(x) = x^2 - x - 2 = (x - 2)(x + 1)$

Dec: [-1,2]

Inc: (-∞,-1], [2,∞)

Local max: $\frac{7}{6}$ at -1

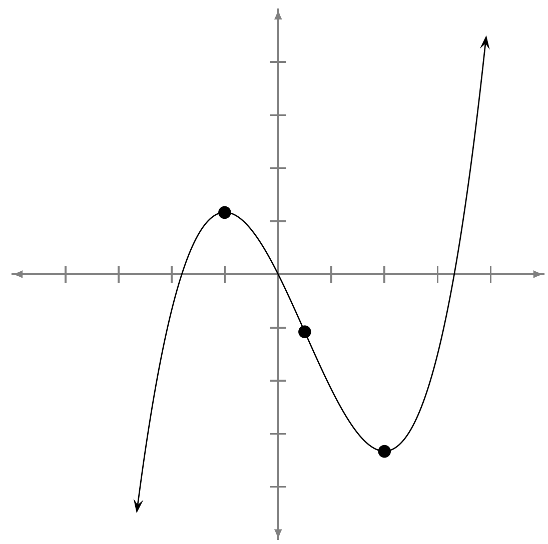
Local min: -$\frac{10}{3}$ at 2

$f''(x) = 2x - 1$

CD: (-∞,$\frac{1}{2}$)

CU: ($\frac{1}{2}$,∞)

IP: ($\frac{1}{2}$,-$\frac{13}{12}$)



$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$

(8) b. $f(x) = \dfrac{x^2 + 1}{x}$

$f(x) = x + \frac{1}{x}$

HA: none

VA: $x = 0$

OA: $y = x$

$f'(x) = 1 - \frac{1}{x^2} = \dfrac{x^2 - 1}{x^2}$

Dec: [-1,0), (0,1])

Inc: (-∞,-1], [1,∞)

Local max: -2 at -1

Local min: 2 at 1

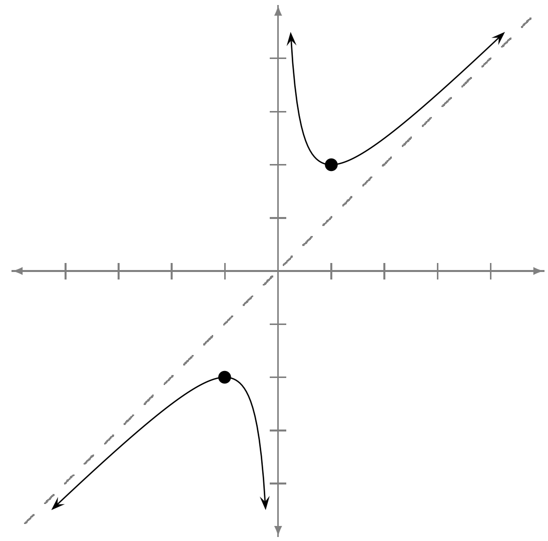
$f''(x) = \frac{2}{x^3}$

CD: (-∞,0)

CU: (0,∞)

IP: none

$f(x) = \dfrac{x^2+1}{x}$



156. A ball is thrown upward with an initial velocity of of 48 ft/sec from a height of 64 ft.

(3) a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 48t + 64$

(3) b. When does the ball reach its maximum height?

$v(t) = h'(t) = \text{-}32t + 48 = \text{-}16(2t - 3)$

$t = \frac{3}{2}$

$\frac{3}{2}$ sec.

(3) c. Find the maximum height of the ball.

$h\left(\frac{3}{2}\right) = 100$

100 ft

(3) d. When does the ball hit the ground?

$\text{-}16t^2 + 48t + 64 = 0$

$\text{-}16(t^2 - 3t - 4) = 0$

$(t - 4)(t + 1) = 0$

$t = 4$

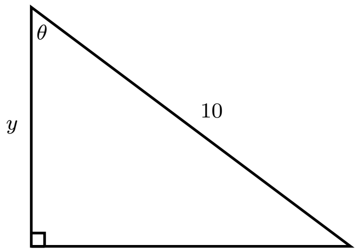
4 sec.

(3) e. Find the velocity of the ball when it hits the ground.

$v(4) = $ -80

-80 ft/sec.

(5) 157. A 10-foot ladder is being pulled away from a wall in such a way that the top of the ladder is sliding down the wall at a rate of 4 ft/sec. At what rate is the angle between the ladder and the wall changing when the top of the ladder is 6 feet above the floor?



$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$

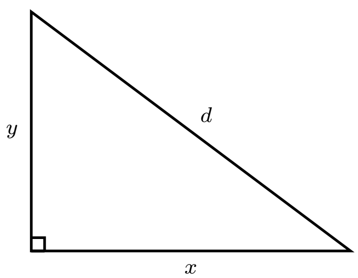
$-\frac{8}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10}(-4)$

$\frac{d\theta}{dt} = \frac{1}{2}$

$\frac{1}{2}$ radians per second

$\cos\theta = \frac{y}{10}$

(5) 158. Two ants on the top of a rectangular table are walking toward the same corner on the edges of two different sides. The ants are both walking at a rate of 1 ft per minute. At what rate is the distance between them changing when one ant is 3 feet from the corner and the other is 4 ft from the corner?



$x^2 + y^2 = d^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

$2(3)(-1) + 2(4)(-1) = (2)(5)\frac{dd}{dt}$

$\frac{dd}{dt} = -1.4$

1.4 ft/min.

(5) 159. Find the absolute maximum and absolute minimum of the function $f(x) = x + \cos x$ on the interval $[0,\pi]$.

$f'(x) = 1 - \sin x$

$f(0) = 1$ (min)

$f(\pi) = \pi - 1$ (max)

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

(5) 160. Find the point on the line $y = x + 1$ that is closest to the origin.

Recall: The distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Hint: Minimize the square of the distance on the interval [-1,1].

The distance between a point $(x,x+1)$ on the line $y = x+1$ and the origin is $\sqrt{x^2 + (x+1)^2} = \sqrt{2x^2 + 2x + 1}$.

Let $d(x) = 2x^2 + 2x + 1$.

$d'(x) = 4x + 2$

$d(\text{-}1) = 1$

$d(1) = 1$

$d\left(\text{-}\frac{1}{2}\right) = \frac{1}{2}$

Point: $\left(\text{-}\frac{1}{2}, \frac{1}{2}\right)$

(5) 161. Use the linearization of $f(x) = \sqrt[3]{x}$ at 8 to approximate $\sqrt[3]{8.01}$.

$f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

$f'(8) = \frac{1}{12}$

$L(x) = \frac{1}{12}(x-8) + 2$

$L(8.01) = 2 + \frac{1}{1200}$

(4) 162. Consider the function $f(x) = x^3 - 3x^2 + 7$ on the interval [1,3]. What does the Mean Value Theorem guarantee?

There is a number $c \in (1,3)$ such that $f'(c) = 1$.

Exam 2 Math 1571 Spring 2008

Name:________________________

E-mail (optional):________________________

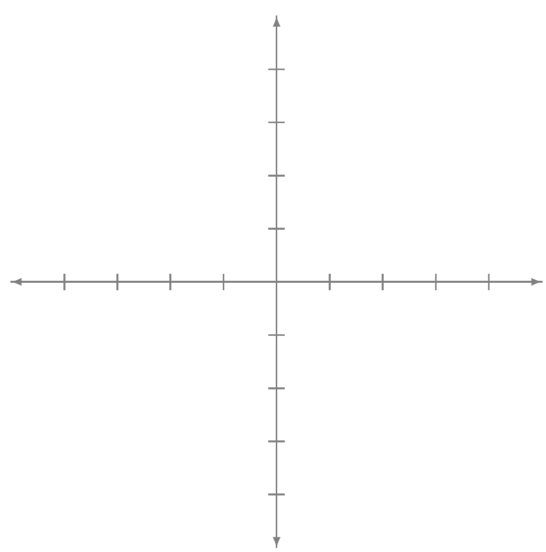
<u>**Directions:**</u> Show all of your work and justify all of your answers. An answer without justification will receive a zero.
Note 1: The final exam is Monday May 5 at 3:00 p.m. in DeBartolo 358.

163. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

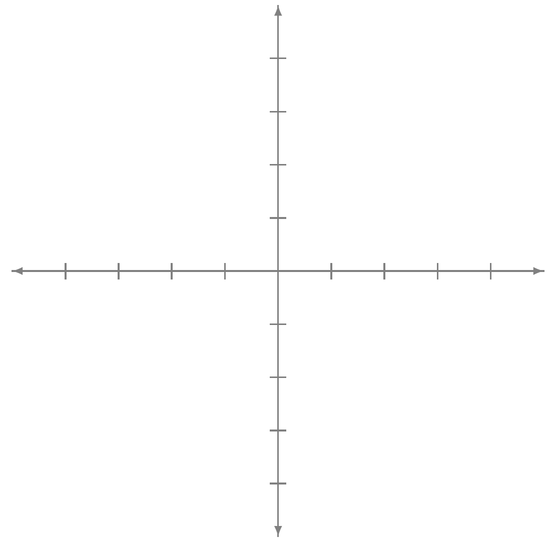
(8) a. $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$

$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$

(8) b. $f(x) = \dfrac{x^2+1}{x}$



$$f(x) = \frac{x^2+1}{x}$$



164. A ball is thrown upward with an initial velocity of of 48 ft/sec from a height of 64 ft.

(3) a. Give the height of the ball as a function of time.

(3) b. When does the ball reach its maximum height?

(3) c. Find the maximum height of the ball.

(3) d. When does the ball hit the ground?

(3) e. Find the velocity of the ball when it hits the ground.

(5) 165. A 10-foot ladder is being pulled away from a wall in such a way that the top of the ladder is sliding down the wall at a rate of 4 ft/sec. At what rate is the angle between the ladder and the wall changing when the top of the ladder is 6 feet above the floor?

(5) 166. Two ants on the top of a rectangular table are walking toward the same corner on the edges of two different sides. The ants are both walking at a rate of 1 ft per minute. At what rate is the distance between them changing when one ant is 3 feet from the corner and the other is 4 ft from the corner?

(5) 167. Find the absolute maximum and absolute minimum of the function $f(x) = x + \cos x$ on the interval $[0,\pi]$.

(5) 168. Find the point on the line $y = x + 1$ that is closest to the origin.

Recall: The distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Hint: Minimize the square of the distance on the interval [-1,1].

(5) 169. Use the linearization of $f(x) = \sqrt[3]{x}$ at 8 to approximate $\sqrt[3]{8.01}$.

(4) 170. Consider the function $f(x) = x^3 - 3x^2 + 7$ on the interval [1,3]. What does the Mean Value Theorem guarantee?

Chapter 5: Math 1571 Fall 2011

Section 5.1: Quiz

Quiz 1 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Give the equation of the line passing through the points (1,6) and (3,4).

$m =$ -1

$y - 6 =$ -$(x - 1)$

$y =$ -$x + 7$

(1) 2. Give the equation of the quadratic function whose graph has vertex (-1,-4), x-intercepts (-3,0) and (1,0), and y-intercept (0,-3).

$(x + 3)(x - 1) = x^2 + 2x - 3$

Note that $f(x) = x^2 + 2x - 3$ has the desired properties.

Quiz 2 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \to 2} \big(5x + 2\big) = 12$.

First, we need to find δ for a given ε.

$|(5x + 2) - 12| < \varepsilon$

$|5x - 10| < \varepsilon$

$5|x - 2| < \varepsilon$

$|x - 2| < \frac{\varepsilon}{5}$

***Proof*:** Let $\varepsilon > 0$ be given. Let $\delta = \frac{\varepsilon}{5}$. If $|x - 2| < \delta$, then

$|x - 2| < \delta$

$|x - 2| < \frac{\varepsilon}{5}$

$5|x - 2| < \varepsilon$

$|5x - 10| < \varepsilon$

$|5x + 2 - 12| < \varepsilon$

$|(5x + 2) - 12| < \varepsilon.$

Therefore, $\lim_{x \to 2} \big(5x + 2\big) = 12$. ■

Quiz 3 Name:________________________

Directions: Show all of your work and justify all of your answers.

Compute the following limits.

(1) 1. $\lim_{x\to 5} \dfrac{x^2 - 3x - 10}{x^2 - 11x + 30}$

$= \lim_{x\to 5} \dfrac{(x+2)(x-5)}{(x-6)(x-5)}$

$= \lim_{x\to 5} \dfrac{x+2}{x-6}$

= -7

(1) 2. $\lim_{x\to\sqrt{3}} \dfrac{x^2 - 3}{x - \sqrt{3}}$

$= \lim_{x\to\sqrt{3}} \dfrac{(x+\sqrt{3})(x-\sqrt{3})}{x-\sqrt{3}}$

$= \lim_{x\to\sqrt{3}} \left(x + \sqrt{3}\right)$

$= 2\sqrt{3}$

Quiz 4 Name:______________________________

Directions: Show all of your work and justify all of your answers.

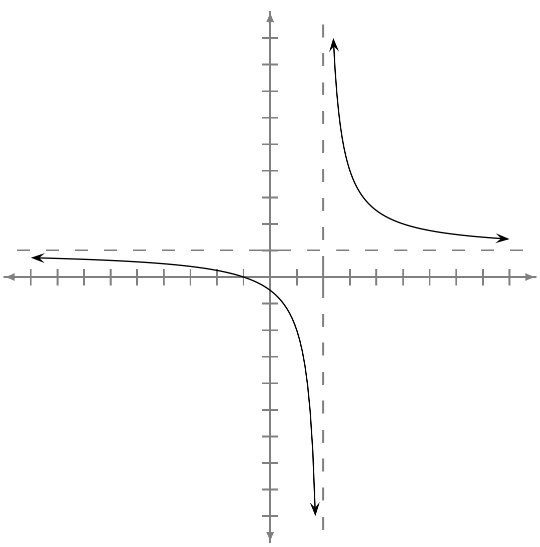
1. For each of the following, find all vertical and horizontal asymptotes of the graph of the given function. Use your answers to sketch the graph.

(1) a. $f(x) = \dfrac{x+1}{x-2}$

HA: $y = 1$

VA: $x = 2$

$f(x) = \dfrac{x+1}{x-2}$



(1) b. $f(x) = \dfrac{x^2-2x-3}{x^2-5x+6}$

$$f(x) = \frac{x^2-2x-3}{x^2-5x+6} = \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$\lim_{x\to\infty} \frac{x^2-2x-3}{x^2-5x+6} = 1$$

$$\lim_{x\to-\infty} \frac{x^2-2x-3}{x^2-5x+6} = 1$$

HA: $y = 1$

$$\lim_{x\to 2^-} \frac{x^2-2x-3}{x^2-5x+6}$$

$$= \lim_{x\to 2^-} \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2^-} \frac{x+1}{x-2}$$

$$= -\infty$$

$$\lim_{x\to 2^+} \frac{x^2-2x-3}{x^2-5x+6}$$

$$= \lim_{x\to 2^+} \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2^+} \frac{x+1}{x-2}$$

$$= \infty$$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-5x+6}$$

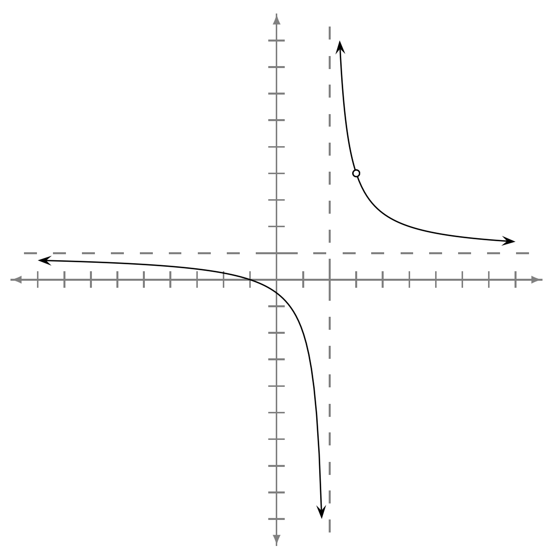
$$= \lim_{x\to 3} \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x\to 3} \frac{x+1}{x-2}$$

$$= 4$$

VA: $x = 2$

$f(x) = \dfrac{x^2-2x-3}{x^2-5x+6}$



Quiz 5 Name:________________________

<u>Directions:</u> Show all of your work and justify all of your answers.

(1) 1. Find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

$g(x) = \dfrac{x^2 - 4}{x}$

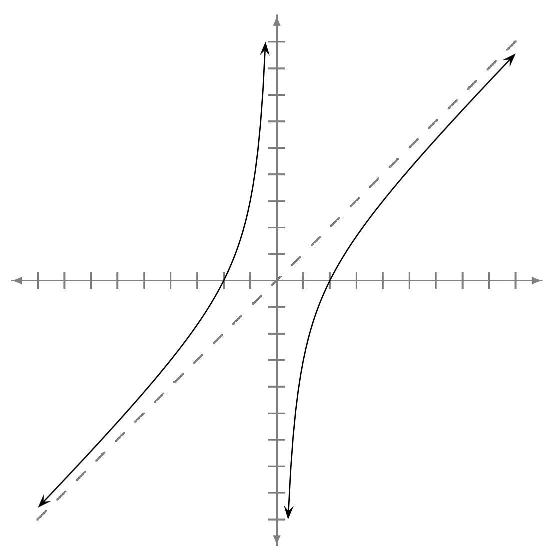
$g(x) = \dfrac{x^2 - 4}{x} = x - \frac{4}{x}$

HA: none

VA: $x = 0$

$g(x) = x - \frac{4}{x}$

OA: $y = x$



Quiz 6 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Use the definition of derivative to differentiate the following function.

$k(x) = x^2 - x + 1$

$$\lim_{h \to 0} \frac{k(x+h) - k(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - (x+h) + 1 - (x^2 - x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h + 1 - x^2 + x - 1}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 1)}{h}$$

$$= \lim_{h \to 0} \big(2x + h - 1\big)$$

$$= 2x - 1$$

Quiz 7 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Differentiate each of the following.

a. $f(x) = x^5 - 22x^3 + 17x + 1$

$f'(x) = 5x^4 - 66x^2 + 17$

b. $h(x) = \dfrac{x^2 + 3x}{4x + 1}$

(1) 2. Find a function whose derivative is $g'(x) = (x^5 + x^4 + 10)^3(5x^4 + 4x^3)$.

$g(x) = \frac{1}{4}(x^5 + x^4 + 10)^4$

Quiz 8 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For the following, determine whether the function is odd, even, or neither. Also, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase, intervals of decrease, and local extrema. Use your answers to sketch the graph.

$g(x)$

$$= \frac{x^2+4x+8}{x+2}$$

$$= x+2+\frac{4}{x+2}$$

$$= x+2+4(x+2)^{-1}$$

$g(1) = \frac{13}{3}$

$g(-1) = 3$

Neither even nor odd.

HA: none

$$\lim_{x\to -2^-} \frac{x^2}{x+2}+4 = -\infty$$

$$\lim_{x\to -2^+} \frac{x^2}{x+2}+4 = \infty$$

VA: $x = -2$

$$g(x) = \frac{x^2}{x+2}+4 = x+2+\tfrac{4}{x+2}$$

OA: $y = x+2$

$g(x) = x+2+4(x+2)^{-1}$

$g'(x)$

$$= 1-4(x+2)^{-2}$$

$$= 1-\frac{4}{(x+2)^2}$$

$$= \frac{(x+2)^2-4}{(x+2)^2}$$

$$= \frac{(x^2+4x+4)-4}{(x+2)^2}$$

$$= \frac{x^2+4x}{(x+2)^2}$$

$$= \frac{x(x+4)}{(x+2)^2}$$

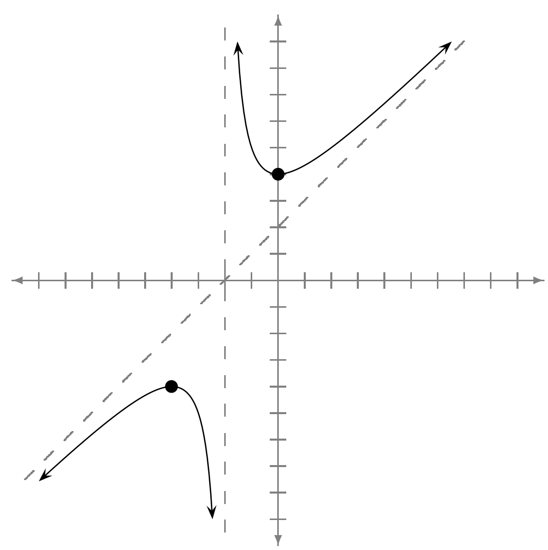
Dec: [-4,-2), (-2,0]

Inc: $(-\infty,-4]$, $[0,\infty)$

Local max: -4 at -4

Local min: 4 at 0

$$g(x) = \frac{x^2+4x+8}{x+2}$$



Quiz 9 Name:________________________

Directions: Show all of your work and justify all of your answers.

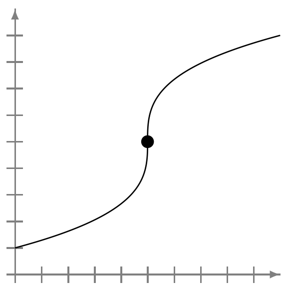
(2) 1. Suppose that f is a differentiable function and $f'(x) = 5^x$. Find the following limit.

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = f'(2) = 5^2 = 25$$

Quiz 10 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Below is the graph of a function that represents the profit of marketing a certain product. The time in months is represented by the x-axis and the profit measured in tens of thousands of dollars is represented by the y-axis. You were in charge of marketing the product for the first five months before moving on to another project. For the past five months, the product has been marketed by your colleague Bill. The solid circle on the graph represents the point at which Bill took charge. You and Bill have both applied for the same promotion which will result in a pay raise of $8000. Your supervisor (also Bill's supervisor) has asked both of you to a meeting. The following is the dialogue of the beginning of the meeting. Give your response.



Supervisor to Bill: What effects has your marketing strategy had on the profits?

Bill: As you can clearly see, the profits have increased as a result of my marketing strategy.

Supervisor to Bill: Maybe the profits have increased as result of being on the market longer. Some of the sales are the result of repeat business from consumers who began using the product in the first five months. After all, the profits have been increasing the entire time that the product has been on the market.

Bill: Yes. I agree that profits have always increased. But since I have taken charge of marketing the product, the rate at which the profits have increased has also increased. This is clearly illustrated by the graph.

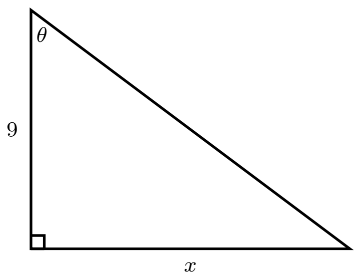
Supervisor: Yes. It would appear so.

You: Bill is wrong. The rate of change of the profit function is given by the derivative. The rate of change of the derivative is given by the second derivative. Since the function changes from concave upward to concave downward at the point where Bill took charge of marketing, the first derivative which measures the rate of change of the profit function has changed from increasing to decreasing. So even though profits are still increasing, the rate at which they are increasing has been decreasing since Bill has been in charge of marketing.

Quiz 11 Name:_______________________

Directions: Show all of your work and justify all of your answers.

(2) 1. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is running away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?



$\tan\theta = \frac{x}{9}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{1}{250}$

$\frac{1}{250}$ radians per second

Quiz 12 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. A gardener has 100 ft of fence with which to enclose a rectangular garden. Show that the maximum area is obtained when all sides are of the same length.

$A(x) = x\left[\frac{1}{2}(100 - 2x)\right] = x(50 - x) = -x^2 + 50x;\ 0 \le x \le 50$

$A'(x) = -2x + 50 = -2(x - 25)$

$A(0) = 0$

$A(50) = 0$

$A(25) = 625$

Quiz 13 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(Page 213: 27) **(1) 1.**

(1) 2. Prove that for $x > 0$, $\sqrt{1+x} < 1 + \frac{1}{2}x$.

Quiz 14 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Compute the following sum.

$$\sum_{i=1}^{10}(2i+1)(i-3)$$

$$= \sum_{i=1}^{10}(2i^2-5i-3)$$

$$= \sum_{i=1}^{10} 2i^2 + \sum_{i=1}^{10} -5i + \sum_{i=1}^{10} -3$$

$$= 2\sum_{i=1}^{10} i^2 - 5\sum_{i=1}^{10} i + \sum_{i=1}^{10} -3$$

$$= 2\left(\frac{10\cdot 11\cdot 21}{6}\right) - 5\left(\frac{10\cdot 11}{2}\right) - 30$$

$$= 770 - 275 - 30$$

$$= 465$$

Quiz 15 Name:_________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Use the definition of integral to evaluate the following.

$$\int_0^1 \left(x^2+x+1\right)dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\tfrac{i}{n}\right) \cdot \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\tfrac{i}{n}\right)^2 + \tfrac{i}{n} + 1\right] \cdot \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{i^2}{n^2} + \tfrac{i}{n} + 1\right) \cdot \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{i^2}{n^3} + \tfrac{i}{n^2} + \tfrac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \tfrac{i^2}{n^3} + \sum_{i=1}^{n} \tfrac{i}{n^2} + \sum_{i=1}^{n} \tfrac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{1}{n^3} \sum_{i=1}^{n} i^2 + \tfrac{1}{n^2} \sum_{i=1}^{n} i + \tfrac{1}{n} \sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{1}{n^3} \cdot \tfrac{n(n+1)(2n+1)}{6} + \tfrac{1}{n^2} \sum_{i=1}^{n} \tfrac{n(n+1)}{2} + 1\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{2n^3+3n^2+n}{6n^3} + \tfrac{n^2+n}{2n^2} + 1\right)$$

$$= \tfrac{1}{3} + \tfrac{1}{2} + 1$$

$$= \tfrac{11}{6}$$

Quiz 16 Name:____________________________

<u>Directions:</u> Show all of your work and justify all of your answers.

1. Integrate.

(1) a. $\displaystyle\int_0^2 \left(x^3 + x + 2\right) dx$

(1) b. $\displaystyle\int \left(2x - \sin x\right) dx$

Section 5.2: Exam 1

Exam 1 Math 1571 Fall 2011

Name:____________________________

<u>Directions:</u> Show all of your work and justify all of your answers. An answer without justification will receive a zero.

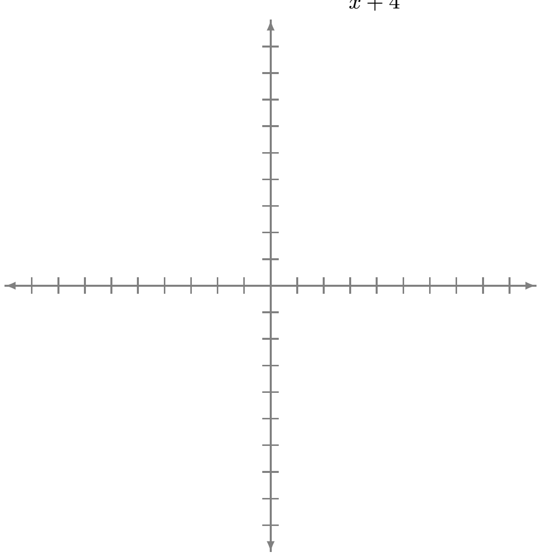
2. Let $f(x) = 3x^2 - 2x + 7$. Calculate f(-2).

3. For each of the following, find the domain and range and sketch the graph.

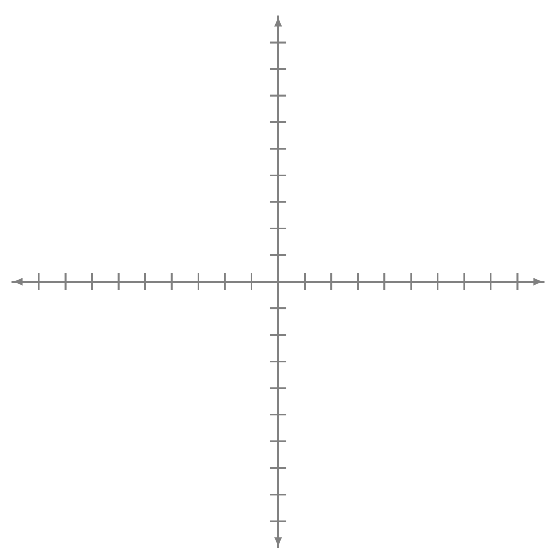
(5) a. $f(x) = \dfrac{x^2 + 3x - 4}{x + 4}$

(5) b. $g(x) = \begin{cases} x^2 - 2, & \text{if } x \le 0; \\ x - 2, & \text{if } x > 0. \end{cases}$

$f(x) = \dfrac{x^2 + 3x - 4}{x + 4}$

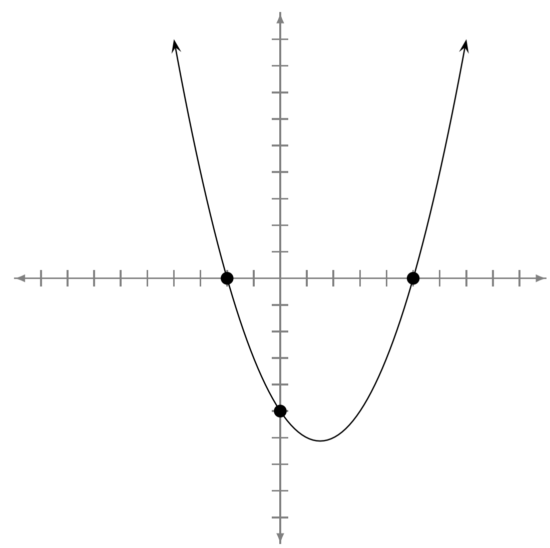


$y = g(x)$



(5) 4. An inspector in a factory earns \$10.00 per hour and works 8 hours per day. The inspector's quota (number of boxes that must be inspected) is 300 boxes. For every box the inspector inspects in excess of 300, he earns \$0.15. Express the inspector's daily wages as a function of the number of boxes he inspects.

(5) 5. Find an expression for the quadratic function with the given graph.



(5) 6. Note that $\lim_{x\to 4}\big(2x-1\big) = 7$.

Given $\varepsilon > 0$, find $\delta > 0$ such that $|(2x-1)-7| < \varepsilon$ whenever $|x-4| < \delta$.

7. Compute the following limits.

(5) a. $\lim_{x\to 1}\left(x^2+3x-1\right)$

(5) b. $\lim_{x\to 5}\dfrac{x^2-2x-15}{x^2-4x-5}$

(5) c. $\lim_{x\to 0}\dfrac{\sin 5x}{x}$

(5) 8. State the Squeeze Theorem.

(5) 9. Use the Intermediate Value Theorem to show that there is a number θ between 0 and $\frac{\pi}{2}$ such that $\sin\theta = \frac{99}{100}$.

10. Let $f(x) = 3x^2 - 2x + 7$. Calculate f(-2).

f(-2) = 23

11. For each of the following, find the domain and range and sketch the graph.

(5) a. $f(x)$

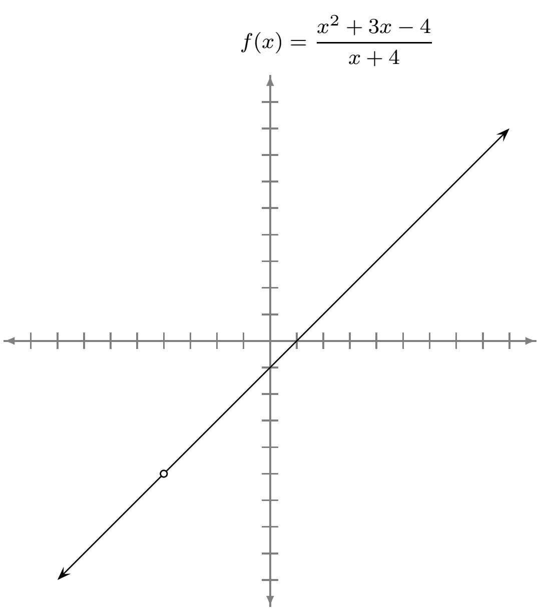
$$= \frac{x^2 + 3x - 4}{x + 4}$$

$$= \frac{(x+4)(x-1)}{x+4}$$

$\neq x - 1$

D: (-∞,-4) ∪ (-4,∞)

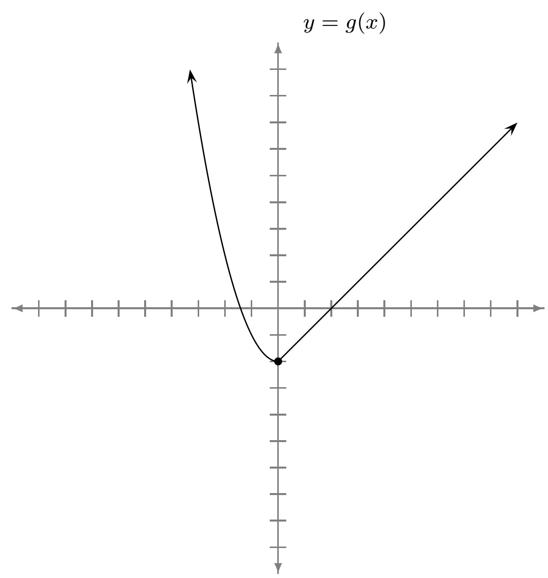
R: (-∞,-5) ∪ (-5,∞)



(5) b. $g(x) = \begin{cases} x^2 - 2, & \text{if } x \leq 0; \\ x - 2, & \text{if } x > 0. \end{cases}$

D: $\mathbb{R}$

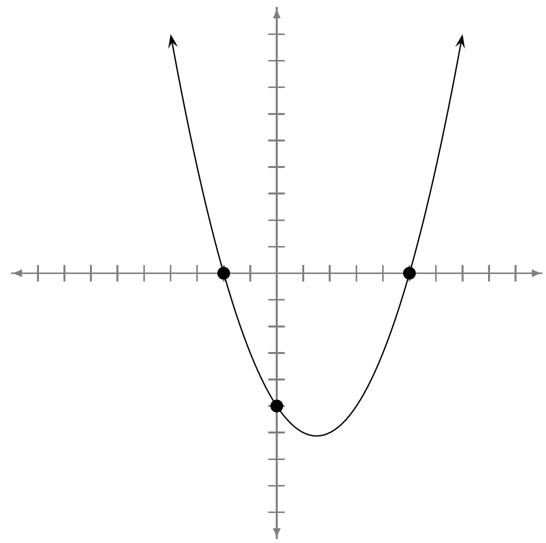
R: [-2,∞)



(5) 12. An inspector in a factory earns \$10.00 per hour and works 8 hours per day. The inspector's quota (number of boxes that must be inspected) is 300 boxes. For every box the inspector inspects in excess of 300, he earns \$0.15. Express the inspector's daily wages as a function of the number of boxes he inspects.

$f(x) = 80 + 0.15(x - 300)$

(5) 13. Find an expression for the quadratic function with the given graph.



Note that the roots are -2 and 5. So $x+2$ and $x-5$ are factors.

$(x+2)(x-5) = x^2 - 3x - 10$

Since the y-intercept is -5, the function is $f(x) = \frac{1}{2}(x+2)(x-5) = \frac{1}{2}x^2 - \frac{3}{2}x - 5$.

(5) 14. Note that $\lim\limits_{x \to 4}\big(2x-1\big) = 7$.

Given $\varepsilon > 0$, find $\delta > 0$ such that $|(2x-1)-7| < \varepsilon$ whenever $|x-4| < \delta$.

$|(2x-1)-7| < \varepsilon$

$|2x-8| < \varepsilon$

$|2(x-4)| < \varepsilon$

$|x-4| < \frac{\varepsilon}{2}$

$\delta = \frac{\varepsilon}{2}$

15. Compute the following limits.

(5) a. $\lim\limits_{x \to 1}\big(x^2 + 3x - 1\big) = 3$

(5) b. $\lim\limits_{x \to 5} \dfrac{x^2 - 2x - 15}{x^2 - 4x - 5}$

$= \lim\limits_{x \to 5} \dfrac{(x+3)(x-5)}{(x+1)(x-5)}$

$= \lim\limits_{x \to 5} \dfrac{x+3}{x+1}$

$= \dfrac{4}{3}$

(5) c. $\lim\limits_{x \to 0} \dfrac{\sin 5x}{x}$

$= \lim\limits_{x \to 0} \left(\dfrac{\sin 5x}{x} \cdot \dfrac{5x}{5x} \right)$

$= \lim\limits_{x \to 0} \left(\dfrac{\sin 5x}{5x} \cdot \dfrac{5x}{x} \right)$

$= 1 \cdot 5$

$= 5$

(5) 16. State the Squeeze Theorem.

(5) 17. Use the Intermediate Value Theorem to show that there is a number θ between 0 and $\frac{\pi}{2}$ such that $\sin\theta = \frac{99}{100}$.

Proof: Recall that sin is a continuous function. Since $\sin 0 = 0 < \frac{99}{100} < 1 = \sin\frac{\pi}{2}$, the Intermediate Value Theorem implies that there is a number θ between 0 and $\frac{\pi}{2}$ such that $\sin\theta = \frac{99}{100}$. ■

Section 5.3: Exam 2

Exam 2 Math 1571 Fall 2011

Name:__________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(2) 18. Use the definition of derivative to differentiate $f(x) = x^3 + 1$.

(2) 19. Calculate the following limit.

$$\lim_{x\to 1} \frac{x^{1000} + x^5 - 2}{x - 1}$$

20. Differentiate.

(2) a. $f(x) = x^7 + 3x^4 + x^2 + 3$

(2) b. $g(x) = \sqrt{x^4 + 1}$

(2) c. $h(x) = \dfrac{x+1}{\sqrt{x}}$

(2) d. $f(x) = \sin x(x^5 + x^3 + x)$

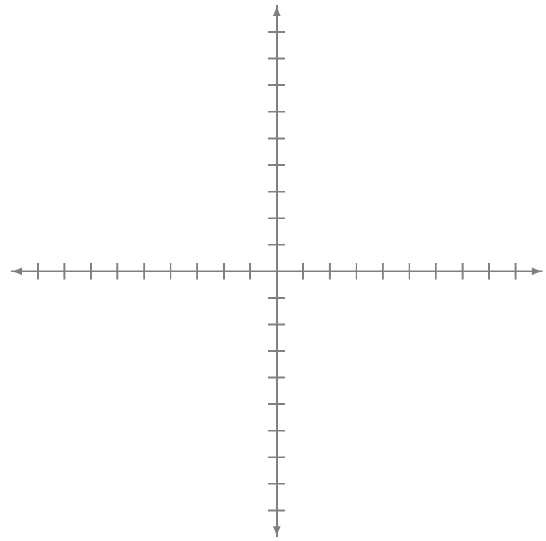
21. For each of the following, find the equation of the line that is tangent to the graph of the given function at the given point.

(2) a. $f(x) = \tan x$; $x = \frac{\pi}{3}$

(2) b. $x^2y^3 + 3y^2 - 2 = 0$; (1,-1)

(4) 22. For the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase, intervals of decrease, and local extrema. Use your answers to sketch the graph.

$$f(x) = \frac{x^2 + 2x + 1}{x + 2}$$



(2) 23. Use the definition of derivative to differentiate $f(x) = x^3 + 1$.

$$\lim_{h \to 0} \left[f(x+h) - f(x) \right] \tfrac{1}{h}$$

$$= \lim_{h \to 0} \left[(x+h)^3 + 1 - (x^3 + 1) \right] \tfrac{1}{h}$$

$$= \lim_{h \to 0} \left[x^3 + 3x^2h + 3xh^2 + h^3 + 1 - x^3 - 1 \right] \tfrac{1}{h}$$

$$= \lim_{h \to 0} \left[3x^2h + 3xh^2 + h^3 \right] \tfrac{1}{h}$$

$$= \lim_{h \to 0} \left[3x^2 + 3xh + h^2 \right]$$

$$= 3x^2$$

(2) 24. Calculate the following limit.

$$\lim_{x \to 1} \frac{x^{1000} + x^5 - 2}{x - 1}$$

$$f(x) = x^{1000} + x^5$$

$$f'(x) = 1000x^{999} + 5x^4$$

$$\lim_{x \to 1} \frac{x^{1000} + x^5 - 2}{x - 1} = f'(1) = 1005$$

25. Differentiate.

(2) a. $f(x) = x^7 + 3x^4 + x^2 + 3$

$$f'(x) = 7x^6 + 12x^3 + 2x$$

(2) b. $g(x) = \sqrt{x^4 + 1}$

$$g(x) = (x^4 + 1)^{\frac{1}{2}}$$

$$g'(x) = \tfrac{1}{2}(x^4 + 1)^{-\frac{1}{2}}(4x^3) = \frac{4x^3}{2\sqrt{x^4 + 1}}$$

(2) c. $h(x) = \dfrac{x+1}{\sqrt{x}}$

$h(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

$h(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$h'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$

(2) d. $f(x) = \sin x(x^5 + x^3 + x)$

$f'(x) = \cos x(x^5 + x^3 + x) + \sin x(5x^4 + 3x^2 + 1)$

26. For each of the following, find the equation of the line that is tangent to the graph of the given function at the given point.

(2) a. $f(x) = \tan x;\ x = \frac{\pi}{3}$

$f'(x) = \sec^2 x$

$y = f\left(\frac{\pi}{3}\right) = \sqrt{3}$

$m = f'\left(\frac{\pi}{3}\right) = 4 >$

$y - \sqrt{3} = 4\left(x - \frac{\pi}{3}\right)$

(2) b. $x^2y^3 + 3y^2 - 2 = 0$; (1,-1)

$\frac{d}{dx}\left(x^2y^3 + 3y^2 - 2\right) = \frac{d}{dx}0$

$2xy^3 + 3x^2y^2y' + 6yy' = 0$

$$y' = \frac{\text{-}2xy^3}{3x^2y^2 + 6y}$$

$x = 1$

$y = $ -1

$y' = $ -$\frac{2}{3}$

$y + 1 = $ -$\frac{2}{3}(x-1)$

(4) 27. For the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase, intervals of decrease, and local extrema. Use your answers to sketch the graph.

$f(x)$

$$= \frac{x^2 + 2x + 1}{x + 2}$$

$$= \frac{(x+1)(x+1)}{x+2}$$

$= x + \frac{1}{x+2}$

HA: none

VA: $x = $ -2

OA: $y = x$

$f'(x) =$

$1 - (x+2)^{\text{-}2}$

$$= \frac{x^2 + 4x + 3}{(x+2)^2}$$

$$= \frac{(x+3)(x+1)}{(x+2)^2}$$

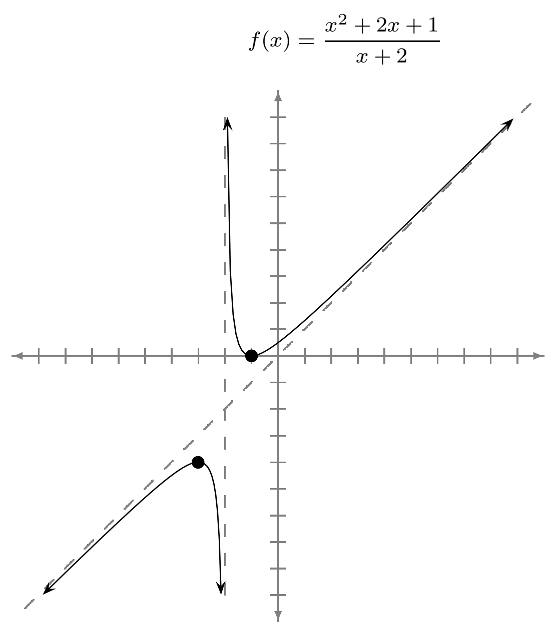
Total Points: 1058

Dec: [-3,-2), (-2,-1])

Inc: (-∞,-3], [-1,∞)

Local max: -4 at -3

Local min: 0 at -1



Section 5.4: Exam 2a

Exam 2 Math 1571 Fall 2011

Name:__________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(2) 28. Use the definition of derivative to differentiate $f(x) = \sqrt{x^2+1}$.

(2) 29. Calculate the following limit.

$$\lim_{\theta \to \frac{\pi}{3}} \frac{\sin\theta - \frac{\sqrt{3}}{2}}{\theta - \frac{\pi}{3}}$$

30. Differentiate.

(2) a. $f(x) = \tan^6 x$

(2) b. $g(x) = \sqrt{\sin x}$

(2) c. $h(x) = \left(\dfrac{x^3 - 2x^2 + 1}{\sqrt{x}}\right)^5$

(2) d. $f(x) = x^5 + x^3 + 4x$

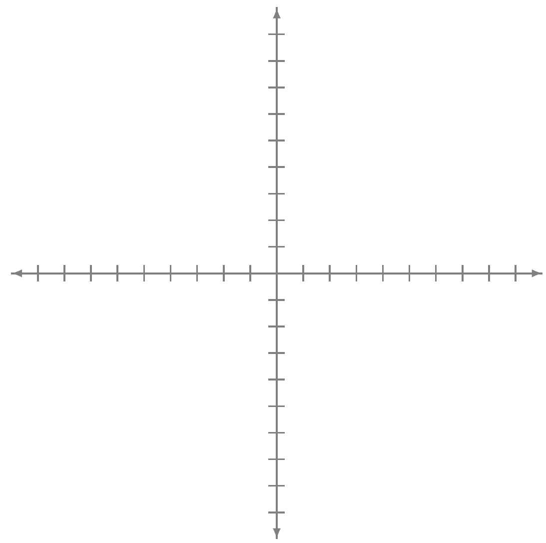
31. For each of the following, find the equation of the line that is tangent to the graph of the given function at the given point.

(2) a. $f(x) = \sin^2 x$; $x = \frac{\pi}{6}$

(2) b. $\sqrt{x - y - 2} = 0$; (1,-1)

(4) 32. For the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase, intervals of decrease, and local extrema. Use your answers to sketch the graph.

$$f(x) = \frac{x+2}{x^2+2x+1}$$



Section 5.5: Exam 3

Exam 3 Math 1571 Fall 2011

Name:____________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(10) 33. A ball is thrown vertically with an initial velocity of 64 ft/sec from a height of 50 ft.

a. Find the function that gives the height of the ball.

b. Find the function that gives the velocity of the ball.

c. When does the ball reach its maximum height?

d. What is the maximum height reached by the ball?

(10) 34. Murray and Maggie begin walking from the same place and at the same time. Maggie walks North at 4 ft/sec and Murray walks East at 3 ft/sec. At what rate is the distance between them increasing after 2 seconds?

(10) 35. For the following, find the absolute maximum and absolute minimum of the given function on the given interval.

$f(x) = x^3 - 3x^2 + 1$; [-2,3]

(10) 36. Find the maximum product of two positive numbers whose sum is 10.

(10) 37. Use a linearization to approximate $\sqrt{401}$.

38. (10) a. State the Mean Value Theorem

(10) b. Suppose that the number of miles traveled in a car t hours after 12:00 P.M. is given by the function $f(t)$. Also, suppose that $f(1) = 30$ and $f(2) = 90$. Use the Mean Value Theorem to show that the car was traveling at precisely 60 mph at some time between 1:00 P.M. and 2:00 P.M.

39. Integrate.

(10) a. $\int_0^{\frac{\pi}{4}} \sec^2 t \, dt$ **(10) b.** $\int_0^3 \left(x^2 - 2x + 1\right) dx$ **(10) c.** $\int 2x\sqrt{x^2+1}\, dx$

(10) 40. A ball is thrown vertically with an initial velocity of 64 ft/sec from a height of 50 ft.

a. Find the function that gives the height of the ball.

$h(t) = \text{-}16t^2 + 64t + 50$

b. Find the function that gives the velocity of the ball.

$v(t) = h'(t) = \text{-}32t + 64$

c. When does the ball reach its maximum height?

$v(t) = 0$

$\text{-}32t + 64 = 0$

$t = 2$

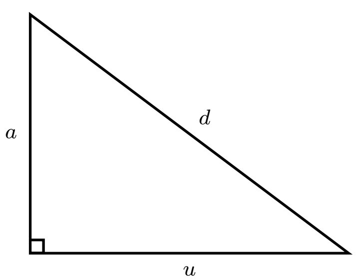
After 2 seconds.

d. What is the maximum height reached by the ball?

$h(2) = 114$

114 ft

(10) 41. Murray and Maggie begin walking from the same place and at the same time. Maggie walks North at 4 ft/sec and Murray walks East at 3 ft/sec. At what rate is the distance between them increasing after 2 seconds?



$a^2 + u^2 = d^2$

$\frac{d}{dt}\left(a^2 + u^2\right) = \frac{d}{dt}\, d^2$

$2a\frac{da}{dt} + 2u\frac{du}{dt} = 2d\frac{dd}{dt}$

$2(8)(4) + 2(6)(3) = 2(10)\frac{dd}{dt}$

$\frac{dd}{dt} = 5$

5 ft/sec

(10) 42. For the following, find the absolute maximum and absolute minimum of the given function on the given interval.

$f(x) = x^3 - 3x^2 + 1$; [-2,3]

$f'(x) = 3x^2 - 6x = 3x(x-2)$

$f(\text{-}2) = \text{-}19$ (min)

$f(3) = 1$ (max)

$f(0) = 1$ (max)

$f(2) = \text{-}3$

(10) 43. Find the maximum product of two positive numbers whose sum is 10.

$f(x) = x(10 - x) = \text{-}x^2 + 10x$

$f(x) = \text{-}x^2 + 10x$

$f'(x) = \text{-}2x + 10$

$f'(x) = \text{-}2(x - 5)$

$f(0) = 0$

$f(10) = 0$

$f(5) = 25$ (max)

(10) 44. Use a linearization to approximate $\sqrt{401}$.

See class notes from 11-30-11.

(Page 188: 31) **(10) 45.** The edge of a cube was found to be 30 cm with a possible maximum error of 0.1 cm. Use differentials to estimate the maximum possible error of the volume.

46. (10) a. State the Mean Value Theorem

(10) b. Suppose that the number of miles traveled in a car t hours after 12:00 P.M. is given by the function $f(t)$. Also, suppose that $f(1) = 30$ and $f(2) = 90$. Use the Mean Value Theorem to show that the car was traveling at precisely 60 mph at some time between 1:00 P.M. and 2:00 P.M.

***Proof*:** By the Mean Value Theorem, there exists $t \in [1,2]$ such that $f'(t) = \frac{f(2)-f(1)}{2-1} = \frac{90-30}{2-1} = 60$. ■

(10) 47. Calculate the following sum.

$$\sum_{i=1}^{20} i(i^2+1)$$

$$= \sum_{i=1}^{20} (i^3+i)$$

$$= \sum_{i=1}^{20} i^3 + \sum_{i=1}^{20} i$$

$$= \left(\frac{20 \cdot 21}{2}\right)^2 + \frac{20 \cdot 21}{2}$$

$$= 210^2 + 210$$

(10) 48. Use the definition of integral to evaluate the following.

$$\int_0^1 (x^2+x+1)\,dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\tfrac{i}{n}\right) \cdot \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\tfrac{i}{n}\right)^2 + \tfrac{i}{n} + 1\right] \cdot \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{i^2}{n^2} + \tfrac{i}{n} + 1\right) \cdot \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{i^2}{n^3} + \tfrac{i}{n^2} + \tfrac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \tfrac{i^2}{n^3} + \sum_{i=1}^{n} \tfrac{i}{n^2} + \sum_{i=1}^{n} \tfrac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{1}{n^3} \sum_{i=1}^{n} i^2 + \tfrac{1}{n^2} \sum_{i=1}^{n} i + \tfrac{1}{n} \sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{1}{n^3} \cdot \tfrac{n(n+1)(2n+1)}{6} + \tfrac{1}{n^2} \sum_{i=1}^{n} \tfrac{n(n+1)}{2} + 1\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{2n^3+3n^2+n}{6n^3} + \tfrac{n^2+n}{2n^2} + 1\right)$$

$$= \tfrac{1}{3} + \tfrac{1}{2} + 1$$

$$= \tfrac{11}{6}$$

49. Integrate.

(Page 318: 31) **(10) a.** $\int_0^{\frac{\pi}{4}} \sec^2 t\, dt$

(10) b. $\int_0^3 \left(x^2 - 2x + 1\right) dx$

$= \left(\frac{1}{3}x^3 - x^2 + x\right)\Big|_0^3$

$= 9 - 9 + 3 - 0$

$= 3$

(10) c. $\int 2x\sqrt{x^2+1}\, dx$

Let $u = x^2 + 1$ and $du = 2x\, dx$.

$\int 2x\sqrt{x^2+1}\, dx$

$= \int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}}\, du$

$= \frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$

Section 5.6: Exam 3a

Exam 3 Math 1571 Fall 2011

Name:____________________________

<u>Directions:</u> Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(10) 50. A ball is thrown vertically with an initial velocity of 64 ft/sec from a height of 50 ft.

a. Find the function that gives the height of the ball.

b. Find the function that gives the acceleration of the ball.

c. What is the maximum height reached by the ball?

d. With what velocity does the ball hit the ground?

(10) 51. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is running away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?

(10) 52. For the following, find the absolute maximum and absolute minimum of the given function on the given interval.

$f(x) = x^3 + 6x^2 - 10$; [-5,1]

(10) 53. Find the maximum product of two positive numbers whose sum is 10.

(Page 188: 31) **(10) 54.** The edge of a cube was found to be 30 cm with a possible maximum error of 0.1 cm. Use differentials to estimate the maximum possible error of the surface area.

55. (10) a. State the Mean Value Theorem

(10) b. Suppose that the number of miles traveled in a car t hours after 12:00 P.M. is given by the function $f(t)$. Also, suppose that $f(1) = 30$ and $f(2) = 90$. Use the Mean Value Theorem to show that the car was traveling at precisely 60 mph at some time between 1:00 P.M. and 2:00 P.M.

56. Integrate.

(10) a. $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 t\, dt$ **(10) b.** $\displaystyle\int_{-3}^{1} \left(x^2 - 2x + 1\right) dx$ **(10) c.** $\displaystyle\int \cos x \sqrt{\sin}\, dx$

Chapter 6: Math 1571 Fall 2014

Section 6.1: Quiz

Quiz 17 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For each of the following, give the domain and range. Use your answers to sketch the graph.

a. $f(x) = \dfrac{x^2 - x - 12}{x - 4}$

b. $f(x) = \sqrt{9 - x}$

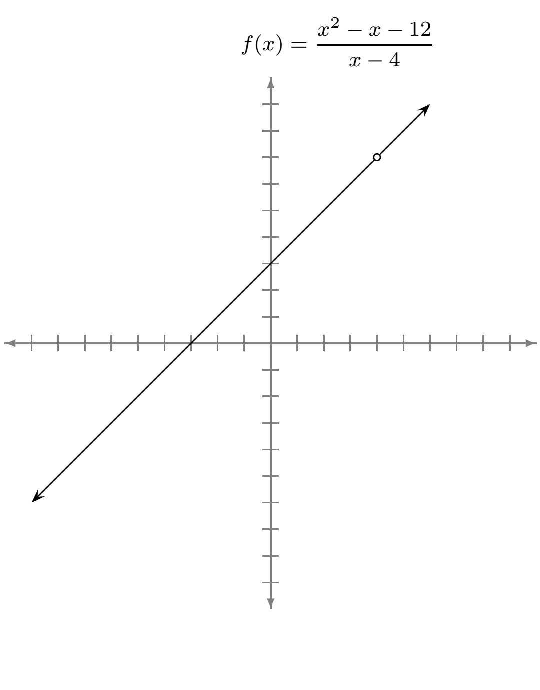
D: (-∞,4) ∪ (4,∞)

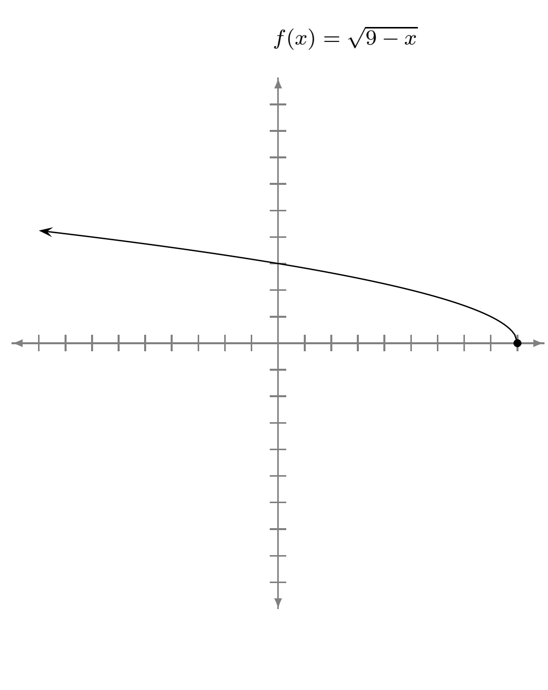
$$f(x) = \frac{x^2 - x - 12}{x - 4} = \frac{(x-4)(x+3)}{x-4}$$

R: (-∞,7) ∪ (7,∞)

D: (-∞,9]

R: [0,∞)





Quiz 18 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Let $f(x) = x^2 + 2x + 1$ and $g(x) = \sqrt{x}$.

a. Find the domain and range of $f \circ g$.

D: $[0,\infty)$

R: $[1,\infty)$

b. Simplify $f \circ g$.

$f \circ g$

$= \left(\sqrt{x}\right)^2 + 2\sqrt{x} + 1$

$= x + 2\sqrt{x} + 1$

c. Find the domain and range of $g \circ f$.

D: $\mathbb{R}$

R: $[0,\infty)$

d. Simplify $g \circ f$.

$g \circ f$

$= \sqrt{x^2 + 2x + 1}$

$= \sqrt{(x+1)^2}$

$= |x+1|$

Quiz 19 Name:____________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, determine whether the function is odd, even, or neither.

(1) a. $f(x) = 2x^3 - 2x + 1$

Neither.

$f(2) = 13$

$f(\text{-}2) = \text{-}11$

(1) b. The function $(f + g)(x)$ where f and g are even.

Even.

Note that

$(f + g)(\text{-}x)$

$= f(\text{-}x) + g(\text{-}x)$

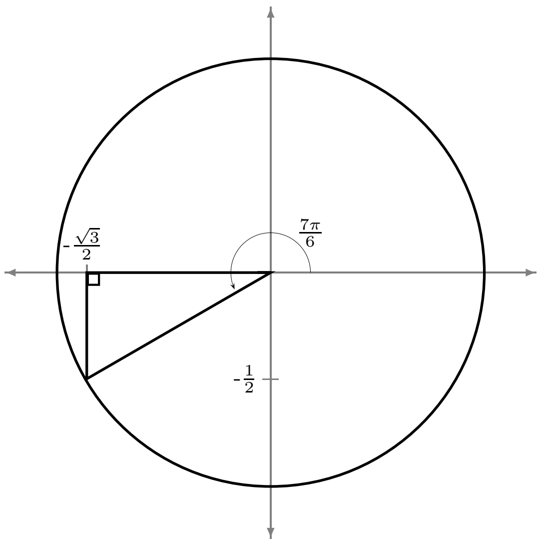
$= f(x) + g(x)$ (f and g are even)

$= (f + g)(x)$.

Quiz 20 Name:______________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Calculate $\sin\frac{7\pi}{6}$, $\cos\frac{7\pi}{6}$, $\tan\frac{7\pi}{6}$, $\cot\frac{7\pi}{6}$, $\sec\frac{7\pi}{6}$, and $\csc\frac{7\pi}{6}$.



$\sin\frac{7\pi}{6} = -\frac{1}{2}$

$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

$\cot\frac{7\pi}{6} = \sqrt{3}$

$\sec\frac{7\pi}{6} = -\frac{2}{\sqrt{3}}$

$\csc\frac{7\pi}{6} = -2$

(1) 2. Use the $\varepsilon - \delta$ definition of limit to prove $\lim\limits_{x\to 2}\big(5x+2\big) = 12$.

First, we need to find δ for a given ε.

$|(5x+2) - 12| < \varepsilon$

$|5x - 10| < \varepsilon$

$5|x-2| < \varepsilon$

$|x-2| < \frac{\varepsilon}{5}$

***Proof*:** Let $\varepsilon > 0$ be given. Let $\delta = \frac{\varepsilon}{5}$. If $|x-2| < \delta$, then

$|x-2| < \delta$

$|x-2| < \frac{\varepsilon}{5}$

$5|x-2| < \varepsilon$

$|5x-10| < \varepsilon$

$|5x+2-12| < \varepsilon$

$|(5x+2)-12| < \varepsilon.$

Therefore, $\lim\limits_{x\to 2}\big(5x+2\big) = 12$. $\blacksquare$

Quiz 21 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Compute the following limits.

(1) a. $\lim_{x\to 3} \dfrac{2x^2 - x - 15}{x^2 - x - 6}$

$$= \lim_{x\to 3} \frac{(2x+5)(x-3)}{(x+2)(x-3)}$$

$$= \lim_{x\to 3} \frac{2x+5}{x+2}$$

$$= \frac{11}{5}$$

(1) b. $\lim_{x\to\sqrt{3}} \dfrac{x^2 - 3}{x - \sqrt{3}}$

$$= \lim_{x\to\sqrt{3}} \frac{(x+\sqrt{3})(x-\sqrt{3})}{x-\sqrt{3}}$$

$$= \lim_{x\to\sqrt{3}} \left(x+\sqrt{3}\right)$$

$$= 2\sqrt{3}$$

Quiz 22 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Suppose that a,$b > 0$. Show (correctly) that $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$.

$$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0}\left(\frac{\sin ax}{\sin bx}\cdot\frac{bx}{ax}\cdot\frac{a}{b}\right) = \lim_{x\to 0}\left(\frac{a}{b}\cdot\frac{\sin ax}{ax}\cdot\frac{bx}{\sin bx}\right) = \frac{a}{b}\cdot 1\cdot 1 = \frac{a}{b}$$

(1) 2. For the following, assign a value to α to make the given function continuous (if possible).

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ \alpha, & \text{if } x = 1 \end{cases}$$

$$\lim_{x\to 1} f(x)$$

$$= \lim_{x\to 1} \frac{x^2-1}{x-1}$$

$$= \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1}$$

$$= \lim_{x\to 1}(x+1)$$

$$= 2$$

Therefore, f is continuous if $\alpha = 2$.

Quiz 23 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. Determine whether or not each of the following equations has a solution.

(1) a. $x^7 + x^3 + 1 = 0$

Let $f(x) = x^7 + x^3 + 1$. Note that $f(1) = 3$ and $f(\text{-}1) = \text{-}1$. Since f is continuous on [-1,1], the Intermediate Value Theorem implies that there exists a number $c \in$ [-1,1] such that $f(c) = 0$.

(1) b. 2. $x^4 + x^2 + 1 = 0$

This equation has no solution since $x^4 + x^2 + 1 \geq 1$ for all $x \in \mathbb{R}$.

Quiz 24 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. For the following, find all vertical and horizontal asymptotes of the graph of the given function. Use your answers to sketch the graph.

$$f(x) = \frac{x^2-2x-3}{x^2-5x+6}$$

$$f(x) = \frac{x^2-2x-3}{x^2-5x+6} = \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$\lim_{x\to\infty} \frac{x^2-2x-3}{x^2-5x+6} = 1$$

$$\lim_{x\to-\infty} \frac{x^2-2x-3}{x^2-5x+6} = 1$$

HA: $y = 1$

$$\lim_{x\to 2^-} \frac{x^2-2x-3}{x^2-5x+6}$$

$$= \lim_{x\to 2^-} \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2^-} \frac{x+1}{x-2}$$

$$= -\infty$$

$$\lim_{x\to 2^+} \frac{x^2-2x-3}{x^2-5x+6}$$

$$= \lim_{x\to 2^+} \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2^+} \frac{x+1}{x-2}$$

$$= \infty$$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-5x+6}$$

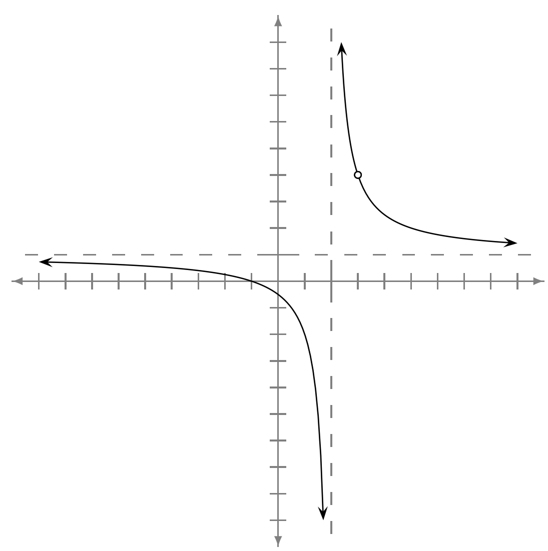
$$= \lim_{x\to 3} \frac{(x+1)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x\to 3} \frac{x+1}{x-2}$$

$$= 4$$

VA: $x = 2$

$f(x) = \dfrac{x^2-2x-3}{x^2-5x+6}$



Quiz 25 Name:________________________

<u>Directions:</u> Show all of your work and justify all of your answers.

For the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

(1) 1. $g(x) = \dfrac{x^2 - 4}{x}$

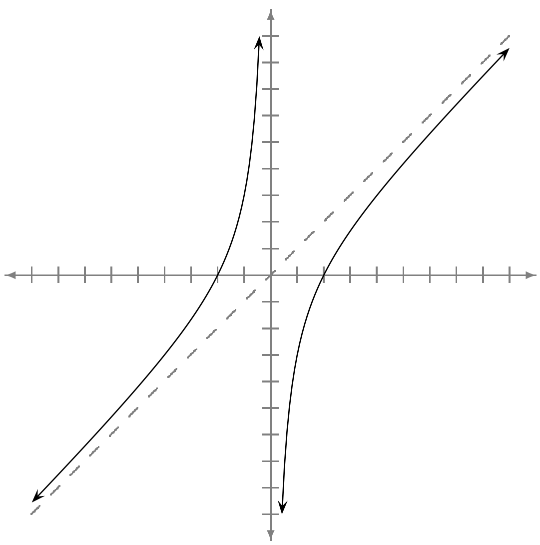
$g(x) = \dfrac{x^2 - 4}{x} = x - \frac{4}{x}$

HA: none

VA: $x = 0$

$g(x) = x - \frac{4}{x}$

OA: $y = x$



Quiz 26 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Use the definition of derivative to differentiate each of the following.

a. $f(x) = x^3 + x$

$\lim\limits_{h\to 0}\big[f(x+h) - f(x)\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[(x+h)^3 + (x+h) - (x^3+x)\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[x^3 + 3x^2h + 3xh^2 + h^3 + x + h - x^3 - x\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[3x^2h + 3xh^2 + h^3 + h\big]\,\frac{1}{h}$

$= \lim\limits_{h\to 0}\big[3x^2 + 3xh + h^2 + 1\big]$

$= 3x^2 + 1$

b. $f(x) = \sqrt{x+1}$

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}\right)$$

$$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \frac{1}{2\sqrt{x+1}}$$

Quiz 27 Name:__________________________

<u>**Directions:**</u> Show all of your work and justify all of your answers.

(1) 1. Differentiate.

$$f(x) = \sqrt{\frac{x^2+1}{x+1}}$$

$$f(x) = \left(\frac{x^2+1}{x+1}\right)^{\frac{1}{2}}$$

$$f'(x) = \tfrac{1}{2}\left(\frac{x^2+1}{x+1}\right)^{-\frac{1}{2}}\left[\frac{2x(x+1)-(x^2+1)(1)}{(x+1)^2}\right]$$

Quiz 28 Name:________________________

Directions: Show all of your work and justify all of your answers.

1.

For each of the following, find the equation of the line that is tangent to the graph of the given function at the specified point.

(1) a. $g(x) = \cos(\sin x)$; $x = \pi$

$g'(x) = -\sin(\sin x) \cdot \cos(x)$

$g(\pi) = \cos(\sin \pi) = \cos 0 = 1$

$g'(\pi) = -\sin(\sin \pi) \cdot \cos \pi = 0$

$y - 1 = 0(x - \pi)$

$y = 1$

(1) b. $\sqrt{x^2 + y^2} = xy - 7$; (3,4)

$\frac{d}{dx} \sqrt{x^2 + y^2} = \frac{d}{dx} \left(xy - 7\right)$

$\frac{1}{2} \left(x^2 + y^2\right)^{-\frac{1}{2}} (2x + 2yy') = y + xy'$

$$\frac{x + yy'}{\sqrt{x^2 + y^2}} = y + xy'$$

$\frac{3+4y'}{5} = 4 + 3y'$

$3 + 4y' = 20 + 15y'$

$11y' = -17$

$y' = -\frac{17}{11}$

$y - 4 = -\frac{17}{11}(x - 3)$

Quiz 29 Name:____________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. For the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

$g(x)$

$$= \frac{x^2+4x+8}{x+2}$$

$$= x+2+\frac{4}{x+2}$$

$$= x+2+4(x+2)^{-1}$$

HA: none

VA: $x =$ -2

$$g(x) = \frac{x^2}{x+2}+4 = x+2+\tfrac{4}{x+2}$$

OA: $y = x+2$

$$g(x) = x+2+4(x+2)^{-1}$$

$g'(x)$

$$= 1-4(x+2)^{-2}$$

$$= 1-\frac{4}{(x+2)^2}$$

$$= \frac{(x+2)^2-4}{(x+2)^2}$$

$$= \frac{(x^2+4x+4)-4}{(x+2)^2}$$

$$= \frac{x^2+4x}{(x+2)^2}$$

$$= \frac{x(x+4)}{(x+2)^2}$$

Dec: [-4,-2), (-2,0]

Inc: (-∞,-4], [0,∞)

Local max: -4 at -4

Local min: 4 at 0

$$g'(x) = 1-4(x+2)^{-2}$$

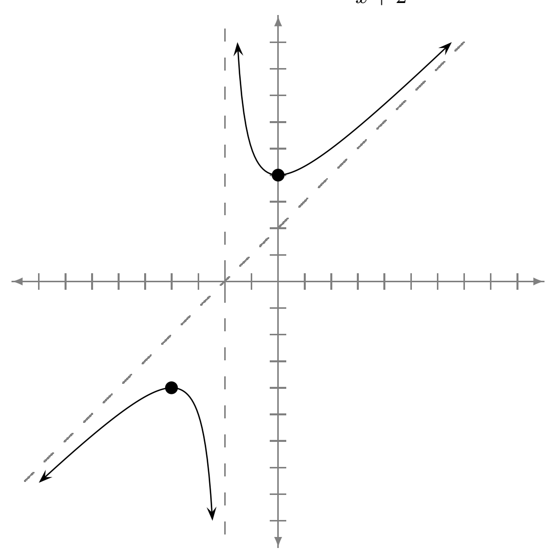
$$g'(x) = 8(x+2)^{-3} = \frac{8}{(x+2)^3}$$

CD: (-∞,-2)

CU: (-2,∞)

IP: none

$$g(x) = \frac{x^2+4x+8}{x+2}$$



Quiz 30 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. A ball is thrown upward with an initial velocity of of 64 ft/sec from a height of 80 ft.

a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 64t + 80$

(1) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 64$

$\text{-}32t + 64 = 0$

$\text{-}32(t - 2) = 0$

$t = 2$

$h(2) = \text{-}64 + 128 + 80 = 144$

144 feet

(1) c. What is the velocity of the ball when the ball hits the ground?

$\text{-}16t^2 + 64t + 80 = 0$

$\text{-}16(t^2 - 4t - 5) = 0$

$\text{-}16(t - 5)(t + 1) = 0$

$t = 5$

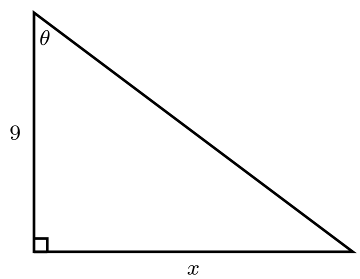
$v(5) = \text{-}96$

-96 ft/sec

Quiz 31 Name:______________________

Directions: Show all of your work and justify all of your answers.

(1) 1. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is running away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?



$\tan\theta = \frac{x}{9}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{1}{10}$

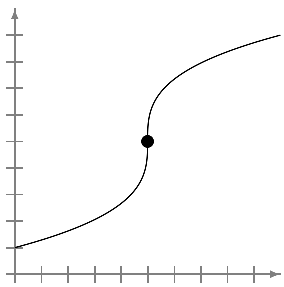
$\frac{d\theta}{dt} = \frac{1}{250}$

$\frac{1}{250}$ radians per second

Quiz 32 Name:_________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Below is the graph of a function that represents the profit of marketing a certain product. The time in months is represented by the x-axis and the profit measured in tens of thousands of dollars is represented by the y-axis. You were in charge of marketing the product for the first five months before moving on to another project. For the past five months, the product has been marketed by your colleague Bill. The solid circle on the graph represents the point at which Bill took charge. You and Bill have both applied for the same promotion which will result in a pay raise of \$8000. Your supervisor (also Bill's supervisor) has asked both of you to a meeting. The following is the dialogue of the beginning of the meeting. Give your response.



Supervisor to Bill: What effects has your marketing strategy had on the profits?

Bill: As you can clearly see, the profits have increased as a result of my marketing strategy.

Supervisor to Bill: Maybe the profits have increased as result of being on the market longer. Some of the sales are the result of repeat business from consumers who began using the product in the first five months. After all, the profits have been increasing the entire time that the product has been on the market.

Bill: Yes. I agree that profits have always increased. But since I have taken charge of marketing the product, the rate at which the profits have increased has also increased. This is clearly illustrated by the graph.

Supervisor: Yes. It would appear so.

You: Bill is wrong. The rate of change of the profit function is given by the derivative. The rate of change of the derivative is given by the second derivative. Since the function changes from concave upward to concave downward at the point where Bill took charge of marketing, the first derivative which measures the rate of change of the profit function has changed from increasing to decreasing. So even though profits are still increasing, the rate at which they are increasing has been decreasing since Bill has been in charge of marketing.

Quiz 33 Name:________________________

<u>Directions:</u> Show all of your work and justify all of your answers.

Two ants are walking on a piece of graph paper. The first ant starts at the point (0,-7) and walks to the point (5,8) along the path $y = 3x - 7$. The second ant starts at the point (0,4) and walks to the point (5,41) along the path $y = 2x^2 - x + 4$. The ants are walking in such a way that their positions on the graph paper always have the same x-coordinate. Give the minimum and maximum distances between the two ants.

$D(x) = 2x^2 - x + 4 - (3x - 7)$; [0,5]

$D(x) = 2x^2 - 4x + 11$

$D'(x) = 4x - 4 = 4(x - 1)$

$D(0) = 11$

$D(5) = 41$ (max)

$D(1) = 9$ (min)

Quiz 34 Name:______________________

Directions: Show all of your work and justify all of your answers.

(1) 1. The radius of a circle is measured to be 2 cm with a possible maximum error of .1 cm. Use differentials to approximate the maximum possible error of the area of the circle.

$A = \pi r^2$

$dA = 2\pi r dr$

$dA = 2\pi \cdot 2 \cdot .1 = .4\pi$

$.4\pi$ cm^2

Quiz 35 Name:__________________________

<u>**Directions:**</u> Show all of your work and justify all of your answers.

(1) 1. Suppose that $g : \mathbb{R} \to \mathbb{R}$ is differentiable, $g(1) = 2$, and $g(13) = 16$. Let $f(x) = g(x)\sqrt{x+3}$. Use the Mean Value Theorem to show that there exists $c \in [1,13]$ such that $f'(c) = 5$.

***Proof*:** Note that f satisfies the hypothesis of the Mean Value Theorem, $f(1) = 4$, and $f(13) = 64$. So the Mean Value Theorem implies that there exists $c \in [1,13]$ such that $f'(c) = \frac{f(13)-f(1)}{13-1} = \frac{64-4}{13-1} = 5$. ■

Total Points: 1436

Due Friday December 5, 2014 at 4:00

Directions: Show your work in the space provided. You may not work with other students. You may ask your instructor and only your instructor for assistance.

2. Calculate the following limits.

(1) a. $\lim_{x \to -1} \dfrac{x^3 + 2x^2 - x - 2}{x + 1}$

(1) b. $\lim_{\theta \to \frac{\pi}{3}} \dfrac{\tan\theta - \sqrt{3}}{x - \frac{\pi}{3}}$

(1) 3. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with the following properties:

i. $f(0) = 1$;

ii. $f'(0) = 1$;

iii. $f(x + y) = f(x) \cdot f(y)$.

Prove that $f'(x) = f(x)$ for all $x \in \mathbb{R}$.

4. Find the equation of the line that is tangent to the given curve at the specified point.

(1) a. $y = x^3 + \sqrt{x} + \sec x$; $x = \frac{\pi}{3}$

(1) b. $x^4y^3 - 3xy^3 + 6x^2y^2 = 8$; (1,2)

(1) 5. For the following, find the intervals of increase and decrease, local extrema, intervals of concavity, and inflection points.

$f(x) = 3x^5 - 5x^4 - 20x^3 + 4$

(1) 6. The hour hand and minute hand of a watch have lengths .5 cm and 1 cm, respectively. At what rate is the distance between the tips of the hands changing at 3:00? Hint: Law of Cosines.

(1) 7. A dog is at the edge of a creek that is 10 feet wide and wants to reach a point on the other side of the creek that is 20 feet upstream. She can run four times faster than she can swim. What route should she take to minimize her travel time?

8. Integrate.

(1) a. $\displaystyle\int_1^4 \left(x^5 + \sqrt{x} + 1\right) dx$

(1) b. $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x \tan x \, dx$

(1) c. $\displaystyle\int_1^2 \left(x^2 + \tfrac{2}{3}x\right) \sqrt[3]{x^3 + x^2 + 1} \, dx$

9. For each of the following, find the area of the region enclosed by the given curves.

(1) a. $y = \sqrt{x}$ and $y = x^2$

(1) b. $x = y^2 - 2y - 3$ and $y = x - 1$

10. Let $\mathcal{R}$ be the region bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$. For each of the following, find the volume of the solid formed by revolving $\mathcal{R}$ about the given line.

(1) a. y-axis

(1) b. $x =$ -1

11. A solid has as its base the upper half of the unit circle. Every cross section taken perpendicular to the x-axis is a square. Find the volume of the solid.

12. Let $\mathcal{R}$ be the region bounded by the curves $y = \sec x$, $x = 0$, and $x = \frac{\pi}{4}$. Find the volume of the solid formed by revolving $\mathcal{R}$ about the x-axis.

Section 6.2: Exam 1

Exam 1 Math 1571 Fall 2014

Name:________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(20) 13. For the following, give the domain and range. Use your answers to sketch the graph.

$f(x)$

$= \dfrac{2x^2 - 3x - 2}{x - 2}$

$= \dfrac{(2x+1)(x-2)}{x-2}$

$\neq 2x + 1$

D: (-∞,2) ∪ (2,∞)

R: (-∞,5) ∪ (5,∞)

$f(x)$

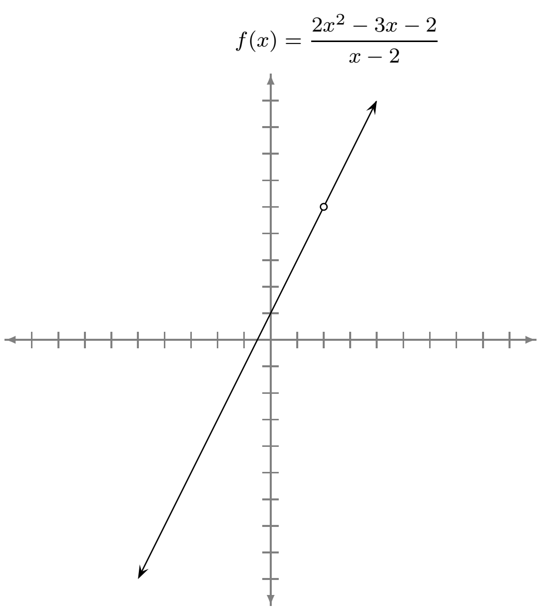
$= \dfrac{3x^2 - 2x - 1}{x - 1}$

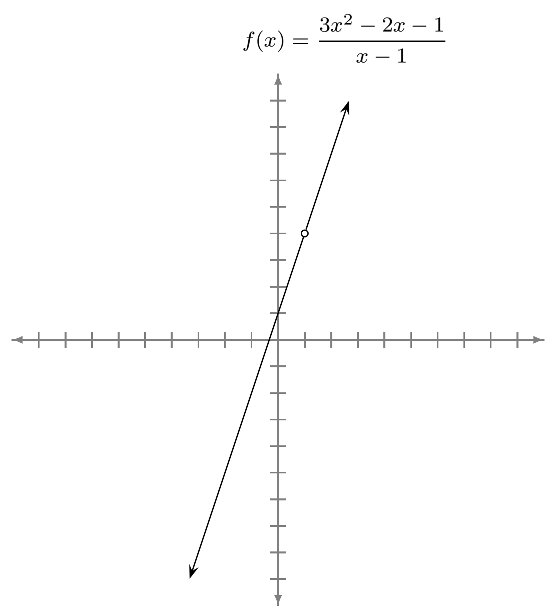
$= \dfrac{(3x+1)(x-1)}{x-1}$

$\neq 3x + 1$

D: (-∞,1) ∪ (1,∞)

R: (-∞,4) ∪ (4,∞)





(20) 14. Let $f(x) = x^2 + 9$ and $g(x) = \sqrt{x}$.

Let $f(x) = \sqrt{x+9}$ and $g(x) = x^2$.

Simplify (correctly) the following as completely as possible.

a. $(g \circ f)(x) = \sqrt{x^2 - 9}$

b. $(f \circ g)(x) = x + 9$

c. $(g \circ f)(x) = x + 9$

d. $(f \circ g)(x) = \sqrt{x^2 + 9}$

(20) 15. List the transformations needed to transform the graph of $y = f(x)$ into the graph of

$y = f(x+2)+3.$

(i) Horizontal shift left 2 units.

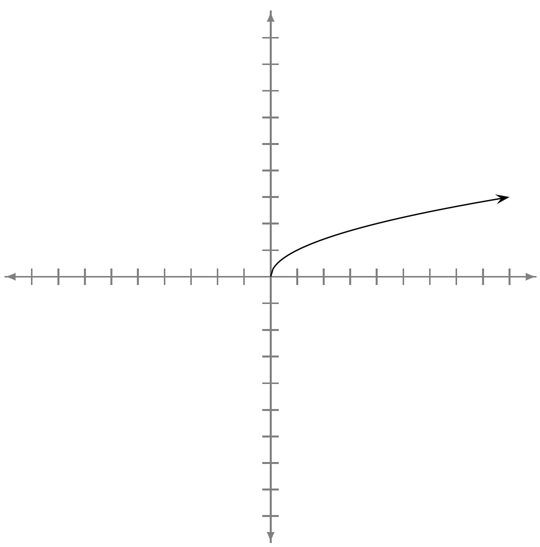
(ii) Vertical shift up 3 units.

$f(x+3)+2.$

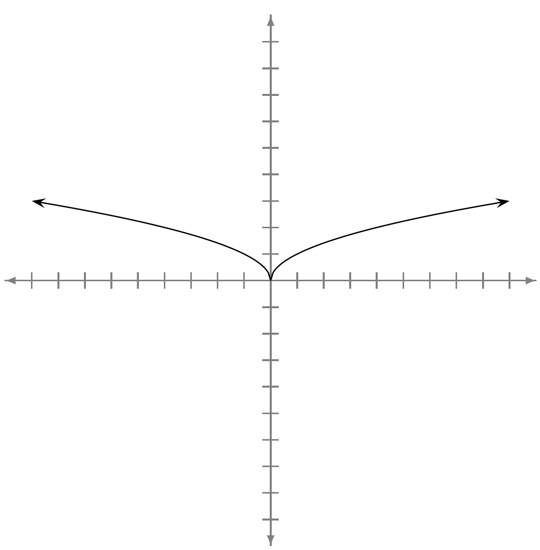
(iii) Horizontal shift left 3 units.

(iv) Vertical shift up 2 units.

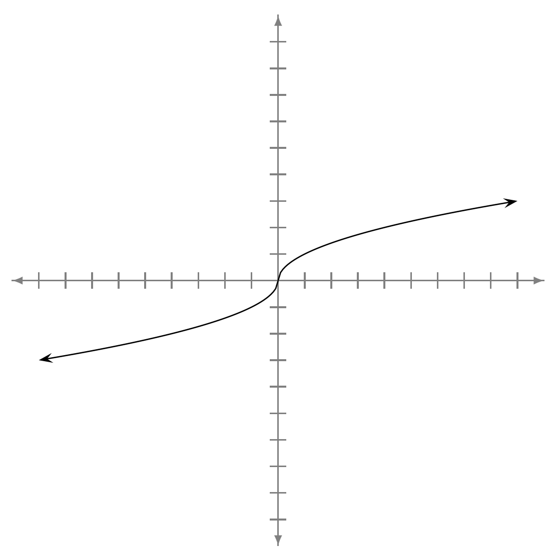
(20) 16. Below is part of the graph of a function $f : \mathbb{R} \to \mathbb{R}$. For each of the following, complete the graph under the given assumptions.

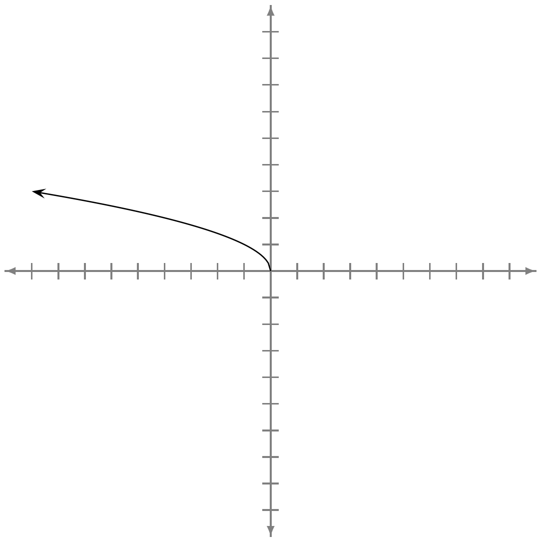


a. The function is even.

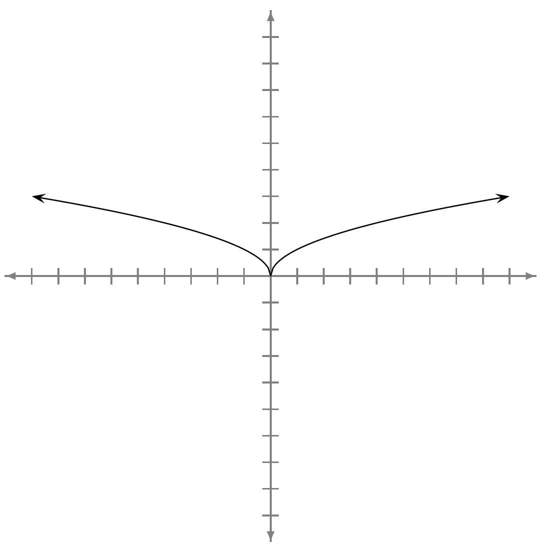


b. The function is odd.

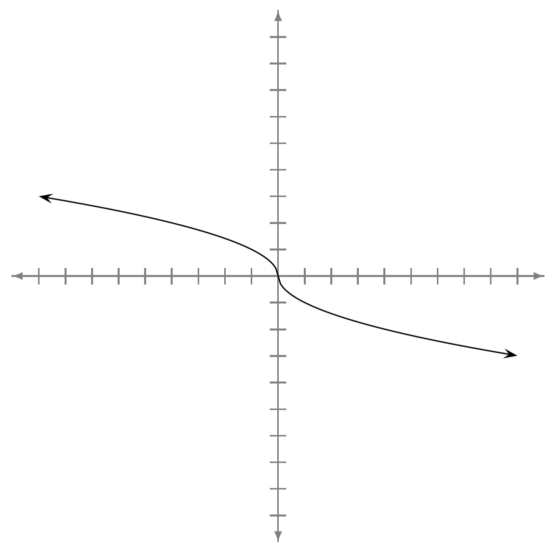




c. The function is even.



d. The function is odd.



17. Calculate the following limits.

(20) a. $\lim\limits_{x \to -2} \dfrac{x^2 + x - 2}{x^2 + 5x + 6}$

$= \lim\limits_{x \to -2} \dfrac{(x+2)(x-1)}{(x+2)(x+3)}$

$= \lim\limits_{x \to -2} \dfrac{x-1}{x+3}$

$=$ -3

$\lim\limits_{x \to -3} \dfrac{x^2 + 2x - 3}{x^2 + 5x + 6}$

$= \lim\limits_{x \to -3} \dfrac{(x+3)(x-1)}{(x+2)(x+3)}$

$= \lim\limits_{x \to -3} \dfrac{x-1}{x+2}$

$= 4$

(20) b. $\lim\limits_{x \to 5} \dfrac{\sqrt{x} - \sqrt{5}}{x - 5}$

$= \lim\limits_{x \to 5} \left(\dfrac{\sqrt{x} - \sqrt{5}}{x-5} \cdot \dfrac{\sqrt{x} + \sqrt{5}}{\sqrt{x} + \sqrt{5}} \right)$

$= \lim\limits_{x \to 5} \dfrac{x-5}{(x-5)(\sqrt{x} + \sqrt{5})}$

$= \lim_{x\to 5} \dfrac{1}{\sqrt{x}+\sqrt{5}}$

$= \dfrac{1}{2\sqrt{5}}$

$\lim_{x\to 7} \dfrac{\sqrt{x}-\sqrt{7}}{x-7}$

$= \lim_{x\to 7} \left(\dfrac{\sqrt{x}-\sqrt{7}}{x-7} \cdot \dfrac{\sqrt{x}+\sqrt{7}}{\sqrt{x}+\sqrt{7}} \right)$

$= \lim_{x\to 7} \dfrac{x-7}{(x-7)(\sqrt{x}+\sqrt{7})}$

$= \lim_{x\to 7} \dfrac{1}{\sqrt{x}+\sqrt{7}}$

$= \dfrac{1}{2\sqrt{7}}$

(10) c. $\lim_{x\to 0} \dfrac{|x|}{x}$

$\lim_{x\to 0^+} \dfrac{|x|}{x} = \lim_{x\to 0^+} \dfrac{x}{x} = 1$

$\lim_{x\to 0^-} \dfrac{|x|}{x} = \lim_{x\to 0^-} \dfrac{-x}{x} = -1$

$\lim_{x\to 0} \dfrac{|x|}{x}$ DNE

(10) d. $\lim_{\theta\to 0} \dfrac{\tan\theta}{\theta}$

$= \lim_{\theta\to 0} \dfrac{\frac{\sin\theta}{\cos\theta}}{\theta}$

$= \lim_{\theta\to 0} \dfrac{\sin\theta}{\theta\cos\theta}$

$= \lim_{\theta\to 0} \left(\dfrac{\sin\theta}{\theta} \cdot \dfrac{1}{\cos\theta} \right)$

$= 1 \cdot 1$

$= 1$

(10) 18. Use the Intermediate Value Theorem to prove that the following equation has a solution.

$\sin x + x = 4$

$\cos x + x = 4$

Proof: Let $f(x) = \sin x + x$ and note that f is continuous on $\mathbb{R}$. Since $f(0) = 0$ and $f(6) = \sin 6 + 6 \geq 6 - 1 = 5$, the Intermediate Value Theorem implies that there is a number $c \in [0{,}6]$ such that $f(c) = 4$. Hence, the equation $\sin x + x = 4$ has a solution between 0 and 6. ■

Proof: Let $f(x) = \cos x + x$ and note that f is continuous on $\mathbb{R}$. Since $f(0) = 1$ and $f(6) = \cos 6 + 6 \geq 6 - 1 = 5$, the Intermediate Value Theorem implies that there is a number $c \in [0{,}6]$ such that $f(c) = 4$. Hence, the equation $\cos x + x = 4$ has a solution between 0 and 6. ■

(10) 19. Note that $\lim_{x\to 1} (3x+4) = 7$. For an arbitrary $\varepsilon > 0$, how close to 1 do we have to take x so that $3x+4$ is within a distance of ε of 7? (i.e., find δ).

$|(3x+4) - 7| < \varepsilon$

$|(3x-3| < \varepsilon$

$|3(x-1)| < \varepsilon$

$3|(x-1)| < \varepsilon$

$|(x-1)| < \frac{\varepsilon}{3}$

Note that $\lim_{x\to 1} (4x+3) = 7$.

For an arbitrary $\varepsilon > 0$, how close to 1 do we have to take x so that $4x+3$ is within a distance of ε of 7? (i.e., find δ).

$|(4x+3)-7| < \varepsilon$

$|(4x-4| < \varepsilon$

$|4(x-1)| < \varepsilon$

$4|(x-1)| < \varepsilon$

$|(x-1)| < \frac{\varepsilon}{4}$

Total Points: 1611

Section 6.3: Exam 2

Exam 2 Math 1571 Fall 2014

20. Differentiate each of the following.

(20) a. $f(x) = \sqrt{x^3+1}$

$f(x) = (x^3+1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2)$

$= \dfrac{3x^2}{2\sqrt{x^3+1}}$

(20) b. $g(x) = \dfrac{x^3}{x^2+1}$

$g'(x) = \dfrac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2}$

$= \dfrac{x^4+3x^2}{(x^2+1)^2}$

(20) c. $h(x) = \tan x + \sec x$

$h'(x) = \sec^2 x + \sec x \tan x$

(20) 21. Find the equation of the line that is tangent to the curve $x^2y^2 + y = 5$ at the point (2,1).

$\frac{d}{dx}\left(x^2y^2+y\right) = \frac{d}{dx}5$

$2xy^2 + x^2(2y)y' + y' = 0$

$4 + 8y' + y' = 0$

$9y' = -4$

$y' = -\frac{4}{9}$

$y - 1 = -\frac{4}{9}(x-2)$

(10) 22. Suppose that f is a differentiable function such that $f(1) = 3$ and $f'(1) = 5$. Calculate the following limit.

$$\lim_{x\to 1} \frac{f(x)-3}{x-1} = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = f'(1) = 5$$

23. A snowball with radius 4 in is melting so that its radius is decreasing at a rate of $\frac{1}{8}$ in per hour.

(10) a. Give the area of the largest cross section of the snowball as a function of time.

$A(t) = \pi\left(4 - \frac{1}{8}t\right)^2$

(10) b. At what rate is the area of the cross section decreasing after 8 hours?

$A'(t) = 2\pi\left(4 - \frac{1}{8}t\right)\left(\text{-}\frac{1}{8}\right) = \text{-}\frac{\pi}{4}\left(4 - \frac{1}{8}t\right)$

$A'(8) = \text{-}\frac{3\pi}{4}$

-$\frac{3\pi}{4}$ in/hr

(10) c. How long will it take the snowball to melt completely?

$4 - \frac{1}{8}t = 0$

$t = 32$

32 hours

24. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(20) a. $f(x) = x^4 + x^2 - 9$

HA: none

VA: none

OA: none

$f'(x) = 4x^3 + 2x = 2x(2x^2 + 1)$

Dec: (-∞,0]

Inc: [0,∞)

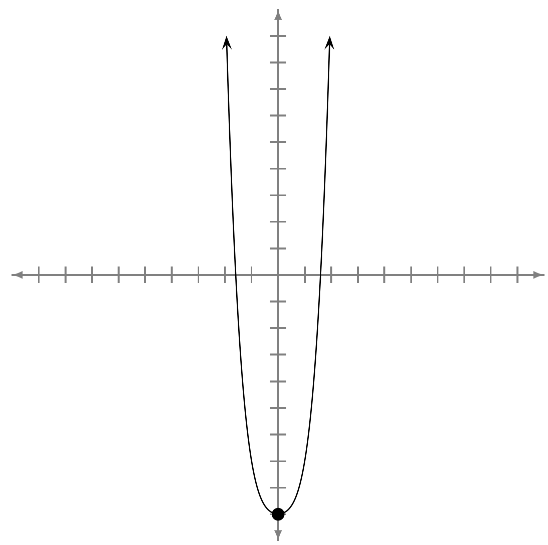
Loc Min: -9 at 0

$f''(x) = 12x^2 + 2 = 2(6x^2 + 1)$

CU: (-∞,∞)

CD: never

IP: none



$f(x) = x^4 + x^2 - 9$

(20) b. $g(x) = \dfrac{x^2+1}{x}$

$g(x) = x + \frac{1}{x}$

$g(x) = x + x^{\text{-}1}$

HA: none

VA: $x = 0$

OA: $y = x$

$g'(x) = 1 - x^{\text{-}2} = \dfrac{x^2 - 1}{x^2} = \dfrac{(x+1)(x-1)}{x^2}$

Dec: [-1,0), (0,1])

Inc: (-∞,-1], [1,∞)

Local max: -2 at -1

Local min: 2 at 1

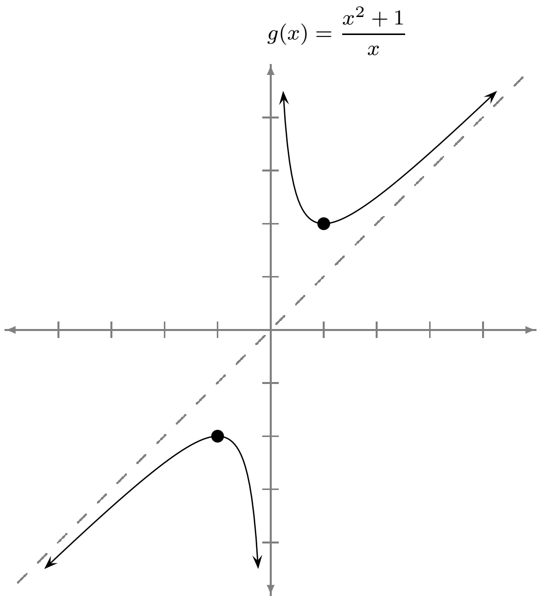
$g'(x) = 1 - x^{-2}$

$g''(x) = \frac{2}{x^3}$

CD: (-∞,0)

CU: (0,∞)

IP: none



(20) c. $h(x) = x - 2\sqrt{x} = x - 2x^{\frac{1}{2}}$

$h'(x) = 1 - x^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{x}} = \frac{\sqrt{x}-1}{\sqrt{x}}$

Dec: (0,1]

Inc: [0,∞)

Local min: -1 at 1

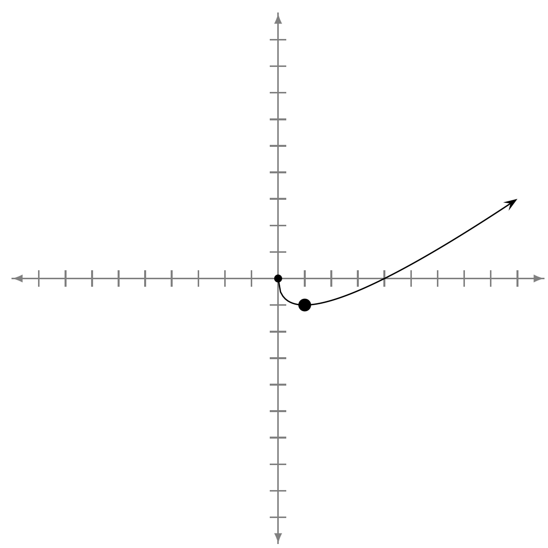
$h'(x) = 1 - x^{-\frac{1}{2}}$

$h''(x) = \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x^3}}$

CU: (0,∞)

CD: never

IP: none



Total Points: 1791

Section 6.4: Exam 3

Exam 3 Math 1571 Fall 2014

Name:________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(1) 25. An object is projected upward from a height of 150 ft at a velocity of 50 ft/sec. Find the time at which the object reaches its maximum height.

$h(t) = \text{-}16t^2 + 50t + 150$

$v(t) = h'(t) = \text{-}32t + 50$

$v(t) = 0$

$\text{-}32t + 50 = 0$

$t = \frac{25}{16}$

(1) 26. A hole in the shape of a right circular cylinder is being dug in such a way that the radius is increasing at a rate of 1 in/min and the height is increasing at a rate of 2 in/min. At what rate is the volume of the hole increasing when the radius is 10 in and the height is 20 in? Recall: $V = \pi r^2 h$

$V = \pi r^2 h$

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

$\frac{dV}{dt} = 2\pi \cdot 10 \cdot 1 \cdot 20 + \pi \cdot 100 \cdot 2 = 600\pi$

600π in^2/min

(1) 27. The edges of a triangle are increasing in such a way that the base and height are equal and the area is increasing at a rate of 10 cm^2/min. At what rate is the base increasing when the base is 5 cm? Recall: $A = \frac{1}{2}bh$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}b^2$

$\frac{dA}{dt} = b\frac{db}{dt}$

$10 = 5\frac{db}{dt}$

$\frac{db}{dt} = 2$

2 cm/min

(1) 28. Find the maximum and minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on the interval [-1,2].

$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x + 2)(x - 1)$

CN: -2, 1

Note: -2 $\notin$ [-1,2]

$f(\text{-}1) = 13$ (max)

$f(2) = 4$

$f(1) = \text{-}7$ (min)

(1) 29. A farmer has 1000 yd of fence with which to enclose a rectangular field.

a. Find the dimensions that will maximize the area of the field.

Let w be the width of the field. Then the length is $\frac{1}{2}(1000 - 2w)$.

$A(w) = w\frac{1}{2}(1000 - 2w) = \text{-}w^2 + 500w$

$A'(w) = \text{-}2w + 500 = \text{-}2(w - 250)$

$A(0) = 0$

$A(500) = 0$

$A(250) = 62500$

250 yd by 250 yd

b. Would the area be greater if the field were in the shape of circle?

Yes.

$C = 2\pi r$

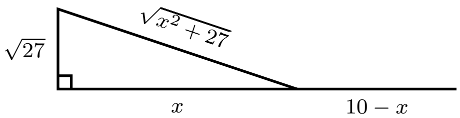
$1000 = 2\pi r$

$r = \frac{500}{\pi}$

$A = \pi r^2$

$A = \pi \cdot \left(\frac{500}{\pi}\right)^2 = \frac{250000}{\pi}$

30. A power line is to be strung from a pumping station at the edge of a river to a factory ten miles downstream and on the other side of the river from the station. The cost of running the lines under water is twice that of running the lines over land. The river is $\sqrt{27}$ miles wide. How should the line be strung in order to minimize the cost?



$C(x) = 2\sqrt{x^2+27} + 10 - x$

$C'(x) = \dfrac{2x}{\sqrt{x^2+27}} - 1$

$C'(x) = 0$

$\dfrac{2x}{\sqrt{x^2+27}} = 1$

$2x = \sqrt{x^2+27}$

$4x^2 = x^2 + 27$

$3x^2 = 27$

$x^2 = 9$

$x = 3$

$C(0) = 6\sqrt{3} + 10$

$C(10) = 2\sqrt{127}$

$C(3) = 22$

The line should be run straight across the river and over land for 10 miles.

(1) 31. Use a linear approximation or differential to approximate $\sqrt[3]{8.01}$.

$f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f(8) = 2$

$f'(8) = \frac{1}{12}$

$L(x) = \frac{1}{12}(x-8) + 2$

$L(8.01) = 2 + \frac{1}{1200}$

(1) 32. Calculate the following limit.

$$\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}} \overset{\text{L'H}}{=} \lim_{x \to \frac{\pi}{4}} \frac{\cos x + \sin x}{1} = \sqrt{2}$$

(1) 33. Calculate the following limit. You may choose to write the limit as an integral and use the Fundamental Theorem of Calculus Part II.

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^2}{n^3}$$

$$= \lim_{n \to \infty} \tfrac{1}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n \to \infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \to \infty} \frac{2n^3 + 3n^2 + n}{6n^3}$$

$$= \tfrac{1}{3}$$

Alternatively,

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \cdot \frac{2}{n} = \int_0^1 (x^2)\,dx = \tfrac{1}{3}x^3\Big|_0^1 = \tfrac{1}{3}$$

34. Integrate.

(1) a. $\displaystyle\int_0^1 (x^2 - x)\,dx = \left(\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2\right)\Big|_0^1 = \tfrac{1}{3} - \tfrac{1}{2} = \text{-}\tfrac{1}{6}$

(1) b. $\displaystyle\int \sin^2 x \cos x\,dx = \tfrac{1}{3}\sin^3 x + C$

Let $u = \sin x$ and $du = \cos x\,dx$.

$$\int \sin^2 x \cos x\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\sin^3 x + C$$

Chapter 7: Math 1571 Fall 2015

Section 7.1: Quizzes

Quiz 36 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, give the domain and range. Use your answers to sketch the graph.

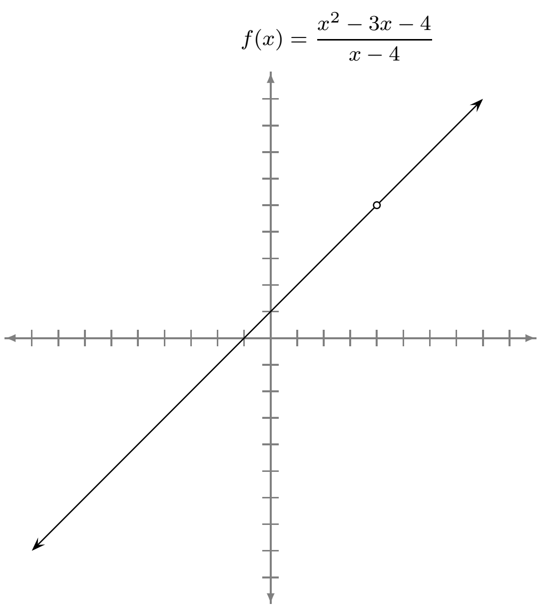
(1) a. $f(x) = \dfrac{x^2 - 3x - 4}{x - 4}$

$= \dfrac{(x+1)(x-4)}{x-4}$

$\neq x + 1$

D: $(-\infty,4) \cup (4,\infty)$

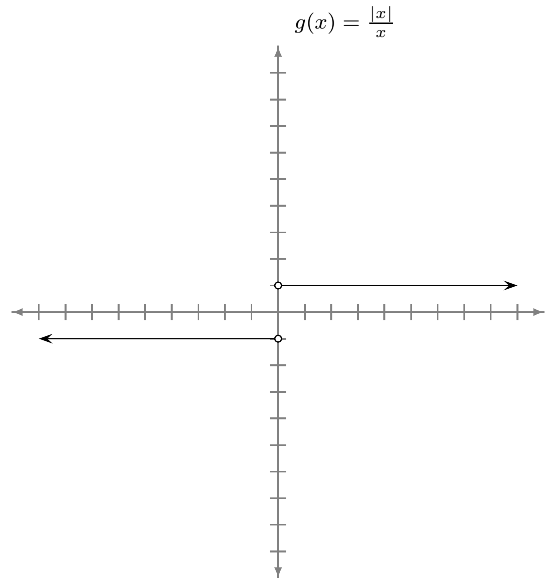
R: $(-\infty,5) \cup (5,\infty)$



(1) b. $g(x) = \frac{|x|}{x}$

D: $(-\infty,0) \cup (0,\infty)$

R: $\{-1,1\}$



Quiz 37 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(Page 43: 41) **(1) 1.** Let $F(x) = (2x + x^2)^4$. Find functions f and g such that $f \circ g = F$.

(1) 2. Given the graph of $y = g(x)$, sketch the graph $y = \text{-}g(x - 4) + 2$.

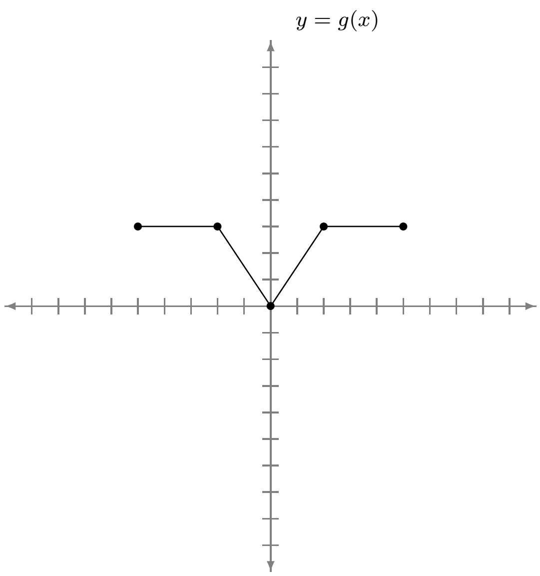
Transformations:

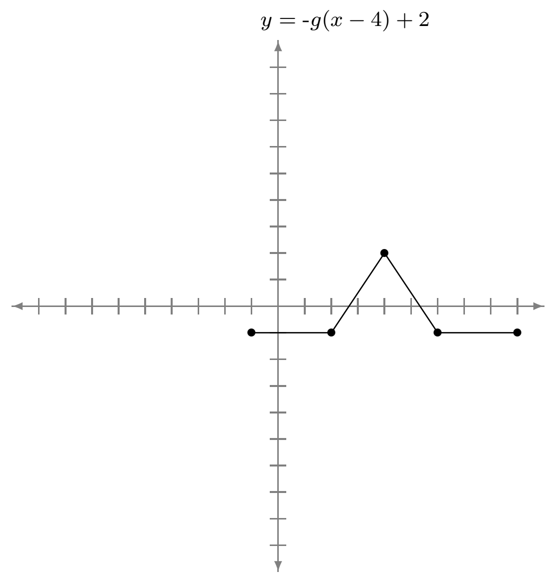
(i) shift right 4 units;

(ii) reflect about x-axis;

(iii) shift up 2 units.

Graphs.





Quiz 38 Name:______________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Prove that the product of two odd functions is an even function.

***Proof*:**

Let f and g be odd functions. Then

$(f \cdot g)(\text{-}x)$

$= [f(\text{-}x)][g(\text{-}x)]$

$= [\text{-}f(x)][\text{-}g(x)]$

$= [f(x)][g(x)]$

$= (f \cdot g)(x)$.

Quiz 39 Name:____________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Give an example of a quadratic function with x-intercepts (-1,0) and (4,0) and y-intercept (0,-8).

$(x+1)(x-4) = x^2 - 3x - 4$

$2(x+1)(x-4) = 2x^2 - 6x - 8$

$f(x) = 2x^2 - 6x - 8$

$f(0) = -8$

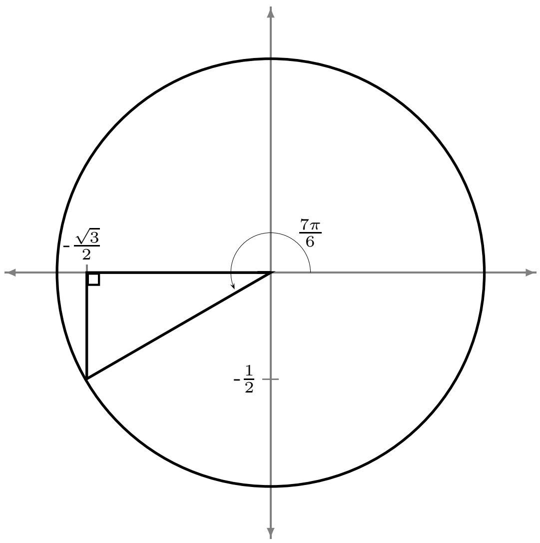
$f(-1) = 0$

$f(4) = 0$

Quiz 40 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Find the value of the six trigonometric functions of $\theta = \frac{7\pi}{6}$.



$\sin\frac{7\pi}{6} = -\frac{1}{2}$

$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

$\cot\frac{7\pi}{6} = \sqrt{3}$

$\sec\frac{7\pi}{6} = -\frac{2}{\sqrt{3}}$

$\csc\frac{7\pi}{6} = -2$

(1) 2. Find all $\theta \in [0,2\pi]$ such that $\tan\theta = -1$.

$\tan\theta = -1$

$\frac{\sin\theta}{\cos\theta} = -1$

$\sin\theta = -\cos\theta$

$\theta = \frac{3\pi}{4}$,

$\theta = \frac{7\pi}{4}$

Quiz 41 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Use the $\varepsilon - \delta$ definition of limit to show that

$\lim\limits_{x\to 3}\big(3x-1\big) = 8.$

$\lim\limits_{x\to -4}\big(2x-1\big) = -9.$

Given $\varepsilon > 0$, find $\delta > 0$ such that $|(2x-1)-7| < \varepsilon$ whenever $|x-4| < \delta$.

$|(2x-1)-7| < \varepsilon$

$|2x-8| < \varepsilon$

$|2(x-4)| < \varepsilon$

$|x-4| < \frac{\varepsilon}{2}$

$\delta = \frac{\varepsilon}{2}$

Proof: Let $\varepsilon > 0$, be given and let $\delta = \frac{\varepsilon}{2}$. Suppose that such that $|x-4| < \delta$ and consider

$|x-4| < \delta$

$|x-4| < \frac{\varepsilon}{2}$

$2|x-4| < \varepsilon$

$|2x-8| < \varepsilon$

$|(2x-1)-7| < \varepsilon$

as desired. ■

Given $\varepsilon > 0$, find $\delta > 0$ such that $|(2x-1)- -9| < \varepsilon$ whenever $|x- -4| < \delta$.

$|(2x-1)- -9| < \varepsilon$

$|2x+8| < \varepsilon$

$|2(x+4)| < \varepsilon$

$|2(x- -4)| < \varepsilon$

$|x- -4| < \frac{\varepsilon}{2}$

$\delta = \frac{\varepsilon}{2}$

Proof: Let $\varepsilon > 0$, be given and let $\delta = \frac{\varepsilon}{2}$. Suppose that such that $|x- -4| < \delta$ and consider

$|x- -4| < \delta$

$|x+4| < \delta$

$|x+4| < \frac{\varepsilon}{2}$

$2|x+4| < \varepsilon$

$|2x+8| < \varepsilon$

$|(2x-1)- -9| < \varepsilon$

as desired. ■

Quiz 42 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

$$g(x) = \frac{x^2 - 4}{x}$$

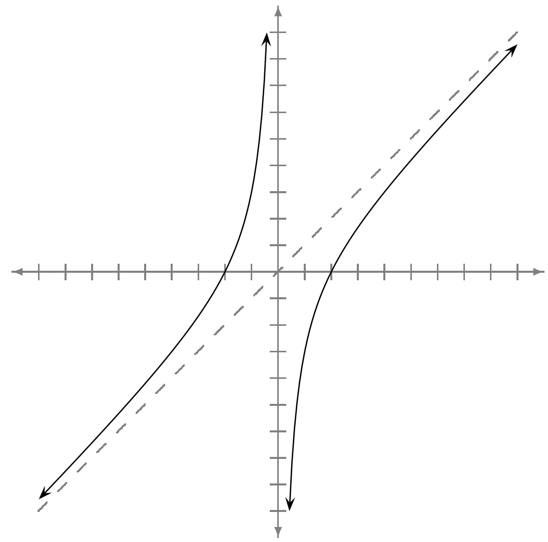
$$g(x) = \frac{x^2 - 4}{x} = x - \tfrac{4}{x}$$

HA: none

VA: $x = 0$

$g(x) = x - \frac{4}{x}$

OA: $y = x$



$$g(x) = \frac{x^2 - 4}{x}$$

Quiz 43 Name:__________________________

Directions: Show all of your work and justify all of your answers.

Calculate the following limits.

(2) 1. $\lim_{\theta\to 0} \frac{\sin 3\theta}{\sin 4\theta}$

$= \lim_{\theta\to 0}\left(\frac{\sin 3\theta}{\sin 4\theta} \cdot \frac{\theta}{\theta} \cdot \frac{3}{3} \cdot \frac{4}{4}\right)$

$= \lim_{\theta\to 0}\left(\frac{\sin 3\theta}{3\theta} \cdot \frac{4\theta}{\sin 4\theta} \cdot \frac{3}{4}\right)$

$= 1 \cdot 1 \cdot \frac{3}{4}$

$= \frac{3}{4}$

(2) 2. $\lim_{x\to 0}\left(x^4 \left|\sin \frac{1}{x}\right|\right)$

$0 \le x^4 \left|\sin \frac{1}{x}\right| \le x^4$

$\lim_{x\to 0} 0 = 0$

$\lim_{x\to 0} x^4 = 0$

By the Squeeze Theorem, $\lim_{x\to 0}\left(x^4 \left|\sin \frac{1}{x}\right|\right) = 0$.

Quiz 44 Name:________________________

Directions: Show all of your work and justify all of your answers.

Use the definition of derivative to differentiate each of the following.

(1) 1. $f(x) = 2x^2 + 5$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 + 5 - (2x^2 + 5)}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 5 - 2x^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2}{h}$$

$$= \lim_{h \to 0} \big(4x + 2h\big)$$

$$= 4x$$

(1) 2. $f(x) = \sqrt{x}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \tfrac{1}{2\sqrt{x}}$$

Quiz 45 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For each of the following, decide if the given function is continuous at the specified value of x.

a. $g(x) = \dfrac{x^3 - 4x^2 - 25x + 2}{x^2 - 4x + 3}$; $x = 1$

$f(1)$ DNE

The function is not continuous at 1.

b. $g(x) = \begin{cases} 4x + 1, & \text{if } x \leq -1 \\ x^2 - 4, & \text{if } x > -1 \end{cases}$; $x = -1$

$g(-1) = -3$

$\lim\limits_{x \to -1^-} g(x) = \lim\limits_{x \to -1^-} \left(4x + 1\right) = -3$

$\lim\limits_{x \to -1^+} g(x) = \lim\limits_{x \to -1^+} \left(x^2 - 4\right) = -3$

$\lim\limits_{x \to -1} g(x) = -3$

The function is continuous at -1.

Quiz 46 Name:________________________

Directions: Show all of your work and justify all of your answers.

Differentiate each of the following.

(1) 1. $f(x) = 2x^4 - 6x^2 + x - 11$

$f'(x) = 8x^3 - 12x + 1$

(1) 2. $h(x) = \dfrac{x^2 + 3x}{4x + 1}$

$$h'(x) = \frac{(2x+3)(4x+1) - (x^2+3x)(4)}{(4x+1)^2}$$

Quiz 47 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Suppose that $f : [0,1] \to [0,1]$ is continuous, $f(0) = 1$, and $f(1) = 0$. Prove that there is a number $c \in [0,1]$ such that $f(c) = c$. Hint: Let $g(x) = f(x) - x$ and use the IVT.

***Proof*:** Let $g(x) = f(x) - x$ and note that g is continuous since the difference of continuous functions is continuous. Then $g(0) = f(0) - 0 = 1$ and $g(1) = f(1) - 1 =$ -1. By the IVT, there is a number $c \in [0,1]$ such that $g(c) = 0$. Since $g(c) = f(c) - c = 0$, $f(c) = c$. ■

(2) 2. Suppose that $f : [0,1] \to [0,1]$ is continuous. Prove that there is a number $c \in [0,1]$ such that $f(c) = c$.

***Proof*:** If $f(0) = 0$, let $c = 0$ and if $f(1) = 1$, let $c = 1$. So we may assume that $f(0) \neq 0$ and $f(1) \neq 1$. Since $f : [0,1] \to [0,1]$, $f(0) > 0$ and $f(1) < 1$. Let $g(x) = f(x) - x$ and note that g is continuous since the difference of continuous functions is continuous. Then $g(0) = f(0) - 0 > 0 - 0 = 0$ and $g(1) = f(1) - 1 < 1 - 1 = 0$. Since $g(0) > 0$ and $g(1) < 0$, the IVT implies that there exists $c \in [0,1]$ such that $g(c) = f(c) - c = 0$. Since $f(c) - c = 0$, $f(c) = c$. ■

Quiz 48 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Find the equation of the line that is tangent to the graph of the function $y = \sin^2 x$ at the point $x = \frac{\pi}{6}$.

$\frac{dy}{dx} = 2\sin x \cos x$

$x = \frac{\pi}{6}$

$y = \frac{1}{4}$

$\frac{dy}{dx} = \frac{\sqrt{3}}{2}$

$y - \frac{1}{4} = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$

(2) 2. Calculate the following limit.

$$\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$$

$f(x) = \tan x$

$$f'\left(\frac{\pi}{4}\right) = \lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$$

$f'(x) = \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right) = 2$

Quiz 49 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

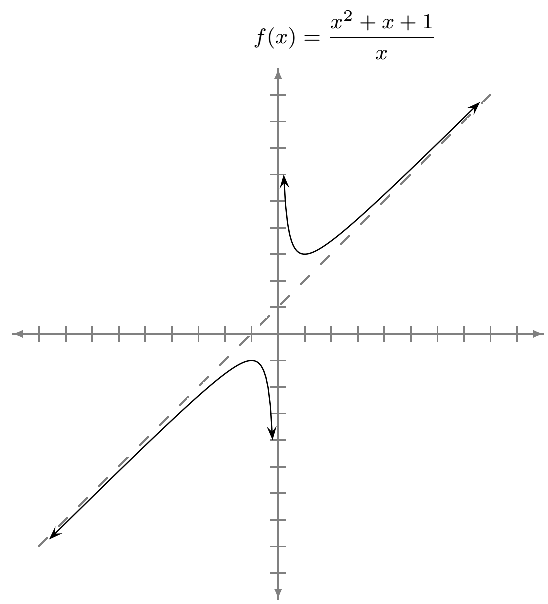
$f(x) = \dfrac{x^2 + x + 1}{x}$

$= x + 1 + \frac{1}{x}$

HA: none

VA: $x = 0$

OA: $y = x + 1$



Quiz 50 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For the following, determine whether the function is odd, even or neither. Also, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase, intervals of decrease, and local extrema. Use your answers to sketch the graph.

$$f(x) = \frac{x^2+x+1}{x}$$

$$= \frac{x^2+x+1}{x}$$

$f(1) = 3$

$f(\text{-}1) = \text{-}1$

Neither even nor odd.

HA: none

VA: $x = 0$

OA: $y = x + 1$

$$f'(x) = \frac{(2x+1)(x) - (x^2+x+1)(1)}{x^2}$$

$$= \frac{2x^2+x-x^2-x-1}{x^2}$$

$$= \frac{x^2-1}{x^2}$$

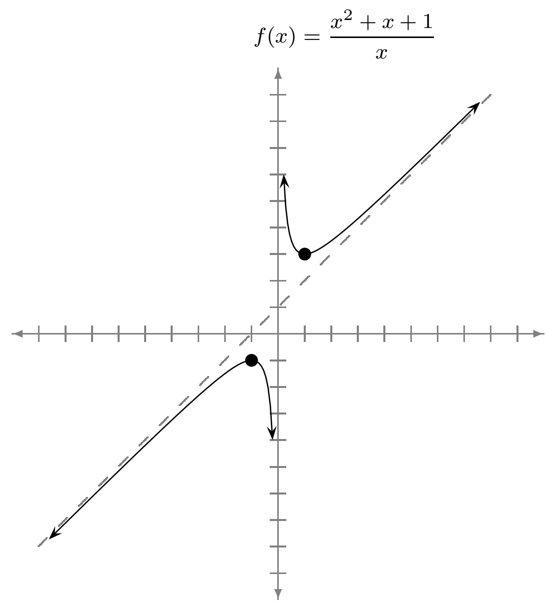
$$= \frac{(x+1)(x-1)}{x^2}$$

Dec: [-1,0), (0,1]

Inc: (-∞,-1], [1,∞)

Local max: -1 at -1

Local min: 3 at 1



Quiz 51 Name:______________________

Directions: Show all of your work and justify all of your answers.

Use the definition of derivative to differentiate the following.

(2) 1. $f(x) = \dfrac{x+1}{x-1}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{(x+h)+1}{(x+h)-1} - \dfrac{x+1}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{(x+h+1)(x-1) - (x+1)(x+h-1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{x^2 + xh - h - 1 - (x^2 + xh + h - 1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{x^2 + xh - h - 1 - x^2 - xh - h + 1}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{-2h}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{-2}{(x+h-1)(x-1)}$$

$$= \frac{-2}{(x-1)(x-1)}$$

$$= \frac{-2}{(x-1)^2}$$

Quiz 52 Name:________________________

Directions: Show all of your work and justify all of your answers.

Find the equation of the line that is tangent to the given curve at the specified point.

(2) 1. $3x^2y^3 + xy = 2$; (-1,1)

$3x^2y^3 + xy = 2$

$\frac{d}{dx}\left(3x^2y^3 + xy\right) = \frac{d}{dx}2$

$6xy^3 + 3x^2 3y^2\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{\text{-}6xy^3 - y}{9x^2y^2 + x}$$

$x = \text{-}1$

$y = 1$

$\frac{dy}{dx} = \frac{6-1}{9-1} = \frac{5}{8}$

$y - 1 = \frac{5}{8}(x + 1)$

Quiz 53 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. A ball is thrown upward with an initial velocity of of 64 ft/sec from a height of 80 ft.

(1) a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 64t + 80$

(1) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 64$

$\text{-}32t + 64 = 0$

$\text{-}32(t - 2) = 0$

$t = 2$

$h(2) = \text{-}64 + 128 + 80 = 144$

144 feet

(1) c. What is the velocity of the ball when the ball hits the ground?

$\text{-}16t^2 + 64t + 80 = 0$

$\text{-}16(t^2 - 4t - 5) = 0$

$\text{-}16(t - 5)(t + 1) = 0$

$t = 5$

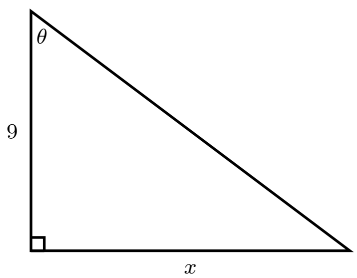
$v(5) = \text{-}96$

-96 ft/sec

Quiz 54 Name:________________________

Directions: Show all of your work and justify all of your answers.

(3) 1. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is running away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?



$\tan\theta = \frac{x}{9}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{1}{250}$

$\frac{1}{250}$ radians per second

Quiz 55 Name:________________________

Directions: Show all of your work and justify all of your answers.

(3) 1. For the following, find the intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

$f(x) = \frac{4}{27}x^3 - 4x - 1$

$f(1) = $ -$\frac{16}{27}$

$f($-1$) = $ -$\frac{8}{9}$

Neither even nor odd.

HA: none

VA: none

OA: none

$f'(x)$

$= \frac{4}{9}x^2 - 4$

$= \frac{4}{9}(x^2 - 9)$

$= \frac{4}{9}(x + 3)(x - 3)$

Dec: [-3,3]

Inc: (-∞,-3], [3,∞)

Local max: 7 at -3

Local min: -9 at 3

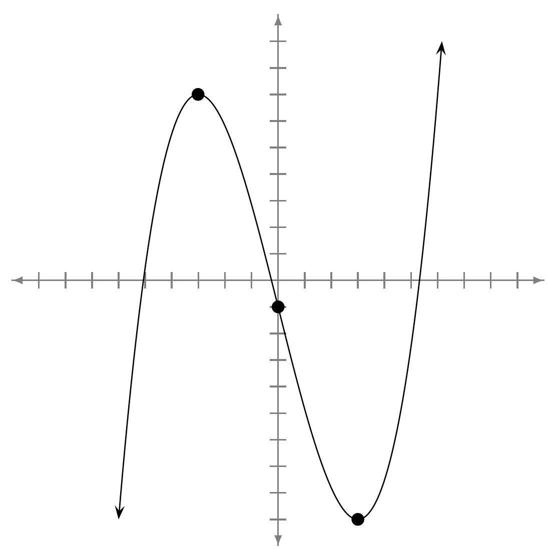
$f''(x) = \frac{8}{9}x$

CD: (-∞,0)

CU: (0,∞)

IP: (0,-1)

$f(x) = \frac{4}{27}x^3 - 4x - 1$



Quiz 56 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Find the absolute maximum and absolute minimum of the given function on the given interval.

$g(x) = x^3 - 3x^2 + 1$; [-1,1]

$g'(x) = 3x^2 - 6x = 3x(x-2)$

$g(-1) = -3$ (min)

$g(1) = -1$

$g(0) = 1$ (max)

(2) 2. The sum of two positive numbers is 30. Can their product be 250?

No.

$f(x) = x(30-x) = -x^2 + 30x$

$f(x) = -x^2 + 30x$

$f'(x) = -2x + 30$

$f'(x) = -2(x-15)$

$f(0) = 0$

$f(30) = 0$

$f(15) = 225$ (max)

Quiz 57 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. Let $g(x) = \sqrt[3]{x}$.

(1) a. Find the linearization of g at 1000.

$g(x) = x^{\frac{1}{3}}$

$g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$$g'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

$g'(1000) = \frac{1}{300}$

$g(1000) = 10$

$L(x) = \frac{1}{300}(x - 1000) + 10$

(1) b. Use your answer from the previous part to approximate $\sqrt[3]{1001}$.

$\sqrt[3]{1001} \approx L(1001) = \frac{1}{300}(1001 - 1000) + 10 = 10\frac{1}{300}$

Quiz 58 Name:_______________________

Directions: Show all of your work and justify all of your answers.

1. A ball is thrown upward with an initial velocity of of 32 ft/sec from a height of 128 ft.

(1) a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 32t + 128$

(1) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 32$

$\text{-}32t + 32 = 0$

$\text{-}32(t - 1) = 0$

$t = 1$

$h(1) = \text{-}16 + 32 + 128 = 144$

144 feet

(1) c. What is the velocity of the ball when the ball hits the ground?

$\text{-}16t^2 + 32t + 128 = 0$

$\text{-}16(t^2 - 2t - 8) = 0$

$\text{-}16(t - 4)(t + 2) = 0$

$t = 4$

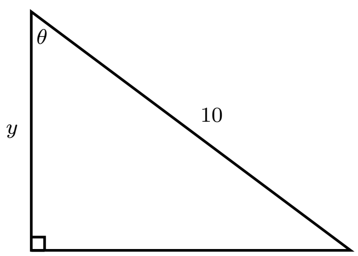
$v(4) = \text{-}96$

-96 ft/sec

Quiz 59 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. A 10-foot ladder is being pulled away from a wall in such a way that the top of the ladder is sliding down the wall at a rate of 4 ft/sec. At what rate is the angle between the ladder and the wall changing when the top of the ladder is 6 feet above the floor?



$\cos\theta = \frac{y}{10}$

$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$

$-\frac{x}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$

$-\frac{8}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10}(-4)$

$\frac{d\theta}{dt} = \frac{1}{2}$

$\frac{1}{2}$ radians per second

Quiz 60 Name:_________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Integrate.

$$\int_2^4 \left(x^2+x\right)dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(2+\tfrac{2i}{n}\right)\cdot \tfrac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(2+\tfrac{2i}{n}\right)^2 + \left(2+\tfrac{2i}{n}\right)\right]\cdot \tfrac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\tfrac{4i^2}{n^2}+\tfrac{8i}{n}+4\right) + \left(2+\tfrac{2i}{n}\right)\right]\cdot \tfrac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{4i^2}{n^2}+\tfrac{10i}{n}+6\right)\cdot \tfrac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{8i^2}{n^3}+\tfrac{20i}{n^2}+\tfrac{12}{n}\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \tfrac{8i^2}{n^3}+\sum_{i=1}^{n}\tfrac{20i}{n^2}+\sum_{i=1}^{n}\tfrac{12}{n}\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{8}{n^3}\sum_{i=1}^{n} i^2+\tfrac{20}{n^2}\sum_{i=1}^{n} i+\tfrac{12}{n}\sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{8}{n^3}\cdot\tfrac{n(n+1)(2n+1)}{6}+\tfrac{20}{n^2}\cdot\tfrac{n(n+1)}{2}+\tfrac{12}{n}\cdot n\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{8}{n^3}\cdot\tfrac{2n^3+3n^2+n}{6}+\tfrac{20}{n^2}\cdot\tfrac{(n^2+n)}{2}+\tfrac{12}{n}\cdot n\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{16n^3+24n^2+8n}{6n^3}+\tfrac{20n^2+20n}{2n^2}+12\right)$$

$$= \tfrac{16}{6}+\tfrac{20}{2}+12$$

$$= 24\tfrac{2}{3}$$

Quiz 61 Name:______________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Use the definition of definite integral to integrate the following.

$$\int_0^1 (x^4 + x^3 + x^2 + x)dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0 + \tfrac{i}{n}\right) \cdot \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\tfrac{i}{n}\right)^4 + \left(\tfrac{i}{n}\right)^3 + \left(\tfrac{i}{n}\right)^2 + \tfrac{i}{n}\right] \tfrac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{i^4}{n^5} + \tfrac{i^3}{n^4} + \tfrac{i^2}{n^3} + \tfrac{i}{n^2}\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{1}{n^5} \sum_{i=1}^{n} i^4 + \tfrac{1}{n^4} \sum_{i=1}^{n} i^3 + \tfrac{1}{n^3} \sum_{i=1}^{n} i^2 + \tfrac{1}{n^2} \sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{1}{n^5} \cdot \tfrac{n(n+1)(6n^3+9n^2+n-1)}{30} + \tfrac{1}{n^4} \cdot \left(\tfrac{n(n+1)}{2}\right)^2 + \tfrac{1}{n^3} \cdot \tfrac{n(n+1)(2n+1)}{6} + \tfrac{1}{n^2} \cdot \tfrac{n(n+1)}{2} 1\right)$$

$$= \lim_{n\to\infty} \left(\tfrac{n(n+1)(6n^3+9n^2+n-1)}{30n^5} + \tfrac{[n(n+1)]^2}{4n^4} + \tfrac{n(n+1)(2n+1)}{6n^3} + \tfrac{n(n+1)}{2n^2}\right)$$

$$= \tfrac{6}{30} + \tfrac{1}{4} + \tfrac{2}{6} + \tfrac{1}{2}$$

$$= \tfrac{12+15+20+30}{60}$$

$$= \tfrac{77}{60}$$

Quiz 62 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Let $\mathcal{R}$ be the region bounded by the curves $y = 2x\sqrt{x^2+1}$, $x = 0$, $x = 1$, and the x-axis. Also, let $\mathcal{S}$ be the solid formed by rotating $\mathcal{R}$ about the x-axis.

(2) a. Find the area of $\mathcal{R}$.

$$\int_0^1 2x\sqrt{x^2+1}\,dx$$

$$= \int_0^1 2x(x^2+1)^{\frac{1}{2}}\,dx$$

$$= \tfrac{2}{3}(x^2+1)^{\frac{3}{2}}\Big|_0^1$$

$$= \tfrac{4\sqrt{2}}{3} - \tfrac{2}{3}$$

$$= \tfrac{2}{3}(2\sqrt{2}-1)$$

(2) b. Find the volume of $\mathcal{S}$.

$$\pi\int_0^1 \left(2x\sqrt{x^2+1}\right)^2\,dx$$

$$= \pi\int_0^1 4x^2(x^2+1)\,dx$$

$$= \pi\int_0^1 \left(4x^4+4x^2\right)dx$$

$$= \pi\left(\tfrac{4}{5}x^5 + \tfrac{4}{3}x^3\right)\Big|_0^1$$

$$= \pi\left(\tfrac{4}{5} + \tfrac{4}{3}\right)$$

$$= \tfrac{32\pi}{15}$$

Section 7.2: Exam 1

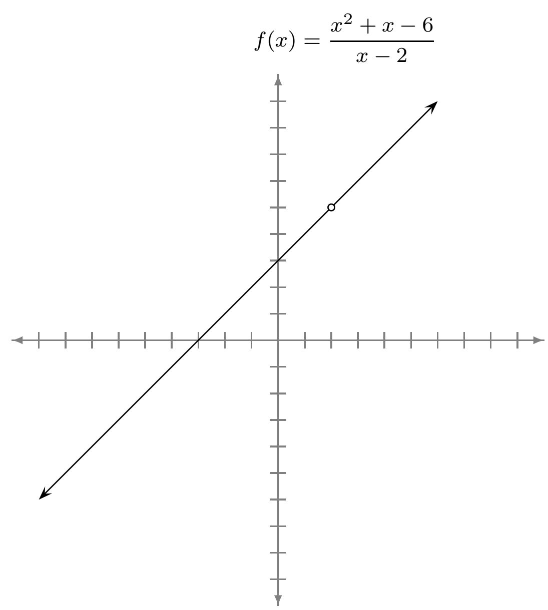
Exam 1 Math 1571 Fall 2015

(10) 2. For the following function, find the domain and range and sketch the graph.

$$f(x) = \frac{x^2+x-6}{x-2} = \frac{(x+3)(x-2)}{x-2} \neq x+3$$

D: $(-\infty,2) \cup (2,\infty)$

R: $(-\infty,5) \cup (5,\infty)$



(10) 3. Let $f(x) = \sqrt{x}$ and $g(x) = \cos x$. Find each of the following.

a. $(f \circ g)(x) = \sqrt{\cos x}$

b. $(g \circ f)(x) = \cos\left(\sqrt{x}\right)$

(10) 4. List the transformations (in a proper order) needed to transform the graph of $y = f(x)$ into the graph of $y = \text{-}2f(x+1)+3$.

(i) Horizontal shift left 1 unit.

(ii) Vertical stretch by a factor of 2.

(iii) Reflect about the x-axis.

(iv) Vertical shift up 3 units.

(10) 5. Prove the following identity.

$\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$\sin\left(x + \frac{\pi}{2}\right)$

$= \sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}$

$= \sin x \cdot 0 + \cos x \cdot 1$

$= \cos x$

(10) 6. Determine whether the following function is odd, even, or neither.

$f(x) = x^2 - x - 1$

Neither.

$f(2) = 1$

$f(\text{-}2) = 5$

7. Calculate the following limits.

(10) a. $\displaystyle\lim_{x \to \text{-}1} \frac{x^2 - 2x - 3}{x^2 + 3x + 2}$

$\displaystyle= \lim_{x \to \text{-}1} \frac{(x-3)(x+1)}{(x+2)(x+1)}$

$\displaystyle= \lim_{x \to \text{-}1} \frac{x-3}{x+2}$

$\displaystyle= \lim_{x \to \text{-}1} \frac{x-3}{x+2}$

-4

(10) b. $\displaystyle\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$

$\displaystyle= \lim_{x \to 16} \frac{\sqrt{x} - 4}{(\sqrt{x} - 4)(\sqrt{x} + 4)}$

$\displaystyle= \lim_{x \to 16} \frac{1}{\sqrt{x} + 4}$

(10) c. $\displaystyle\lim_{x \to 1} \frac{x-1}{|x-1|}$

$\displaystyle\lim_{x \to 1^-} \frac{x-1}{|x-1|}$

$\displaystyle= \lim_{x \to 1^-} \frac{x-1}{-(x-1)}$

$= \text{-}1$

$\displaystyle\lim_{x \to 1^+} \frac{x-1}{|x-1|}$

$\displaystyle= \lim_{x \to 1^+} \frac{x-1}{x-1}$

$= 1$

$\displaystyle\lim_{x \to 1} \frac{x-1}{|x-1|}$ DNE

(10) 8. Calculate the following limit.

$$\lim_{\theta\to 0}\frac{\sin^2\theta}{\theta} = \lim_{\theta\to 0}\frac{(\sin\theta)(\sin\theta)}{\theta} = \lim_{\theta\to 0}\left(\sin\theta\cdot\frac{\sin\theta}{\theta}\right) = 0\cdot 1 = 0$$

(10) 9. Prove that there is a number that is exactly 1 more than its cube.

$x^3 + 1 = x$

$x^3 - x + 1 = 0$

Let $f(x) = x^3 - x + 1$ and note that f is continuous since it is a polynomial.

$f(0) = 1$

$f(\text{-}2) = \text{-}5$

By the Intermediate Value Theorem, there is a solution between -2 and 0.

(10) 10. Let $f(x) = x^2 + 1$ and calculate the following limit.

$$\lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{(3+h)^2 + 1 - 10}{h}$$

$$= \lim_{h\to 0} \frac{9 + 6h + h^2 + 1 - 10}{h}$$

$$= \lim_{h\to 0} \frac{6h + h^2}{h}$$

$$= \lim_{h\to 0} \big(6 + h\big)$$

$$= 6$$

(10) a. Use the $\varepsilon - \delta$ definition of limit to prove that $\lim\limits_{x\to 4} \big(\frac{1}{2}x + 1\big) = 3$.

First, we need to find δ for a given ε.

$\left|\big(\frac{1}{2}x + 1\big) - 3\right| < \varepsilon$

$\left|\frac{1}{2}x - 2\right| < \varepsilon$

$\frac{1}{2}|x - 4| < \varepsilon$

$|x - 4| < 2\varepsilon$

Proof: Let $\varepsilon > 0$ be given. Let $\delta = 2\varepsilon$. If $|x - 4| < \delta$, then

$|x - 4| < \delta$

$|x - 4| < 2\varepsilon$

$\frac{1}{2}|x - 4| < \varepsilon$

$\left|\frac{1}{2}x - 2\right| < \varepsilon$

$\left|\big(\frac{1}{2}x + 1\big) - 3\right| < \varepsilon.$

Therefore, $\lim\limits_{x\to 2} \big(5x + 2\big) = 12.$ ■

Section 7.3: Exam 2

Exam 2 Math 1571 Fall 2015

11. Use the definition of derivative to differentiate the following.

(10) a. $f(x) = \sqrt{x}$

$\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim\limits_{h\to 0} \left(\frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \right)$

$= \lim\limits_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim\limits_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim\limits_{h\to 0} \frac{1}{(\sqrt{x+h}+\sqrt{x})}$

$= \frac{1}{2\sqrt{x}}$

(10) b. $g(x) = \frac{1}{x}$

$\lim\limits_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$= \lim\limits_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

$= \lim\limits_{h\to 0} \frac{x-(x+h)}{x(x+h)h}$

$= \lim\limits_{h\to 0} \frac{-h}{x(x+h)h}$

$= \lim\limits_{h\to 0} \frac{-1}{x(x+h)}$

$= -\frac{1}{x^2}$

12. Differentiate.

(10) a. $f(x) = x^4 + x^3 + x + 1$

$f'(x) = 4x^3 + 3x^2 + 1$

(10) b. $g(x) = \frac{x^2+x+1}{2x+1}$

$g'(x) = \frac{(2x+1)(2x+1)-(x^2+x+1)(2)}{(2x+1)^2}$

$g'(x) = \frac{2x^2+2x-1}{(2x+1)^2}$

(10) c. $h(x) = \sqrt{x^3+x+1}$

$h(x) = (x^3+x+1)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(x^3+x+1)^{-\frac{1}{2}}(3x^2+1)$

$h'(x) = \frac{3x^2+1}{2\sqrt{x^3+x+1}}$

(10) d. $f(x) = \tan x + \sin x$

$f'(x) = \sec^2 x + \cos x$

(10) 13. Calculate the following limit.

$\lim\limits_{x\to 1} \frac{x^{100}-1}{x-1}$

$f(x) = x^{100}$

$f'(x) = 100x^{99}$

$\lim\limits_{x\to 1} \frac{x^{100}-1}{x-1} = f'(1) = 100$

(10) 14. Find the equation of the line that is tangent to the graph of the given function at the specified point.

$f(x) = x^3 + 1$; $x = 2$

$f'(x) = 3x^2$

$x = 2$

$y = f(2) = 9$

$m = f'(2) = 12$

$y - 9 = 12(x - 2)$

$y = 12x - 15$

(10) 15. Find the equation of the line that is tangent to the graph of the given curve at the specified point.

$xy + x^2 + y^2 = 3$; (1,1)

$\frac{d}{dx}\left(xy + x^2 + y^2\right) = \frac{d}{dx}3$

$y + xy' + 2x + 2yy' = 0$

$y' = \frac{-2x-y}{x+2y}$

$x = 1$

$y = 1$

$m = y' = -1$

$y - 1 = -(x - 1)$

$y = -x + 2$

(10) 16. Find the rate of change of the given function at the specified point.

$f(t) = \sqrt{t^2 + 1}$; $t = 3$

$f(t) = (t^2 + 1)^{\frac{1}{2}}$

$f'(t) = \frac{1}{2}\left(t^2 + 1\right)^{-\frac{1}{2}}(2t)$

$f'(t) = \frac{t}{\sqrt{t^2+1}}$

$f'(3) = \frac{3}{\sqrt{10}}$

17. For the following, find the intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(10) a. $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 5$

HA: none

VA: none

OA: none

$f'(x)$

$= 3x^2 - 3x - 6$

$= 3(x^2 - x - 2)$

$= 3(x-2)(x+1)$

Inc: (-∞,-1],[2,∞)

Dec: [-1,2]

Local max: $\frac{17}{2}$ at -1

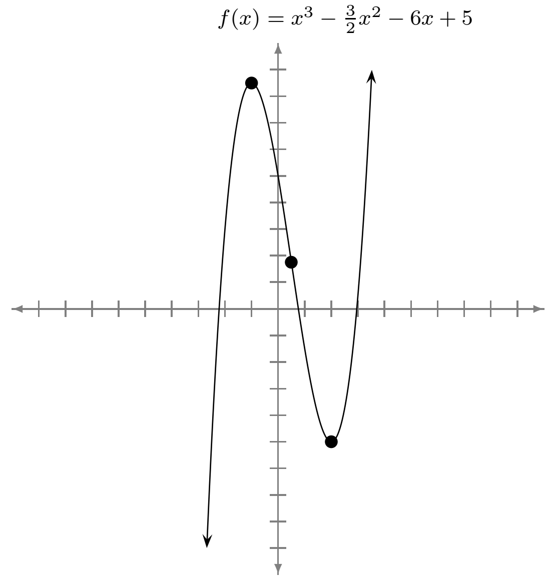
Local min: -5 at 2

$f''(x) = 6x - 3 = 3(2x-1)$

CU: $\left(\frac{1}{2},\infty\right)$

CD: $\left(-\infty,\frac{1}{2}\right)$

IP: $\left(\frac{1}{2},\frac{7}{4}\right)$



(10) b. $f(x) = \frac{1}{x^3+1} = (x^3+1)^{-1}$

HA: $y = 0$

VA: $x = -1$

OA: none

$f'(x) = -1\,(x^3+1)^{-2}\,(3x^2) = \frac{-3x^2}{(x^3+1)^2}$

Inc: never

Dec: $(-\infty,-1),(-1,\infty)$

Local max: none

Local min: none

$f''(x)$

$= \frac{-6x\left(x^3+1\right)^2 - \left(-3x^2\right)2\left(x^3+1\right)(3x^2)}{(x^3+1)^4}$

$= \frac{\left(x^3+1\right)\left[-6x\left(x^3+1\right)+18x^4\right]}{(x^3+1)^4}$

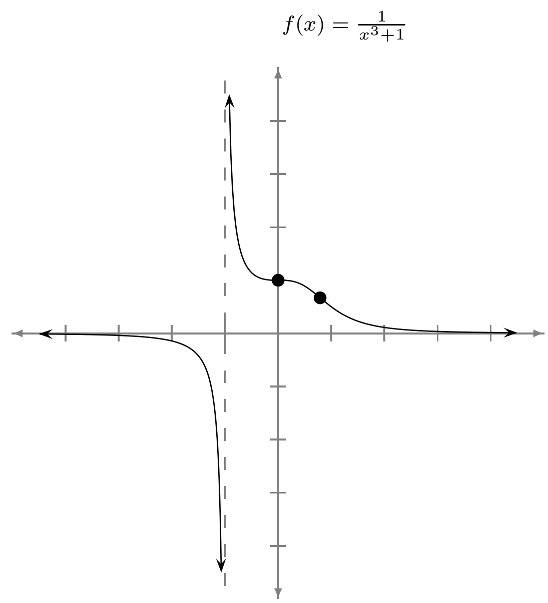
$= \frac{\left(x^3+1\right)\left(12x^4-6x\right)}{(x^3+1)^4}$

$= \frac{6x\left(2x^3-1\right)}{(x^3+1)^3}$

CU: $(-1,0)$, $\left(\sqrt[3]{\frac{1}{2}},\infty\right)$

CD: $(-\infty,-1)$, $\left(0,\sqrt[3]{\frac{1}{2}}\right)$

IP: $(0,1)$, $\left(\sqrt[3]{\frac{1}{2}},\frac{2}{3}\right)$



(10) c. $f(x) = \frac{x^2-1}{x+2}$

$= \frac{(x+1)(x-1)}{x+2}$

$= x - 2 + \frac{3}{x+2}$

$= x - 2 + 3\left(x+2\right)^{-1}$

HA: none

VA: $x =$ -2

OA: $y = x - 2$

$f'(x)$

$= 1 - 3\left(x+2\right)^{-2}$

$= 1 - \frac{3}{(x+2)^2}$

$= \frac{(x+2)^2-3}{(x+2)^2}$

$= \frac{x^2+4x+1}{(x+2)^2}$

$= \frac{\left[x-\left(-2-\sqrt{3}\right)\right]\left[x-\left(-2+\sqrt{3}\right)\right]}{(x+2)^2}$

Inc: $\left(-\infty,-2-\sqrt{3}\right], \left[-2+\sqrt{3},\infty\right)$

Dec: $\left[-2-\sqrt{3},-2+\sqrt{3}\right]$

Local max: $-\left(2\sqrt{3}+4\right)$ at $-2-\sqrt{3}$

Local min: $2\sqrt{3}+4$ at $-2+\sqrt{3}$

$f''(x)$

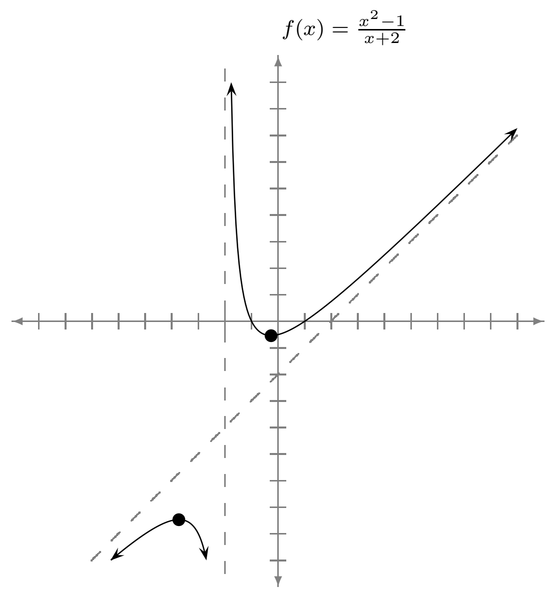
$= 6\left(x+2\right)^{-3}$

$= \frac{6}{(x+2)^3}$

CU: $(-2,\infty)$

CD: $(-\infty,-2)$

IP: none



Section 7.4: Exam 3

Exam 3 Math 1571 Fall 2015

(10) 18. For the following, find the intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use your answers to sketch the graph.

$f(x) = 2x^3 - 3x^2 - 12x + 7$

$f'(x)$

$= 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x-2)(x+1)$

Inc: (-∞,-1],[2,∞)

Dec: [-1,2]

Local max: 14 at -1

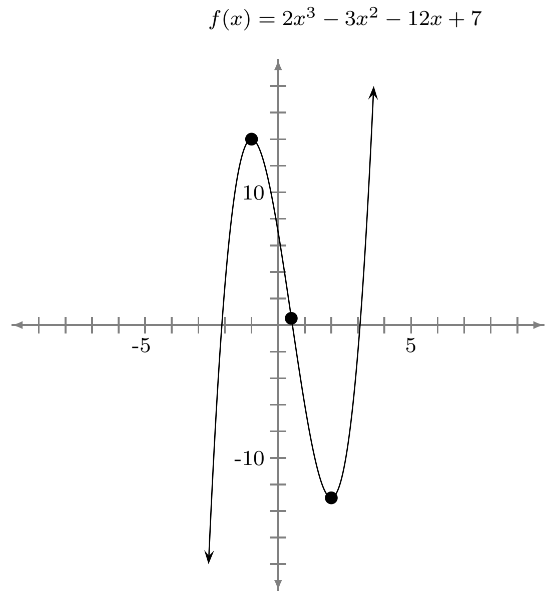
Local min: -13 at 2

$f''(x) = 12x - 6 = 6(2x-1)$

CU: $\left(\frac{1}{2},\infty\right)$

CD: $\left(\text{-}\infty,\frac{1}{2}\right)$

IP: $\left(\frac{1}{2},\frac{1}{2}\right)$



(10) 19. Find the equation of the line that is tangent to the given curve at the specified point.

$x^2y + xy^3 = 6$; (2,1)

$\frac{d}{dx}\left(x^2y + xy^3\right) = \frac{d}{dx}6$

$2xy + x^2y' + y^3 + x \cdot 3y^2y' = 0$

$$y' = \frac{\text{-}2xy - y^3}{x^2 + 3xy^2}$$

$x = 2$

$y = 1$

$y' = \frac{-1}{2}$

$y - 1 = \frac{-1}{2}(x-2)$

20. A ball is thrown upward with an initial velocity of of 48 ft/sec from a height of 160 ft.

(10) a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 48t + 160$

(10) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 48$

$\text{-}32t + 48 = 0$

$t = \frac{3}{2}$

$h\left(\frac{3}{2}\right) = \text{-}36 + 72 + 160 = 196$

196 feet

(10) c. Find the time that the ball hits the ground.

$\text{-}16t^2 + 48t + 160 = 0$

$t^2 - 3t - 10 = 0$

$(t-5)(t+2) = 0$

$t = 5$

(10) 21. A balloon is being inflated so that its volume is increasing at a rate of π in^3 per second. At what rate is the radius of the balloon increasing when the radius is 2 in?

$\frac{dV}{dt} = \frac{d}{dt}(\frac{4}{3}\pi r^3)$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

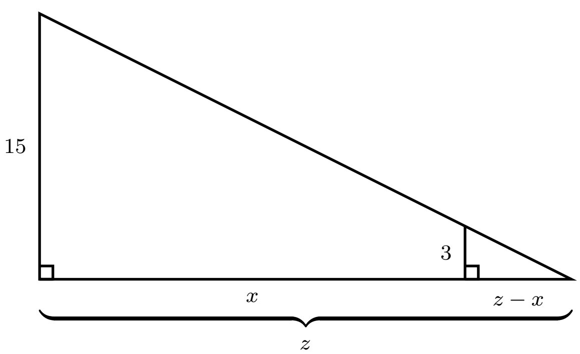
$\pi = 4\pi \cdot 2^2 \frac{dr}{dt}$

$\pi = 16\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{16}$

$\frac{1}{16}$ inches per second

(10) 22. A street light is mounted on top of a 15-ft pole. A child 3 ft tall is running away from the pole in such a way that the tip of the child's shadow is moving at a rate of 10 ft per second. At what rate is the child running?



$\frac{z}{z-x} = \frac{15}{3}$

$\frac{z}{z-x} = 5$

$z = 5(z-x)$

$5x = 4z$

$5\frac{dx}{dt} = 4\frac{dz}{dt}$

$5\frac{dx}{dt} = 4 \cdot 10$

$\frac{dx}{dt} = 8$

8 ft/sec

(10) 23. Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 3x^2 - 12x$ on [0,3].

$f'(x)$

$= 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x-2)(x+1)$

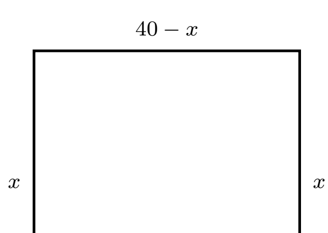
CN: -1, 2

$f(0) = 0$ (max)

$f(3) = \text{-}9$

$f(2) = \text{-}20$ (min)

(10) 24. A rectangle has perimeter 80 cm. What is its maximum possible area?



Let x be the length of one side.

The length of an adjacent side is $\frac{80-2x}{2} = 40 - x$.

$A(x) = x(40-x) = \text{-}x^2 + 40x$; $[0{,}40]$

$A'(x) = \text{-}2x + 40 = \text{-}2(x-20)$

$A(0) = 0$

$A(40) = 0$

$A(20) = 400$ (max)

(10) 25. Use a linear approximation or differential to approximate $\sqrt{65}$.

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{\text{-}\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$x = 64$

$f(64) = 8$

$f'(64) = \frac{1}{16}$

$L(x) = 8 + \frac{1}{16}(x-64)$

$L(65) = 8 + \frac{1}{16} = \frac{129}{16}$

(10) 26. Let $f(x) = \sin x \cdot \cos x + x$. Use the Mean Value Theorem to show that there is a number c between 0 and π such that $f'(c) = 1$.

***Proof*:** Note that f is continuous on $[0{,}\pi]$ and differentiable on $(0{,}\pi)$. By the MVT, there is a number $c \in (0{,}\pi)$ such that $f'(c) = \frac{f(\pi)-f(0)}{\pi-0} = 1$. ∎

27. Integrate.

(10) a. $\displaystyle\int_0^1 \big(3x^2+1\big)dx$

$= \big(x^3 + x\big)\Big|_0^1$

$= 2 - 0$

$= 2$

(10) b. $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$

$= \tan x\Big|_0^{\frac{\pi}{4}}$

$= 1 - 0$

$= 1$

(10) c. $\displaystyle\int (15x^4 + 9x^2 + 3)\sqrt{x^5 + x^3 + x}\,dx$

Let $u = x^5 + x^3 + x$.

Then $du = (5x^4 + 3x^2 + 1)dx$.

$\displaystyle\int (15x^4 + 9x^2 + 3)\sqrt{x^5 + x^3 + x}\,dx$

$= \displaystyle\int 3\sqrt{u}\,du$

$= \displaystyle\int 3\sqrt{u}\,du$

$= \displaystyle\int 3u^{\frac{1}{2}}\,du$

$= 2u^{\frac{3}{2}} + C$

$= 2\sqrt{u^3} + C$

$= 2\sqrt{(x^5 + x^3 + x)^3} + C$

(10) 28. Calculate the following limit. If you choose, you may identify the limit as an integral and use the Fundamental Theorem of Calculus Part II.

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(3+\frac{2i}{n}\right)+1\right]\cdot\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i}{n}+7\right)\frac{2}{n}$

$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} \left(\frac{4i}{n}+7\right)$

$= \lim_{n\to\infty} \frac{2}{n}\left(\sum_{i=1}^{n}\frac{4i}{n}+\sum_{i=1}^{n}7\right)$

$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4}{n}\sum_{i=1}^{n}i+\sum_{i=1}^{n}7\right)$

$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4}{n}\cdot\frac{n(n+1)}{2}+7n\right)$

$= \lim_{n\to\infty} \left(\frac{4n}{n+1}+14\right)$

$= 4+14$

$= 18$

Alternatively,

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(3+\frac{2i}{n}\right)+1\right]\cdot\frac{2}{n}$

$= \int_3^5 \left(2x+1\right)dx$

$= \left(x^2+x\right)\Big|_3^5$

$= 30-12$

$= 18$

(10) 29. Let $F(x) = \displaystyle\int_0^x \left(t^3\sin t\right)dt$. Use the Fundamental Theorem of Calculus Part I to find $F'(x)$.

$F'(x) = x^3\sin x$

Section 7.5: Others

Exam 2 Math 1571 Fall 2015

Name:________________________

<u>**Directions:**</u> Show all of your work and justify all of your answers. An answer without justification will receive a zero.

30. Differentiate.

(10) a. $f(x) = \sqrt[3]{x^4}+x^2+x-7$

(10) b. $g(x) = \dfrac{x^3+1}{\sin x}$

(10) c. $h(x) = \sqrt[5]{x^3+\sqrt{x}+1}$

(10) d. $f(x) = \tan x\cdot\sin x$

(10) 31. Calculate the following limit.

$\lim_{x\to\frac{\pi}{6}} \dfrac{\sin x-\frac{1}{2}}{x-\frac{\pi}{6}}$

32. Find the equation of the line that is tangent to the graph of the given curve at the specified point.

(10) a. $f(x) = \sqrt[3]{x+1}$; $x = 8$

(10) b. $xy + x^2 + y^2 = 1$; (1,-1)

(10) 33. Find the rate of change of the given function at the specified point.

$f(t) = \sqrt[3]{t^2+1}$; $t = \sqrt{26}$

34. For the following, find the intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(10) a. $f(x) = 2x^3 - 3x^2 - 12x$

(10) b. $f(x) = \dfrac{x^2+1}{x}$

Exam 3 Math 1571 Fall 2015

Name:______________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(10) 35. A can in the shape of a right circular cylinder is being filled with water at a rate of 18 cm^3 per second. The radius of the base of the can is 3 cm. At what rate is the water level rising? Recall: $V = \pi r^2 h$.

(10) 36. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100cm^2? Recall: $A = \frac{1}{2}bh$.

(10) 37. Find the absolute maximum and absolute minimum of $f(x) = \tan x$ on $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

(10) 38. A cardboard box manufacturer wishes to make open boxes from pieces of cardboard 12 in square by cutting equal squares from the four corners and turning up the sides. Find the length of the side of the square to be cut out in order to obtain a box of the largest possible volume.

(10) 39. Use a linear approximation or differential to approximate 3.999^2

(10) 40. Let $f(x) = \sqrt[5]{\sin x + \cos x} + x$. Use the Mean Value Theorem to show that there is a number c between 0 and $\frac{\pi}{2}$ such that $f'(c) = 1$.

41. Integrate.

(10) a. $\displaystyle\int_0^1 \big(3\sqrt{x}+2\big)dx$

(10) b. $\displaystyle\int \sec x \tan x \, dx$

(10) c. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$

(10) 42. Identify a, b, and $f(x)$ such that

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(1+\tfrac{2i}{n}\right)^2+\tan\left(1+\tfrac{2i}{n}\right)\right]\cdot\tfrac{2}{n}.$$

(10) 43. Let $F(x) = \displaystyle\int_0^{x^3}\big(t\sin t\big)dt$. Use the Fundamental Theorem of Calculus Part I to find $F'(x)$.

Exam 3 Math 1571 Fall 2015

Name:________________________

Directions: Show all of your work and justify all of your answers. An answer without justification will receive a zero.

(10) 44. A balloon is being inflated so that its volume is increasing at a rate of π in^3 per second. At what rate is the radius of the balloon increasing when the radius is 2 in? Recall: $V = \frac{4}{3}\pi r^3$.

(10) 45. A street light is mounted on top of a 15-ft pole. A child 3 ft tall is running away from the pole in such a way that the tip of the child's shadow is moving at a rate of 10 ft per sec. At what rate is the child running?

(10) 46. Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 3x^2 - 12x$ on [0,3].

(10) 47. A rectangle has perimeter 80 cm. What is its maximum possible area?

(10) 48. Use a linear approximation or differential to approximate $\sqrt{65}$.

(10) 49. Let $f(x) = \sin x \cdot \cos x + x$. Use the Mean Value Theorem to show that there is a number c between 0 and π such that $f'(c) = 1$.

50. Integrate.

(10) a. $\displaystyle\int_0^1 \left(3x^2+1\right) dx$

(10) b. $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$

(10) c. $\displaystyle\int (15x^4+9x^2+3)\sqrt{x^5+x^3+x}\, dx$

(10) 51. Calculate the following limit. If you choose, you may identify the limit as an integral and use the Fundamental Theorem of Calculus Part II.

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(3 + \frac{2i}{n}\right) + 1\right] \cdot \frac{2}{n}$

(10) 52. Let $F(x) = \displaystyle\int_0^x \left(t^3 \sin t\right) dt$. Use the Fundamental Theorem of Calculus Part I to find $F'(x)$.

Chapter 8: Math 1571 Fall 2016

Section 8.1: Quizzes

Quiz 63 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, give the domain and range. Use your answers to sketch the graph.

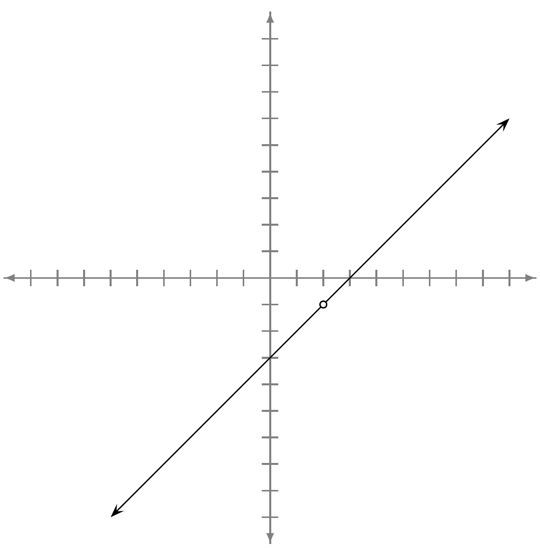
(1) a. $f(x) = \dfrac{x^2 - 5x + 6}{x - 2}$

$f(x) = \frac{(x-3)(x-2)}{x-2} \neq x - 3$

Domain: (-∞,2) $\cup$ (2,∞)

Range: (-∞,-1) $\cup$ (-1,∞)

$f(x) = \dfrac{x^2 - 5x + 6}{x - 2}$



(1) b. $f(x) = \sqrt{4 - x^2}$

D: [-2,2]

R: [0,2]

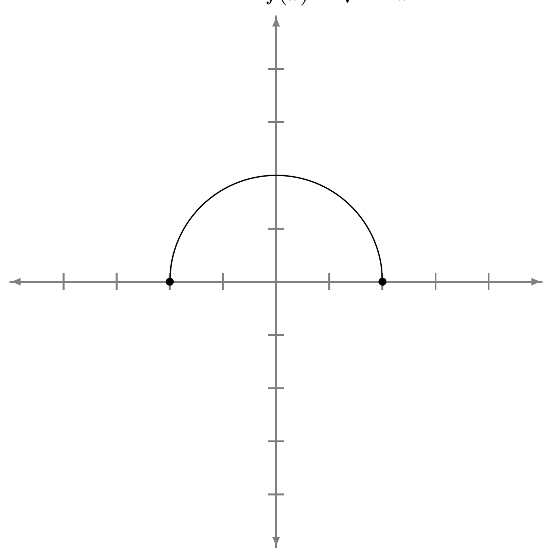
$y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$

Top half of circle with center (0,0) and radius 2.

$f(x) = \sqrt{4 - x^2}$



Quiz 64 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For the functions below, calculate and simplify $(f \circ g)(x)$ and $(g \circ f)(x)$. Give the domain, range, and graph of each. Hint: The functions $(f \circ g)(x)$ and $(g \circ f)(x)$ are different and have different domains.

$f(x) = x^2$

$g(x) = \sqrt{x}$

$(f \circ g)(x)$

$= (\sqrt{x})^2$

$= x$

D: $[0,\infty)$

R: $[0,\infty)$

$(g \circ f)(x)$

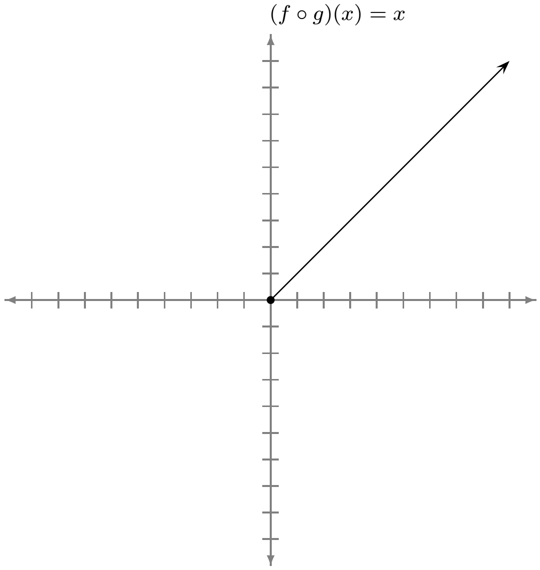
$= \sqrt{x^2}$

$= |x|$

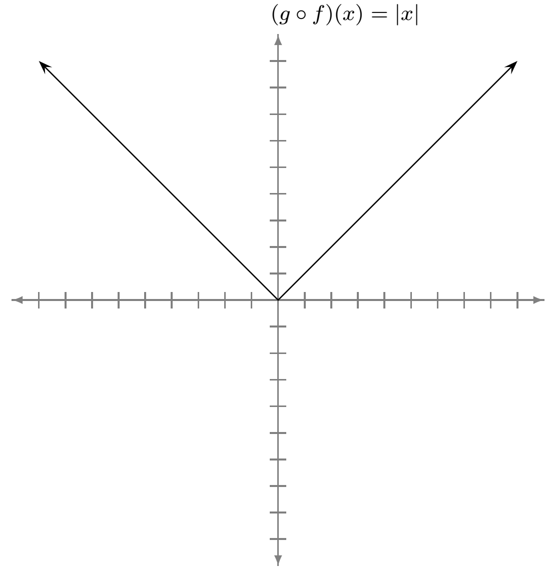
D: $\mathbb{R}$

R: $[0,\infty)$

$(f \circ g)(x) = x$



$(g \circ f)(x) = |x|$



Quiz 65 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For the following, find the amplitude, period, phase shift, and average value. Sketch the graph.

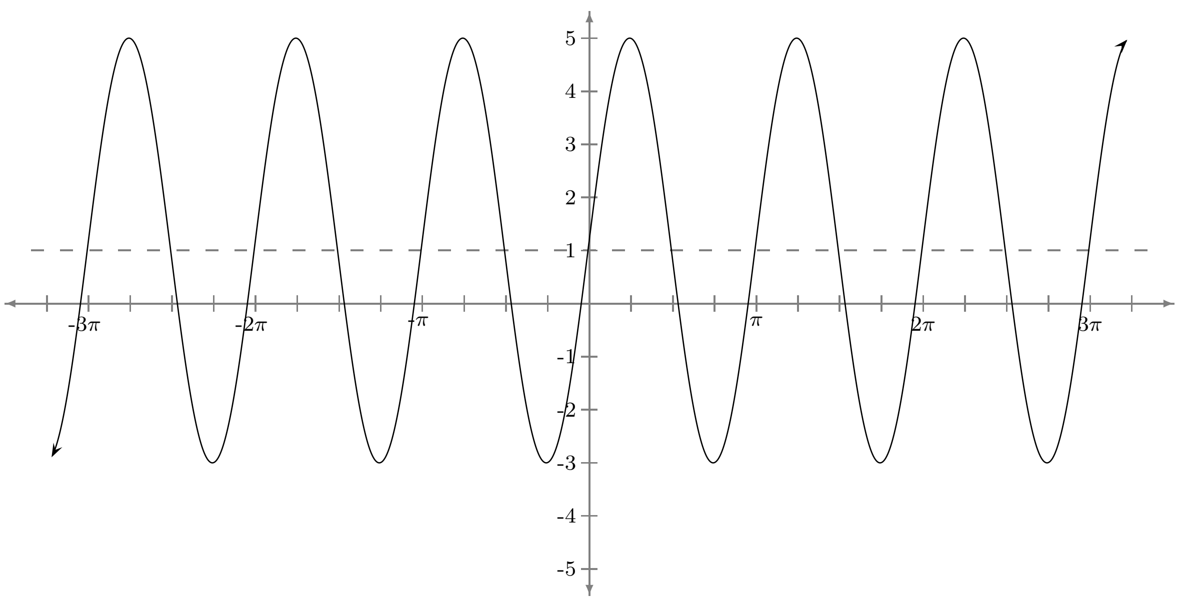
$y = 4\sin(2x+\pi)+1$

Amplitude: 4

Period: π

Phase Shift: $\frac{\pi}{2}$

Average Value: 1



2. Given that $0 \le \alpha \le \frac{\pi}{2}$ and $\sin\alpha = \frac{\sqrt{5}}{3}$, find each of the following.

(1) a. $\cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$

$\frac{5}{9} + \cos^2\alpha = 1$

$\cos^2\alpha = \frac{4}{9}$

$\cos\alpha = \frac{2}{3}$

(1) b. $\sin 2\alpha$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{\sqrt{5}}{3}\cdot\frac{2}{3} = \frac{4\sqrt{5}}{9}$

Quiz 66 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \to 3}\big(5x+1\big) = 16$.

First, we need to find δ for a given ε.

$|(5x+1) - 16| < \varepsilon$

$|5x - 15| < \varepsilon$

$5|x-3| < \varepsilon$

$|x-3| < \frac{\varepsilon}{5}$

***Proof*:** Let $\varepsilon > 0$ be given. Put $\delta = \frac{\varepsilon}{5}$. If $|x-3| < \delta$, then

$|x-3| < \delta$

$|x-3| < \frac{\varepsilon}{5}$

$5|x-3| < \varepsilon$

$|5x-15| < \varepsilon$

$|5x+1-16| < \varepsilon$

$|(5x+1) - 16| < \varepsilon$.

Therefore, $\lim_{x \to 3}\big(5x+1\big) = 16$. ■

Quiz 67 Name:_________________________

Directions: Show all of your work and justify all of your answers.

1. Compute the following limits.

(1) a. $\lim_{x\to 2}\left(x^3+x^2-4\right)=8$

(1) b. $\lim_{x\to 6}\dfrac{x-6}{x^2-5x-6}$

$=\lim_{x\to 6}\dfrac{x-6}{(x-6)(x+1)}$

$=\lim_{x\to 6}\dfrac{1}{x+1}$

$=\dfrac{1}{7}$

(1) c. $\lim_{x\to\sqrt{5}}\dfrac{x-\sqrt{5}}{x^2-5}$

$=\lim_{x\to\sqrt{5}}\dfrac{x-\sqrt{5}}{(x-\sqrt{5})(x+\sqrt{5})}$

$=\lim_{x\to\sqrt{5}}\dfrac{1}{(x+\sqrt{5})}$

$=\dfrac{1}{2\sqrt{5}}$

Quiz 68 Name:________________________

Directions: Show all of your work and justify all of your answers.

For each of the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

(2) 1. $f(x) = \dfrac{x^2+3x-4}{x^2+2x-8}$

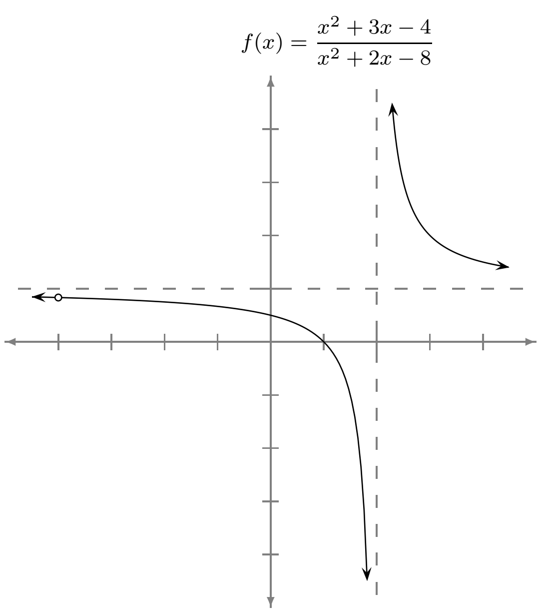
$= \dfrac{(x+4)(x-1)}{(x+4)(x-2)}$

$\neq \dfrac{x-1}{x-2}$

HA: $y = 1$

VA: $x = 2$

OA: none



(2) 2. $g(x) = \dfrac{x^2-9}{x-2}$

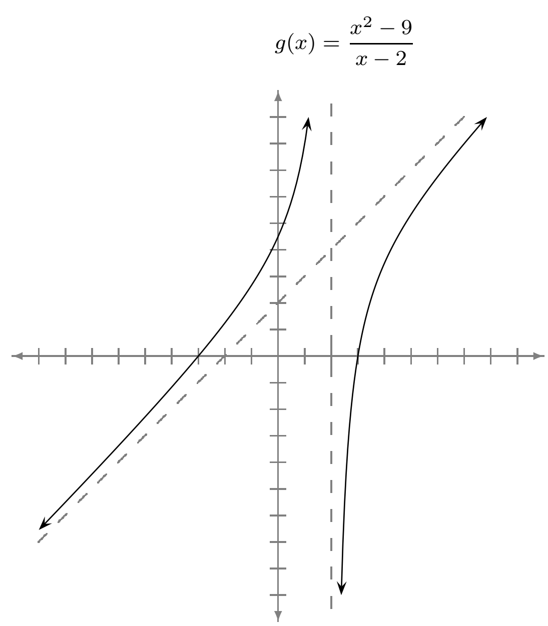
$= \dfrac{(x+3)(x-3)}{x-2}$

$= x + 2 - \frac{5}{x-2}$

HA: none

VA: $x = 2$

OA: $y = x + 2$



Quiz 69 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Use the definition of derivative to differentiate each of the following.

a. $f(x) = x^2 + 1$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h\to 0} \big(2x + h\big)$$

$$= 2x$$

b. $f(x) = x + \sqrt{x}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h) + \sqrt{x+h} - (x + \sqrt{x})}{h}$$

$$= \lim_{h\to 0} \frac{x + h + \sqrt{x+h} - x - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{h + \sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{h}{h} + \frac{\sqrt{x+h} - \sqrt{x}}{h}\right)$$

$$= \lim_{h\to 0} \left(1 + \frac{\sqrt{x+h} - \sqrt{x}}{h}\right)$$

$$= 1 + \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= 1 + \lim_{h\to 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= 1 + \lim_{h\to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= 1 + \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= 1 + \lim_{h\to 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})}$$

$$= 1 + \tfrac{1}{2\sqrt{x}}$$

Quiz 70 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For each of the following find the equation of the line that is tangent to the graph of the given function at the specified point.

a. $y = 4x^2 - x + 2$; $x =$ -2

$\frac{dy}{dx} = 8x - 1$

$x =$ -2

$y = 20$

$\frac{dy}{dx} =$ -17

$y - 20 =$ -17$(x + 2)$

b. $g(x) = \sqrt{x} - \sqrt[3]{x}$; $x = 64$

$g(x) = x^{\frac{1}{2}} - x^{\frac{1}{3}}$

$g'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{3}x^{-\frac{2}{3}}$

$$g'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}}$$

$g(64) = 4$

$g'(64) = \frac{1}{24}$

$y - 4 = \frac{1}{24}(x - 64)$

Quiz 71 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, find the equation of the line that is tangent to the graph of the given curve at the specified point.

(1) a. $f(\theta) = \sin^2\theta$; $\theta = \frac{\pi}{3}$

$f'(\theta) = 2\sin\theta\cos\theta$

$\theta = \frac{\pi}{3}$

$y = f\left(\frac{\pi}{3}\right) = \frac{3}{4}$

$m = f'\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$y - \frac{3}{4} = \frac{\sqrt{3}}{2}\left(\theta - \frac{\pi}{3}\right)$

(1) b. $4x^2y^3 + xy + y = 6$; (1,1)

$\frac{d}{dx}\left(4x^2y^3 + xy + y\right) = \frac{d}{dx}6$

$8xy^3 + 12x^2y^2y' + y + xy' + y' = 0$

$12x^2y^2y' + xy' + y' = \text{-}8xy^3 - y$

$y'(12x^2y^2 + x + 1) = \text{-}8xy^3 - y$

$y' = \frac{\text{-}8xy^3 - y}{12x^2y^2 + x + 1}$

$x = 1$

$y = 1$

$m = y' = \text{-}\frac{9}{14}$

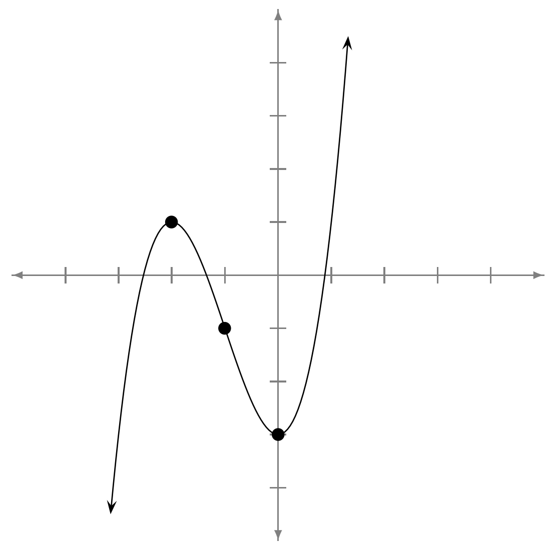
$y - 1 = \text{-}\frac{9}{14}(x - 1)$

Quiz 72 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(2) a. $g(x) = x^3 + 3x^2 - 3$



$g(2) = 17$

$g(-2) = 1$

Neither even nor odd.

HA: none

VA: none

OA: none

$g'(x) = 3x^2 + 6x = 3x(x+2)$

Dec: [-2,0]

Inc: $(-\infty,-2] \cup [0,\infty)$

Local max: 1 at -2

Local min: -3 at 0

$g''(x) = 6x + 6 = 6(x+1)$

CD: $(-\infty,-1)$

CU: $(-1,\infty)$

IP: (-1,-1)

$g(x) = x^3 + 3x^2 - 3$

(2) b. $f(x) = \dfrac{x^2+1}{x}$

$f(x) = x + \frac{1}{x}$

$f(\text{-}x) = \dfrac{x^2+1}{\text{-}x} = \text{-}f(x)$

Odd

HA: none

VA: $x = 0$

OA: $y = x$

$f'(x) = 1 - \frac{1}{x^2} = \dfrac{x^2-1}{x^2}$

Dec: [-1,0), (0,1])

Inc: (-∞,-1], [1,∞)

Local max: -2 at -1

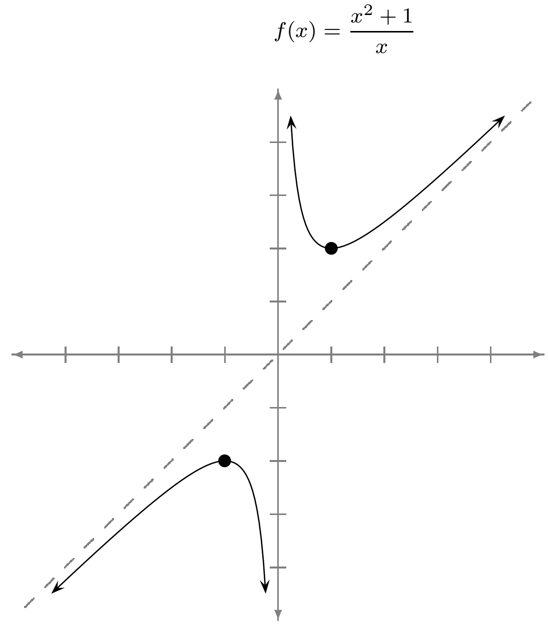
Local min: 2 at 1

$f''(x) = \frac{2}{x^3}$

CD: (-∞,0)

CU: (0,∞)

IP: none



Quiz 73 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. A ball is tossed upward with an initial velocity of of 4 ft/sec from a height of 12 ft.

a. Give the height of the ball as a function of time.

$h(t) = \text{-}16t^2 + 4t + 12$

b. Find the maximum height of the ball.

$h(t) = \text{-}16t^2 + 4t + 12$

$v(t) = h'(t) = \text{-}32t + 4$

$v(t) = 0$

$\text{-}32t + 4 = 0$

$t = \frac{1}{8}$

$h\left(\frac{1}{8}\right) = \frac{25}{2}$

12.5 ft

c. When does the ball hit the ground?

$\text{-}16t^2 + 4t + 12 = 0$

$\text{-}4(4t^2 - t - 3) = 0$

$4t^2 - t - 3 = 0$

$(4t + 3)(t - 1) = 0$

$t = 1$

1 sec.

d. What is the velocity of the ball when the ball hits the ground?

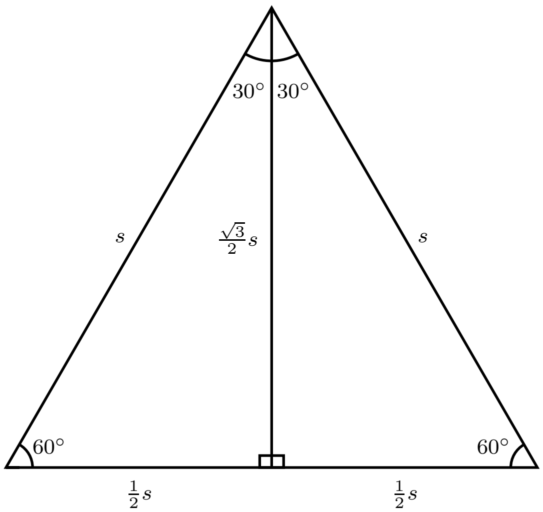
$v(1) = \text{-}28$

-28 ft/sec

Quiz 74 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. An equilateral triangle is expanding in such a way that its area is increasing at a rate of 1 square inch per second. At what rate is the length of a side increasing when the side is 2 inches long?



$A = \frac{1}{2}s \cdot \frac{\sqrt{3}}{2}s$

$A = \frac{\sqrt{3}}{4}s^2$

$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{\sqrt{3}}{4}s^2\right)$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2}s \cdot \frac{ds}{dt}$

$1 = \frac{\sqrt{3}}{2} \cdot 2 \cdot \frac{ds}{dt}$

$\frac{ds}{dt} = \frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}$ in/sec

Quiz 75 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Find the absolute maximum and absolute minimum of the given function on the given interval.

$g(x) = x^3 - 3x^2 + 1$; [-1,1]

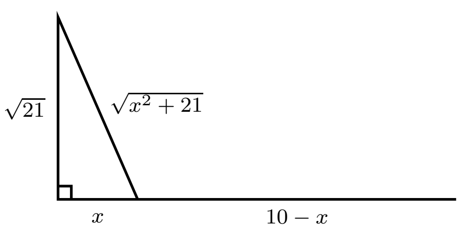
$g'(x) = 3x^2 - 6x = 3x(x-2)$

$g(\text{-}1) = \text{-}3$ (min)

$g(1) = \text{-}1$

$g(0) = 1$ (max)

(2) 2. A man is standing at the edge of a river that is $\sqrt{21}$ miles wide. He needs to reach a point on the other side of the river that is 10 miles upstream. The man can walk 5 miles per hour and row 9 miles per hour against a 7 mile-per-hour current. How far upstream should he row in order to reach his destination in the least amount of time?



$$T(x) = \frac{\sqrt{x^2+21}}{2} + \frac{10-x}{5}$$

$$T(x) = \tfrac{1}{2}(x^2+21)^{\frac{1}{2}} - \tfrac{1}{5}x + 2$$

$$T'(x) = \tfrac{1}{4}(x^2+21)^{\text{-}\frac{1}{2}}(2x) - \tfrac{1}{5}$$

$$T'(x) = \frac{x}{2\sqrt{x^2+21}} - \frac{1}{5}$$

$$\frac{x}{2\sqrt{x^2+21}} - \frac{1}{5} = 0$$

$$\frac{x}{2\sqrt{x^2+21}} = \frac{1}{5}$$

$$5x = 2\sqrt{x^2+21}$$

$$25x^2 = 4(x^2+21)$$

$$25x^2 = 4x^2 + 81$$

$$21x^2 = 81$$

$$x^2 = 4$$

$$x = 2$$

$$T(0) = 2 + \tfrac{\sqrt{21}}{2}$$

$$T(10) = \tfrac{11}{2}$$

$$T(2) = \tfrac{41}{10} \quad \text{(min)}$$

Quiz 76 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Let $g(x) = \sqrt[3]{x}$. Find the linearization of f at 125 and use it to approximate $\sqrt[3]{126}$.

$f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f(125) = 5$

$f'(125) = \frac{1}{75}$

$L(x) = \frac{1}{75}(x - 125) + 5$

$f(126) \approx L(126) = 5\frac{1}{75}$

(1) 2. The outside diameter of a sphere is 10 meters. If the material used to make the sphere is 1 centimeter thick, use differentials approximate the volume of the material used to make the sphere.

$V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$

$dV = 4\pi 500^2(-1)$

$dV = -1000000\pi$

$1{,}000{,}000\pi$ cm^3

Quiz 77 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Integrate.

(1) a. $\displaystyle\int_1^2 \big(x^2 - x + 1\big)dx$

$= \big(\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + x\big)\Big|_1^2$

$= \tfrac{8}{3} - 2 + 2 - \big(\tfrac{1}{3} - \tfrac{1}{2} + 1\big)$

$= \tfrac{11}{6}$

(1) b. $\displaystyle\int \frac{6x^2+4}{(x^3+2x+1)^7}\,dx$

Let $u = x^3 + 2x + 1$ and $du = (3x^2 + 2)\,dx$.

$\displaystyle\int \frac{6x^2+4}{(x^3+2x+1)^7}\,dx$

$\displaystyle= \int \frac{2}{u^7}\,du$

$\displaystyle= \int 2u^{-7}\,du$

$= -\tfrac{1}{3}u^{-6} + C$

$\displaystyle= \frac{-1}{3(x^3+2x+1)^6} + C$

(1) c. $\displaystyle\int_0^2 \frac{x}{\sqrt{x+1}}\,dx$

Let $u = x + 1$ and $du = dx$.

$\displaystyle\int_0^2 \frac{x}{\sqrt{x+1}}\,dx$

$\displaystyle= \int_1^3 \frac{u-1}{\sqrt{u}}\,du$

$\displaystyle= \int_1^3 \big(\sqrt{u} - \tfrac{1}{\sqrt{u}}\big)du$

$\displaystyle= \int_1^3 \Big(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\Big)du$

$= \Big(\tfrac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\Big)\Big|_1^3$

$= \tfrac{4}{3}$

Quiz 78 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Find the area of the region enclosed by the given curves.

$y = x^2 - x - 3$

$y = x + 5$

$x^2 - x - 3 = x + 5$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$\int_{-2}^{4} \big[(x+5) - (x^2 - x - 3)\big] dx$

$= \int_{-2}^{4} \big(-x^2 + 2x + 8\big) dx$

$= \left(-\frac{1}{3}x^3 + x^2 + 8x\right)\Big|_{-2}^{4}$

$= -\frac{64}{3} + 16 + 32 - (\frac{8}{3} + 4 - 16)$

$= 36$

(1) 2. Find the average value of the given function over the given interval.

$f(x) = 2\sin x \cos x$; $[0,\pi]$

$\frac{1}{\pi}\int_{0}^{\pi} 2\sin x \cos x \, dx$

$= \frac{1}{\pi}\left(\sin^2 x\right)\Big|_{0}^{\pi}$

$= \frac{1}{\pi}\,(1 - -1)$

$= 0$

Quiz 79 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Let $\mathcal{R}$ be the region bounded by the curves $y = x$ and $y = \sqrt{x}$ from 0 to 1.

(2) a. Find the volume of the solid formed by revolving $\mathcal{R}$ about the x-axis.

i. $\frac{\pi}{6}$ ***ii.*** $\frac{\pi}{3}$ ***iii.*** 1 ***iv.*** $\frac{\sqrt{2}}{2}$ ***v.*** $\sqrt{2}$ ***vi.*** None of the above.

(2) b. Find the volume of the solid formed by revolving $\mathcal{R}$ about the y-axis.

i. $\sqrt{3}$ ***ii.*** $2\sqrt{3}$ ***iii.*** $\frac{\pi}{8}$ ***iv.*** $\frac{2\pi}{7}$ ***v.*** $\frac{2\pi}{15}$ ***vi.*** None of the above.

Section 8.2: Exam 1

Exam 1 Math 1571 Fall 2016

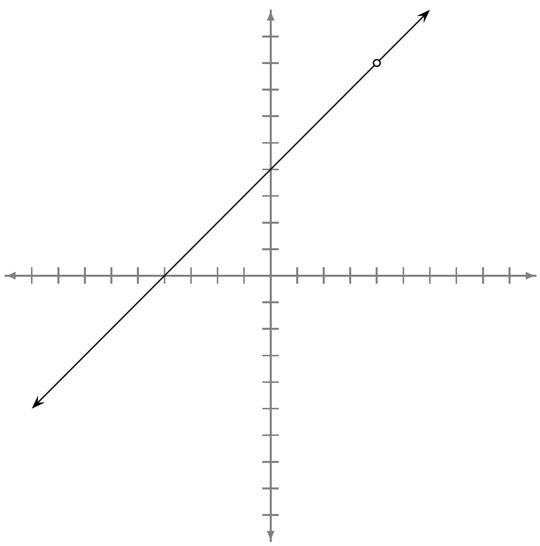
2. For the following, give the domain and range. Sketch the graph.

(10) a. $f(x) = \dfrac{x^2 - 16}{x - 4}$

$$f(x) = \frac{x^2 - 6}{x - 4} = \frac{(x-4)(x+4)}{x-4} \neq x + 4$$

D: (-∞,4) ∪ (4,∞)

R: (-∞,8) ∪ (8,∞)



(10) 3. An inspector in a factory earns \$10.00 per hour and works 8 hours per day. The inspector's quota (number of boxes that must be inspected) is 300 boxes. For every box the inspector inspects in excess of 300, he earns \$0.15. Express the inspector's daily wages as a function of the number of boxes he inspects.

$f(x) = 80 + 0.15(x - 300)$

4. Let $f(x) = \sqrt{x+1}$ and $g(x) = x^2$.

(10) a. Find $(f \circ g)(x)$ and give its domain and range.

$(f \circ g)(x) = f[g(x)] = f(x^2) = \sqrt{x^2+1}$

D: $\mathbb{R}$

R: $[1,\infty)$

(10) b. Find $(g \circ f)(x)$ and give its domain and range.

$(g \circ f)(x) = g[f(x)] = g(\sqrt{x+1}) = (\sqrt{x+1})^2 = x+1$

D: $[-1,\infty)$

R: $[0,\infty)$

(10) 5. List the transformations needed to obtain the graph of $y = -2f(x-6)+4$ from the graph of $y = f(x)$.

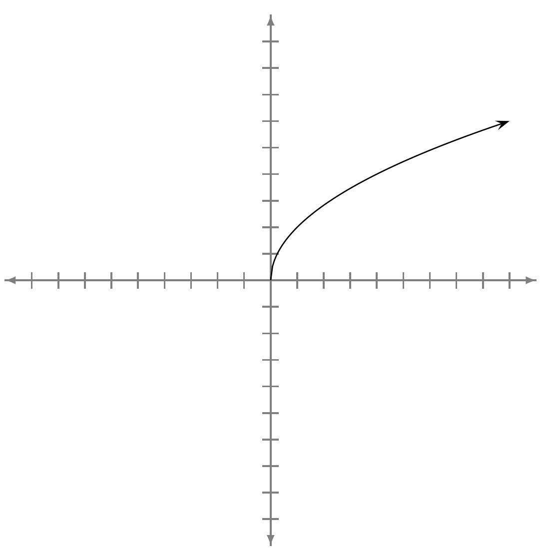
a. Shift right 6 units.

b. Vertical stretch by a factor of 2.

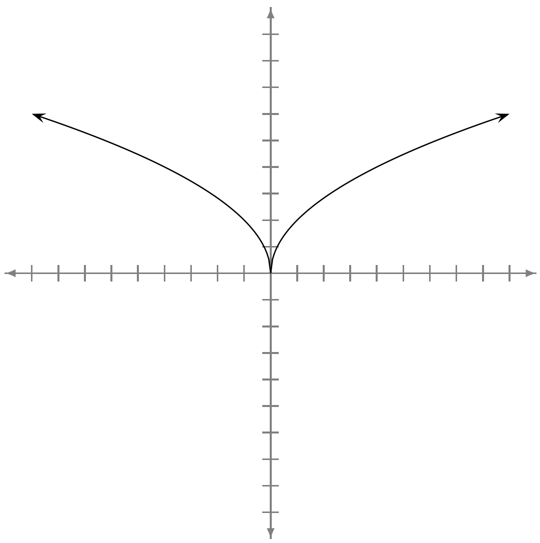
c. Reflect over the x-axis.

d. Shift up 4 units.

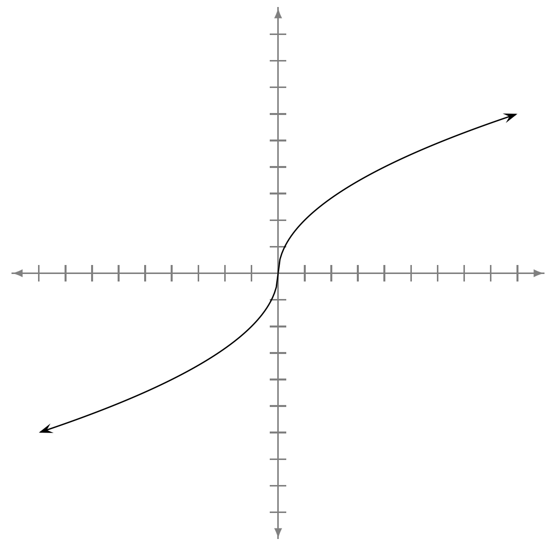
6. Complete the graph of the function under the given conditions.



(10) a. The function is even.



(10) b. The function is odd.



(10) 7. Prove the following identity.

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$\sin\left(\theta + \frac{\pi}{2}\right) = (\sin\theta)(\cos\frac{\pi}{2}) + (\cos\theta)(\sin\frac{\pi}{2}) = (\sin\theta)\cdot 0 + (\cos\theta)\cdot 1 = \cos\theta$

8. Calculate the following limits.

(10) a. $\lim\limits_{x\to 1} \frac{x-1}{\sqrt{x}-1}$

$= \lim\limits_{x\to 1} \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}-1}$

$= \lim\limits_{x\to 1}\left(\sqrt{x}+1\right)$

$= 2$

(10) b. $\lim\limits_{x\to 1^+} \frac{x-1}{\sqrt{x-1}} = \lim\limits_{x\to 1^+} \sqrt{x-1} = 0$

(10) c. $\lim\limits_{x\to 7} \frac{x^2-5x-14}{x^2-4x-21}$

$= \lim\limits_{x\to 7} \frac{(x-7)(x+2)}{(x-7)(x+3)}$

$= \lim\limits_{x\to 7} \frac{x+2}{x+3}$

$= \frac{9}{10}$

(10) d. $\lim\limits_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x^3}\right)$

$-\sqrt{x} \le \sqrt{x}\sin\left(\frac{1}{x^3}\right) \le \sqrt{x}$

$\lim\limits_{x\to 0^+} -\sqrt{x} = 0$

$\lim\limits_{x\to 0^+} \sqrt{x} = 0$

$\lim\limits_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x^3}\right) = 0$ (Squeeze Theorem)

(10) e. $\lim\limits_{\theta\to 0} \frac{\tan 5\theta}{\theta}$

$= \lim\limits_{\theta\to 0} \frac{\sin 5\theta}{\theta\cos 5\theta}$

$= \lim\limits_{\theta\to 0} \frac{5\sin 5\theta}{5\theta\cos 5\theta}$

$= \lim\limits_{\theta\to 0}\left(5\cdot\frac{\sin 5\theta}{5\theta}\cdot\frac{1}{\cos 5\theta}\right)$

$= 5\cdot 1\cdot 1$

$= 5$

(10) 9. Note that $\lim\limits_{x\to 4}\left(\frac{1}{2}x+1\right) = 3$. For a given ε, find δ such that $\left|\left(\frac{1}{2}x+1\right)-3\right| < \varepsilon$ whenever $|x-4| < \delta$.

$\left|\left(\frac{1}{2}x+1\right)-3\right|<\varepsilon$

$\left|\frac{1}{2}x-2\right|<\varepsilon$

$\left|\frac{1}{2}(x-4)\right|<\varepsilon$

$\frac{1}{2}|x-4|<\varepsilon$

$|x-4|<2\varepsilon$

Let $\delta = 2\varepsilon$.

(10) 10. For the following function, define α such that the given function is continuous at 3.

$$g(x) = \begin{cases} x^2+x-2, & \text{if } x \leq 3 \\ \\ \alpha, & \text{if } x > 3 \end{cases}$$

$\lim\limits_{x\to 3^-} g(x) = \lim\limits_{x\to 3^-} \left(x^2+x-2\right) = 10$

Let $\alpha = 10$.

(10) 11. Show that the following equation has a solution between 1 and 2.

$x^5+x^2-3=0$

***Proof*:**

Let $f(x) = x^5+x^2-3$ and note that f is continuous on [1,2] since f is a polynomial. Since $f(1)$ = -1 and $f(2) = 33$, the Intermediate Value Theorem implies that there is a number $c \in$ [1,2] such that $f(c) = 0$. ■

Section 8.3: Exam 2

Exam 2 Math 1571 Fall 2016

12. Use the definition of derivative to differentiate the following.

$f(x) = \dfrac{1}{x+1}$

$\displaystyle\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \displaystyle\lim_{h\to 0} \frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{h}$

$= \displaystyle\lim_{h\to 0} \left[\tfrac{(x+1)-(x+h+1)}{(x+h+1)(x+1)} \cdot \tfrac{1}{h}\right]$

$= \displaystyle\lim_{h\to 0} \tfrac{x+1-x-h-1}{h(x+h+1)(x+1)}$

$= \displaystyle\lim_{h\to 0} \tfrac{-h}{h(x+h+1)(x+1)}$

$= \displaystyle\lim_{h\to 0} \tfrac{-1}{(x+h+1)(x+1)}$

$= -\tfrac{1}{(x+1)^2}$

13. Differentiate.

(10) a. $f(x) = (x^3+4)^7$

$f'(x) = 7\left(x^3+4\right)^6\left(3x^2\right)$

(10) b. $g(x) = \cos(x^8)$

$g'(x) = -\sin(x^8)(8x^7)$

(10) c. $y = \sec\theta$

$y' = \sec\theta\tan\theta$

(10) d. $h(x) = x^2\tan x$

$h'(x) = 2x\tan x + x^2\sec^2 x$

(10) e. $f(x) = \dfrac{x^3+x+1}{x^2-2}$

$f'(x) = \dfrac{(3x^2+1)(x^2-2)-(x^3+x+1)(2x)}{(x^2-2)^2}$

14. For each of the following, find the equation of the line that is tangent to the given curve at the indicated point.

(10) a. $f(x) = x^2+4$; $x = 1$

$f'(x) = 2x$

$x = 1$

$y = f(1) = 5$

$m = f'(1) = 2$

$y - 5 = 2(x-1)$

(10) b. $x^2y^3 + x + y = 7$; (2,1)

$\frac{d}{dx}\left(x^2y^3+x+y\right) = \frac{d}{dx}7$

$2xy^3 + 3x^2y^2y' + 1 + y' = 0$

$y' = \frac{-2xy^3-1}{3x^2y^2+1}$

$x = 2$

$y = 1$

$m = y' = -\frac{5}{13}$

$y - 1 = -\frac{5}{13}(x-2)$

15. A particle moves along a coordinate line in such a way that its position at time t is $s(t) = 2t^3 - 3t^2 - 12t + 12$.

(10) a. Find the velocity at time t.

$v(t) = s'(t) = 6t^2 - 6t - 12$

(10) b. Find the time(s) when the velocity of the object is 0.

$6t^2 - 6t - 12 = 0$

$t^2 - t - 2 = 0$

$(t - 2)(t + 1) = 0$

$t = 2$

(10) c. Find the total distance traveled by the object during the first 3 seconds.

$|s(2) - s(0)| + |s(3) - s(2)| = |\text{-}8 - 12| + |3 - \text{-}8| = 31$

16. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(10) a. $f(x) = x^3 - 3x - 1$

HA: none

VA: none

OA: none

$f'(x)$

$= 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3(x + 1)(x - 1)$

Inc: (-∞,-1],[1,∞)

Dec: [-1,1]

Local max: 1 at -1

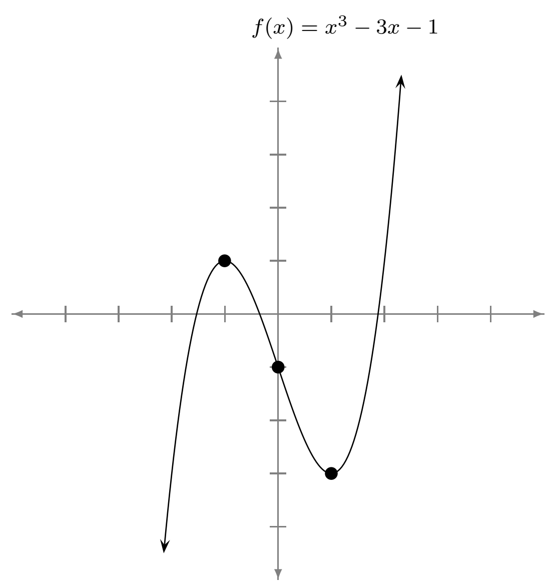
Local min: -3 at 1

$f'(x) = 6x$

CD: (-∞,0)

CU: (0,-∞)

IP: (0,-1)



(10) b. $g(x)$

$= \dfrac{x^2-2x+2}{x-1}$

$= x-1+\frac{1}{x-1}$

$= x-1+(x-1)^{-1}$

HA: none

VA: $x=1$

OA: $y=x-1$

$g'(x)$

$= 1-(x-1)^{-2}$

$= 1-\frac{1}{(x-1)^2}$

$= \frac{(x-1)^2-1}{(x-1)^2}$

$= \frac{x^2-2x+1-1}{(x-1)^2}$

$= \frac{x^2-2x}{(x-1)^2}$

$= \frac{x(x-2)}{(x-1)^2}$

Inc: (-∞,0],[2,∞)

Dec: [0,2]

Local max: -2 at 0

Local min: 2 at 2

$g''(x)$

$= 2(x-1)^{-3}$

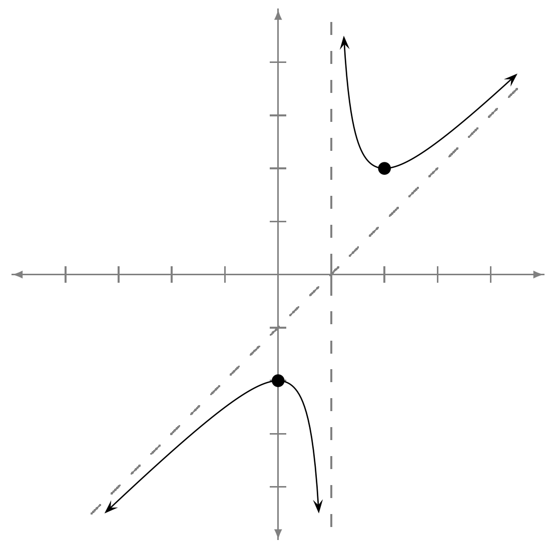
$= \frac{2}{(x-1)^3}$

CD: (-∞,1)

CU: (1,-∞)

IP: none

$g(x) = \dfrac{x^2-2x+2}{x-1}$



Section 8.4: Exam 3

Exam 3 Math 1571 Fall 2016

17. A ball is thrown upward with an initial velocity of of 16 ft/sec from a height of 32 ft.

(10) a. Give the height of the ball as a function of time.

$h(t) = -16t^2 + 16t + 32$

(10) b. What is the velocity of the ball when the ball hits the ground?

$v(t) = h'(t) = -32t + 16$

$-16t^2 + 16t + 32 = 0$

$t^2 - t - 2 = 0$

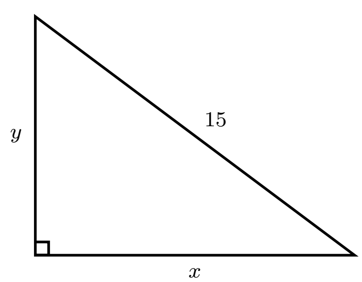
$(t - 2)(t + 1) = 0$

$t = 2$

$v(2) = -48$

-48 ft/sec

(10) 18. A 15-ft ladder leans against the side of a wall. The bottom of the ladder is being pushed toward the wall at a rate of 1 ft/sec. How fast is the top of the ladder moving up the wall when the top of the ladder is 12 feet above the ground?



$x^2 + y^2 = 225$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2 \cdot 9 \cdot (-1) + 2 \cdot 12 \cdot \frac{dy}{dt} = 0$

$24\frac{dy}{dt} = 18$

$\frac{du}{dt} = \frac{3}{4}$

$\frac{3}{4}$ ft/sec

(10) 19. Find the maximum and minimum values of $f(x) = 2x^3 + 9x^2 - 24x + 48$ on the interval [0,2].

$f'(x)$

$= 6x^2 + 18x - 24$

$= 6(x^2 + 3x - 4)$

$= 6(x + 4)(x - 1)$

$f(0) = 48$

$f(2) = 52$ (max)

$f(1) = 35$ (min)

(10) 20. What is the largest possible area of a rectangle with perimeter 40?

Let w represent the width of the rectangle. Then the length is $\frac{40-2w}{2} = 20-w$ and the area is $A(w) = w(20-w)$.

$A(w) = w(20 - w) = -w^2 + 20w)$

$A'(w) = \text{-}2w + 20 = \text{-}2(w - 10)$

$A(0) = 0$

$A(20) = 0$

$A(10) = 100$

(10) 21. Use a linear approximation to approximate $\sqrt[3]{26}$.

$f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$x = 27$

$y = f(27) = 3$

$m = f'(27) = \frac{1}{27}$

$L(x) = \frac{1}{27}(x - 27) + 3$

$L(26) = 3 - \frac{1}{27} = \frac{80}{27}$

(10) 22. The outside radius of a ball is 3 inches. The material used to make the ball is $\frac{1}{9}$ in thick. Use differentials to approximate the volume of the material used to make the ball. Recall: $V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 \cdot dr$

$dV = 4\pi \cdot 9 \left(\text{-}\frac{1}{9}\right) = \text{-}4\pi$

4π cm^3

(10) 23. Use the Mean Value Theorem to show that the equation $x^5 + x + 4 = 0$ has at most one solution.

***Proof*:** Let $f(x) = x^5 + x + 4$. Toward a contradiction, suppose that there are real numbers a and b such that $a < b$ and $f(a) = f(b) = 0$. Since f is a polynomial, f is continuous on $[a,b]$ and differentiable on (a,b). By the MVT, there exists $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a} = 0$. However, $f'(x) = 5x^4 + 1 \geq 1$. This is a contradiction. Therefore, there is at most one real number c such that $f(c) = 0$. ■

(10) 24. Use the definition of integral to evaluate the following.

$$\int_0^4 \left(x^2+x+1\right)dx$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\tfrac{4i}{n}\right)\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} \left[\left(\tfrac{4i}{n}\right)^2+\tfrac{4i}{n}+1\right]\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} \left(\tfrac{16i^2}{n^2}+\tfrac{4i}{n}+1\right)\cdot\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} \left(\tfrac{64i^2}{n^3}+\tfrac{16i}{n^2}+\tfrac{4}{n}\right)$$

$$= \lim_{n\to\infty}\left(\sum_{i=1}^{n}\tfrac{64i^2}{n^3}+\sum_{i=1}^{n}\tfrac{16i}{n^2}+\sum_{i=1}^{n}\tfrac{4}{n}\right)$$

$$= \lim_{n\to\infty}\left(\tfrac{64}{n^3}\sum_{i=1}^{n}i^2+\tfrac{16}{n^2}\sum_{i=1}^{n}i+\tfrac{4}{n}\sum_{i=1}^{n}1\right)$$

$$= \lim_{n\to\infty}\left(\tfrac{64}{n^3}\cdot\tfrac{n(n+1)(2n+1)}{6}+\tfrac{16}{n^2}\cdot\tfrac{n(n+1)}{2}+4\right)$$

$$= \tfrac{128}{6}+8+4$$

$$= \tfrac{100}{3}$$

Chapter 9: Math 1571 Fall 2021

Section 9.1: Quizzes

Quiz 80 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Give the equation of the line passing through the points (1,-3) and (2,1).

$m = 4$

$y - 1 = 4(x - 2)$

(1) 2. Suppose that $|a| \leq 2$ and $|b| \leq 3$.

a. Is it true that $|a + b| \leq 5$?

Yes by the Triangle Inequality.

b. Is it true that $|b - a| \leq 1$?

No. Set $a =$ -2 and $b = 3$.

(1) 3.

a. Convert $\frac{3\pi}{2}$rad to degree measure.

270°

b. Convert -135° to radian measure.

-$\frac{3\pi}{4}$rad

(1) 4. Give the center and radius of the following circle.

$x^2 + y^2 + 6x - 2y + 6 = 0$

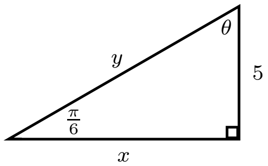
$x^2 + 6x + y^2 - 2y + 6 = 0$

$(x + 3)^2 + (y - 1)^2 =$ -6 $+ 10 = 4$

Center: (-3, 1)

Radius: 2

(1) 5. In the triangle below, solve for θ, x, and y.



$\sin \frac{\pi}{6} = \frac{1}{2} = \frac{5}{y}$

$y = 10$

$x^2 + 25 = 100$

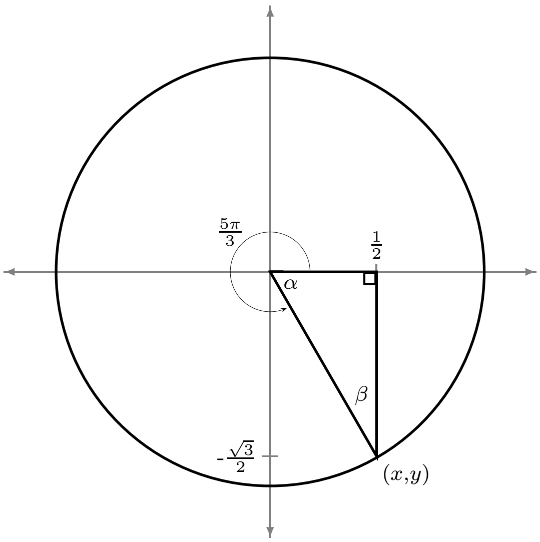
$x = 5\sqrt{3}$

$\frac{\pi}{2} + \frac{\pi}{6} + \theta = \pi$

$\theta = \frac{\pi}{3}$

(1) 6. Calculate each of the following.

a. $\sin\frac{5\pi}{3}$ **b.** $\cos\frac{5\pi}{3}$ **c.** $\tan\frac{5\pi}{3}$ **d.** $\cot\frac{5\pi}{3}$ **e.** $\sec\frac{5\pi}{3}$ **f.** $\csc\frac{5\pi}{3}$



$\alpha = \frac{\pi}{3}$

$\beta = \frac{\pi}{6}$

$\sin\frac{5\pi}{3} = y = \text{-}\frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{3} = x = \frac{1}{2}$

$\tan\frac{5\pi}{3} = \frac{y}{x} = \text{-}\sqrt{3}$

$\cot\frac{5\pi}{3} = \frac{x}{y} = \text{-}\frac{1}{\sqrt{3}}$

$\sec\frac{5\pi}{3} = \frac{1}{x} = 2$

$\csc\frac{5\pi}{3} = \frac{1}{y} = \text{-}\frac{2}{\sqrt{3}}$

(1) 7. Use the identity $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ to prove the identity $\sin\left(\theta+\frac{\pi}{2}\right) = \cos\theta$.

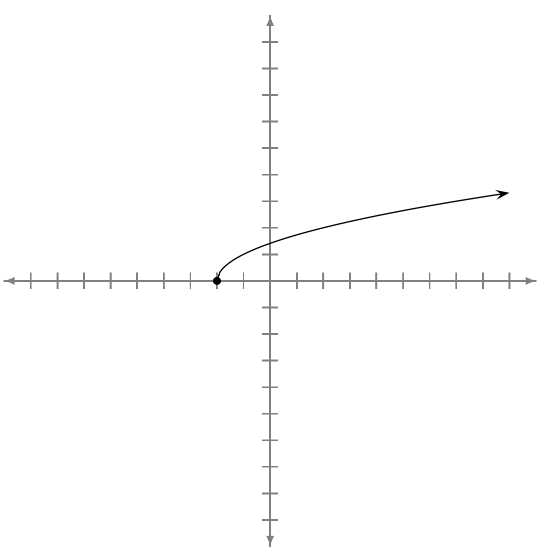
$\sin\left(\theta+\frac{\pi}{2}\right) = \sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2} = \sin\theta\cdot 0 + \cos\theta\cdot 1 = \cos\theta$

(2) 8. For the following function, give the domain, range, and graph.

$f(x) = \sqrt{x+2}$

D: [-2,∞)

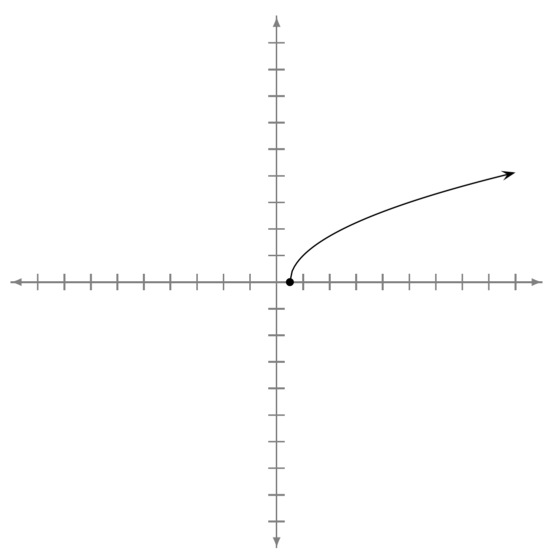
R: [0,∞)



$f(x) = \sqrt{2x-1}$

D: $\left[\frac{1}{2},\infty\right)$

R: [0,∞)



Quiz 81 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Suppose that f is a linear function, $f(1) = 2$, and $f(4) = \text{-}5$. Find $f(x)$.

$m = \frac{2+5}{1-4} = \text{-}\frac{7}{3}$

$y - 2 = \text{-}\frac{7}{3}(x-1)$

$f(x) = \text{-}\frac{7}{3}(x-1) + 2 = \text{-}\frac{7}{3}x + \frac{13}{3}$

(2) 2. For the function below, give the vertex, x-intercept(s) (if any), y-intercept, and sketch the graph.

$f(x) = x^2 - 2x - 15 = (x+3)(x-5)$

x-intercepts: (-3,0) and (5,0)

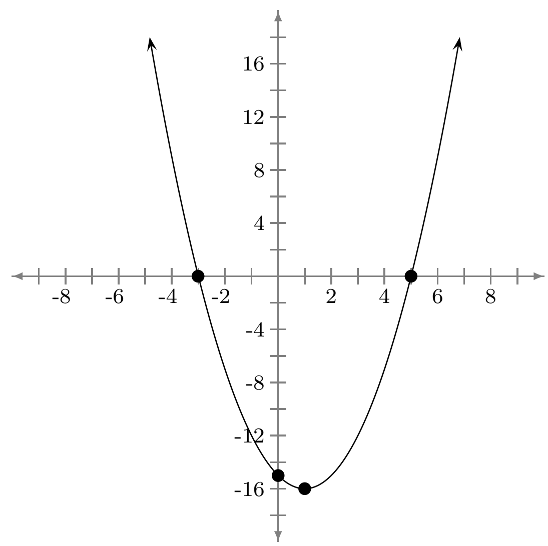
$f(0) = \text{-}15$

y-intercept: (0,-15)

$\frac{\text{-}(\text{-}2)}{2 \cdot 1} = 1$

$f(1) = \text{-}16$

Vertex: (1,-16)



(1) 3. Set $f(x) = x^2 + 1$ and $g(x) = \sin x$. Give the formula, domain, and range of each of the following.

a. $(f \circ g)(x) = \sin^2 x + 1$

Domain: $\mathbb{R}$

Range: [1,2]

b. $(g \circ f)(x) = \sin(x^2 + 1)$

Domain: $\mathbb{R}$

Range: [-1,1]

(1) 4. Answer one of the following. Make your choice clear.

a. Prove that the composition of two odd functions is an odd function.

Proof: Suppose that f and g are odd functions. Then
$(f \circ g)(\text{-}x) = f(g(\text{-}x)) = f(\text{-}g(x)) = \text{-}f(g(x)) = \text{-}(f \circ g)(x)$ as desired. ■

b. Find a cubic function (third degree polynomial) f such that $f(0) = f(\text{-}1) = f(1) = 0$ and $f(2) = 18$.

Since $f(0) = f(\text{-}1) = f(1) = 0$, $x(x-1)(x+1)$ is a factor of $f(x)$. Since $2(2-1)(2+1) = 6$ and $f(2) = 18$, $f(x) = 3x(x-1)(x+1)$.

(1) 5. List the transformations (in correct order) needed to transform the graph of $y = g(x)$ into the graph of $y = \text{-}g(x-4)+2$. Given the graph of $y = g(x)$, sketch the graph $y = \text{-}g(x-4)+2$.

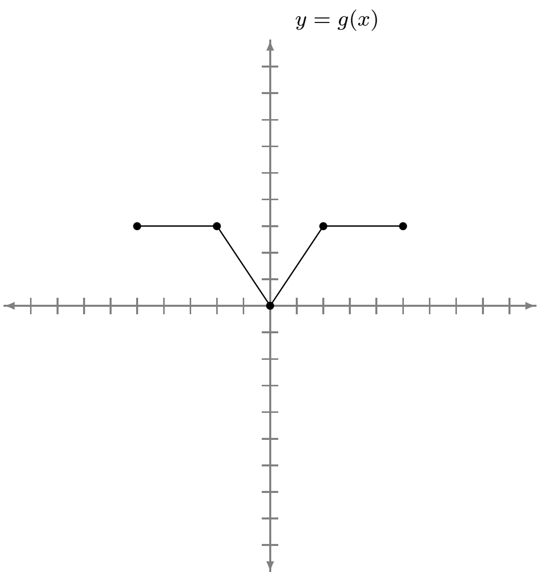
Transformations:

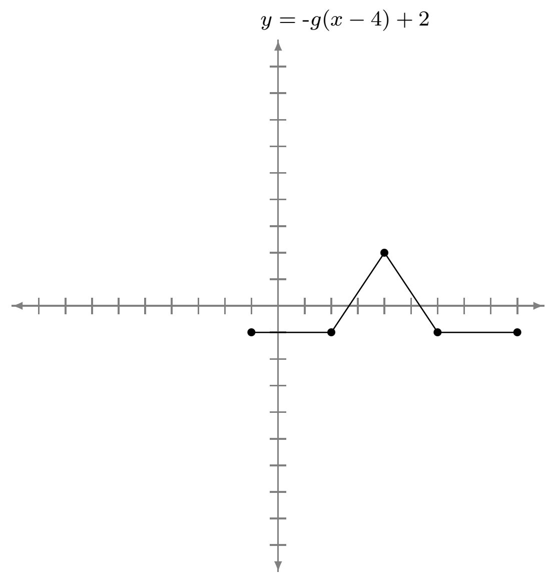
(i) shift right 4 units;

(ii) reflect about x-axis;

(iii) shift up 2 units.

Graphs.





(1) 6. Use the $\varepsilon - \delta$ definition of limit to show that $\lim\limits_{x \to 5}\big(4x-7\big) = 13$.

***Proof*:** Let $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta < \frac{\varepsilon}{4}$. If $|x-5| < \delta$, then

$|x-5| < \frac{\varepsilon}{4}$

$4|x-5| < \varepsilon$

$|4x-20| < \varepsilon$

$|(4x-7)-13| < \varepsilon.$ ■

Quiz 82 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. Calculate the following limits.

(2) a. $\lim_{x\to 1} \frac{x^2+6x-7}{x^2+x-2} = \lim_{x\to 1} \frac{(x+7)(x-1)}{(x+2)(x-1)} = \lim_{x\to 1} \frac{x+7}{x+2} = \frac{8}{3}$

(2) b. $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} = \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \lim_{x\to 9}\left(\sqrt{x}+3\right) = 6$

(2) 2. Set $f(x) = x^2+x-2$. Compute **one** of the following limits. Make your choice clear.

$\lim_{x\to 5} \frac{f(x)-f(5)}{x-5} = \lim_{x\to 5} \frac{x^2+x-2-28}{x-5} = \lim_{x\to 5} \frac{x^2+x-30}{x-5} = \lim_{x\to 5} \frac{(x+6)(x-5)}{x-5} = \lim_{x\to 5}\left(x+6\right) = 11$

$\lim_{h\to 0} \frac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \frac{(5+h)^2+(5+h)-2}{h} = \lim_{h\to 0} \frac{25+10h+h^2+5+h-2-28}{h} = \lim_{h\to 0} (h^2+11h)\cdot\frac{1}{h} = \lim_{h\to 0}\left(h+11\right) = 11$

(2) 3. Determine whether or not the following function is continuous at 2.

$$g(x) = \begin{cases} x+1, & \text{if } x \le 2 \\ x^2+4, & \text{if } x > 2 \end{cases}$$

Since $g(2) = 3$ and $\lim_{x\to 2^+} g(x) = 8$, g is not continuous at 2.

(1) 4. Calculate the following limit.

$\lim_{x\to 0} \left|x\sin\left(\frac{1}{x}\right)\right|$

Since $0 \le \left|x\sin\left(\frac{1}{x}\right)\right| \le |x|$ and $\lim_{x\to 0} |x| = 0$, $\lim_{x\to 0}\left|x\sin\left(\frac{1}{x}\right)\right| = 0$ by the Squeeze Theorem.

Quiz 83 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Set $f(x) = x^2 + \frac{1}{x} - 3$. Show that there is a number $c \in (1,2)$ such that $f(c) = 0$.

Proof: Since f is continuous on the closed interval [1,2], $f(1) =$ -1, and $f(2) = \frac{3}{2}$, the Intermediate Value Theorem implies that there exists a real number c between 1 and 2 such that $f(c) = 0$. ■

2. Calculate the following limits.

(2) a. $\lim\limits_{x\to\infty} \left(\sqrt{x^2+4} - x\right)$

$$= \lim_{x\to\infty} \left[(\sqrt{x^2+4} - x) \cdot \frac{\sqrt{x^2+4}+x}{\sqrt{x^2+4}+x}\right]$$

$$= \lim_{x\to\infty} \frac{x^2+4-x^2}{\sqrt{x^2+4}+x}$$

$$= \lim_{x\to\infty} \frac{4}{\sqrt{x^2+4}+x}$$

$$= 0$$

(1) b. $\lim\limits_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1}$

$$= \lim_{x\to\infty} \frac{x\sqrt{1+\frac{1}{x^2}}}{x\left(1+\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{1}{x}}$$

$$= 1$$

3. For each of the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

(2) a. $f(x)$

$$= \frac{x^2-2x-3}{x^2-x-2}$$

$$= \frac{(x-3)(x+1)}{(x-2)(x+1)}$$

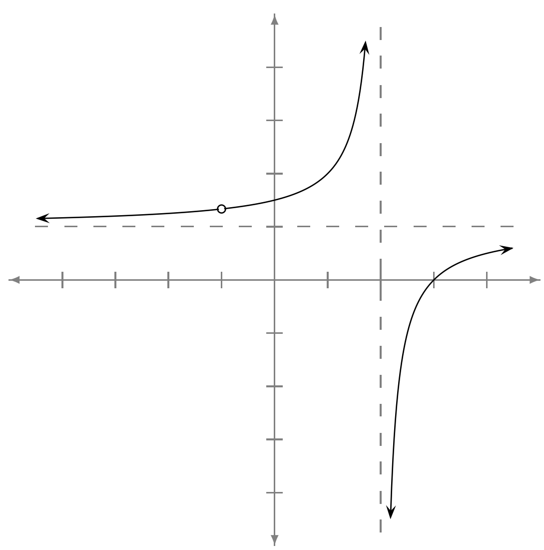
$$\neq \frac{x-3}{x-2}$$

HA: $y = 1$

VA: $x = 2$

$\lim\limits_{x\to -1} f(x) = \frac{4}{3}$

OA: none

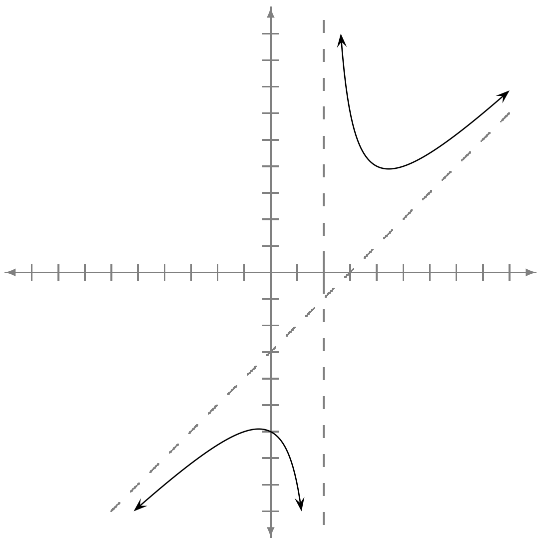


(2) b. $f(x) = \dfrac{x^2 - 5x + 12}{x - 2} = x - 3 + \frac{6}{x-2}$

HA: none

VA: $x = 2$

OA: $y = x - 3$



Quiz 84 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Calculate the following limits.

(2) a. $\lim_{\theta \to \frac{\pi}{3}} \dfrac{\cos\theta + \frac{1}{2}}{\cos\left(\theta - \frac{\pi}{3}\right)} = \dfrac{\cos\frac{\pi}{3} + \frac{1}{2}}{\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right)} = 1$

(1) b. $\lim_{\theta \to 0} \dfrac{\tan\theta}{\theta} = \lim_{\theta \to 0} \dfrac{\sin\theta}{\theta\cos\theta} = \lim_{\theta \to 0}\left(\dfrac{\sin\theta}{\theta} \cdot \dfrac{1}{\cos\theta}\right) = 1$

2. Use the definition of derivative to differentiate each of the following.

(2) a. $f(x) = x^2 + x - 3$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \left[(x+h)^2 + (x+h) - 3 - (x^2 + x - 3)\right] \tfrac{1}{h}$$

$$= \lim_{h\to 0} \left[x^2 + 2xh + h^2 + x + h - 3 - x^2 - x + 3\right] \tfrac{1}{h}$$

$$= \lim_{h\to 0} \left[2xh + h^2 + h\right] \tfrac{1}{h}$$

$$= \lim_{h\to 0} (2x + h + 1)$$

$$= 2x + 1$$

(2) b. $g(x) = \sqrt{x+4}$

$$\lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+4} - \sqrt{x+4}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{x+h+4} - \sqrt{x+4}}{h} \cdot \frac{\sqrt{x+h+4} + \sqrt{x+4}}{\sqrt{x+h+4} + \sqrt{x+4}}\right)$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h+4} + \sqrt{x+4})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+4} + \sqrt{x+4}}$$

$$= \frac{1}{2\sqrt{x+4}}$$

(1) c. $f(x) = \dfrac{1}{x+1}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{x + 1 - (x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-h}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)}$$

$$= \frac{-1}{(x+1)^2}$$

Bonus (1) 3.

Set $f(x) = \sin x$. Prove that $f'(x) = \cos x$.

See class notes.

Quiz 85 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. Differentiate.

(1) a. $f(x) = x^4 + 2x^3 - 5x^2 + x - 6$

$f'(x) = 4x^3 + 6x^2 - 10x + 1$

(1) b. $g(x) = (x^3 + x^2 + 7)(x^2 - x + 1)$

$g'(x) = (3x^2 + 2x)(x^2 - x + 1) + (x^3 + x^2 + 7)(2x - 1)$

(1) c. $f(x) = \dfrac{x^2 + 1}{x^3 + x + 2}$

$$f'(x) = \frac{2x(x^3 + x + 2) - (x^2 + 1)(3x^2 + 1)}{(x^3 + x + 2)^2}$$

(1) d. $h(x) = \sqrt[3]{x} + \frac{1}{x^2} = x^{\frac{1}{3}} + x^{-2}$

$h(x) = \frac{1}{3}x^{-\frac{2}{3}} - 2x^{-3}$

(1) e. $g(x) = (x^5 + x^3 + x^2 - 2)^7$

$g'(x) = 7(x^5 + x^3 + x^2 - 2)^6(5x^4 + 3x^2 + 2x)$

(1) f. $h(x) = \sqrt{x^2 + 1} = (x^2 + 1)^{\frac{1}{2}}$

$$h'(x) = \tfrac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2 + 1}}$$

(1) 2. Set $f(x) = x^3 + 2x^2 + 3x + 5$. Give the equation of the line that is tangent to the graph of f at the point where $x = -1$.

$f'(x) = 3x^2 + 4x + 3$

$x = -1$

$y = f(-1) = 3$

$m = f'(-1) = 2$

$y - 3 = 2(x + 1)$

(1) 3. Set $f(x) = x^4 + 2x^3 - 5x^2 + x - 6$. Find $f'''(x)$.

$f'(x) = 4x^3 + 6x^2 - 10x + 1$

$f''(x) = 12x^2 + 12x - 10$

$f'''(x) = 24x + 12$

Quiz 86 Name:________________________

Directions: Show all of your work and justify all of your answers.

(3) 1. Suppose that an ant is walking on a numbered line in such a way that it's position at time t is $s(t) = t^4 - t^3 + 2t^2 + 1$.

a. Give a formula for the ant's velocity.

$v(t) = s'(t) = 4t^3 - 3t^2 + 4t$

b. Give a formula for the ant's acceleration.

$a(t) = v'(t) = s''(t) = 12t^2 - 6t + 4$

c. Give the ant's average rate of change from time $t = 0$ to time $t = 2$.

$$\frac{s(2) - s(0)}{2 - 0} = 8$$

d. Give the ant's instantaneous rate of change at time $t = 1$.

$s'(1) = 5$

(3) 2. Differentiate. See class notes for solutions.

a. $y = \sin x$ **b.** $y = \cos x$ **c.** $y = \tan x$

d. $y = \cot x$ **e.** $y = \sec x$ **f.** $y = \csc x$

(2) 3. Give the derivative of y with respect to x. In other words, find y'.

a. $y = 5\tan^3 x + 4\cos^2 x + x^5$

$y' = 15\tan^2 x \sec^2 x - 8\cos x \sin x + 5x^4$

b. $x^4y^3 + 3xy = 10$

$\frac{d}{dx}\left(x^4y^3 + 3xy\right) = \frac{d}{dx}10$

$4x^3y^3 + x^4(3y^2)y' + 3y + 3xy' = 0$

$3x^4y^2y' + 3xy' = \text{-}4x^3y^3 - 3y$

$$y' = \frac{\text{-}4x^3y^3 - 3y}{3x^4y^2 + 3x}$$

Quiz 87 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(1) a. $g(x) = x^3 + 3x^2 - 3$

HA: none

VA: none

OA: none

$g'(x) = 3x^2 + 6x = 3x(x+2)$

Dec: [-2,0]

Inc: $(-\infty,-2] \cup [0,\infty)$

Local max: 1 at -2

Local min: -3 at 0

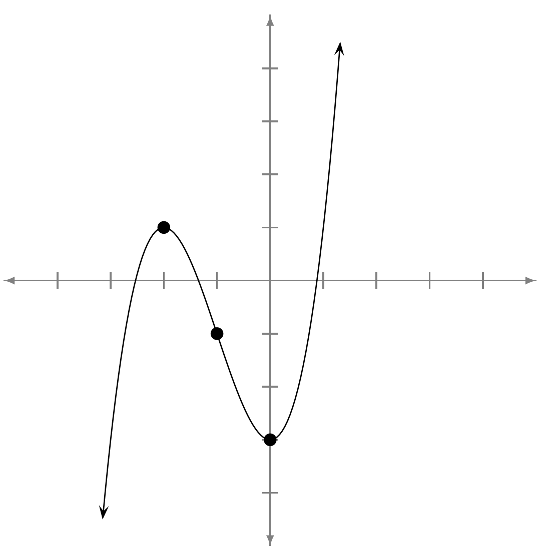
$g''(x) = 6x + 6 = 6(x+1)$

CD: $(-\infty,-1)$

CU: $(-1,\infty)$

IP: (-1,-1)

$g(x) = x^3 + 3x^2 - 3$



(1) b. $f(x) = \dfrac{x^2+x+1}{x} = x + 1 + \frac{1}{x}$

OA: $y = x + 1$

HA: none

VA: $x = 0$

$f(x) = x + 1 + \frac{1}{x}$

$f'(x) = 1 - \frac{1}{x^2}$

$= \frac{x^2-1}{x^2}$

$= \frac{(x+1)(x-1)}{x^2}$

Dec: [-1,0), (0,1]

Inc: $(-\infty,-1]$, $[1,\infty)$

Local max: -1 at -1

Local min: 3 at 1

$f'(x) = 1 - \frac{1}{x^2}$

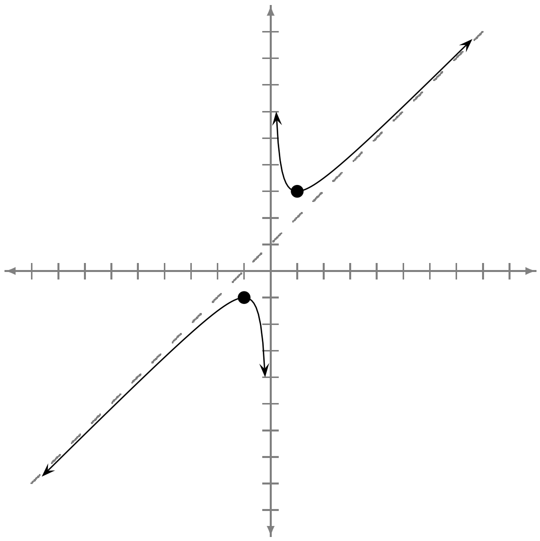
$f''(x) = \frac{2}{x^3}$

CU: $(0,\infty)$

CD: $(-\infty,0)$

IP: none

$f(x) = \dfrac{x^2+x+1}{x}$



(1) c. $k(x) = \sqrt{x^2+1}$

$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$$

$$= \lim_{x\to\infty}\left[\left(\sqrt{x^2+1}-x\right)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right]$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{x^2+1}+x}$$

$$= 0$$

$$\lim_{x\to -\infty}\left(\sqrt{x^2+1}- -x\right)$$

$$= \lim_{x\to -\infty}\left(\sqrt{x^2+1}+x\right)$$

$$= \lim_{x\to -\infty}\left[\left(\sqrt{x^2+1}+x\right)\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}-x}\right]$$

$$= \lim_{x\to -\infty}\frac{1}{\sqrt{x^2+1}-x}$$

$$= 0$$

OA: $y = x$, $y = -x$

HA: none

VA: none

$$k'(x) = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$

Dec: $(-\infty,0]$

Inc: $[0,\infty)$

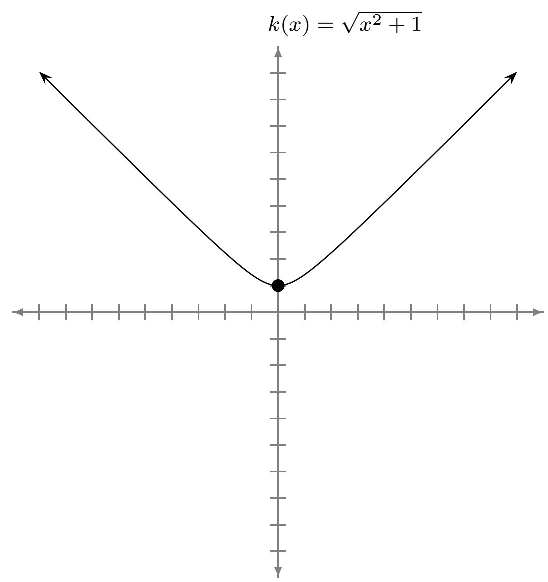
Local min: 1 at 0

$$k''(x) = \frac{1\cdot\sqrt{x^2+1}-x\cdot\frac{x}{\sqrt{x^2+1}}}{x^2+1} = \frac{1}{\sqrt{\left(x^2+1\right)^3}}$$

CU: $(-\infty,\infty)$

CD: never

IP: none



(1) 2. Give an example of a function $f(x)$ such that $f'(x) = 4x^3 + x^2 - x + 2$.

For any real number C, the derivative of $f(x) = x^4 + \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + C$ is $4x^3 + x^2 - x + 2$.

3. A ball is thrown vertically with an initial velocity of 72 ft/sec from a height of 144 ft.

(1) a. Find the function that gives the height of the ball.

$h(t) = -16t^2 + 72t + 144$

(1) b. Find the function that gives the velocity of the ball.

$v(t) = h'(t) = \text{-}32t + 72$

(1) c. When does the ball reach its maximum height?

$v(t) = 0$

$\text{-}32t + 72 = 0$

$t = \frac{9}{4}$

After $\frac{9}{4}$ seconds.

(1) d. When does the ball hit the ground?

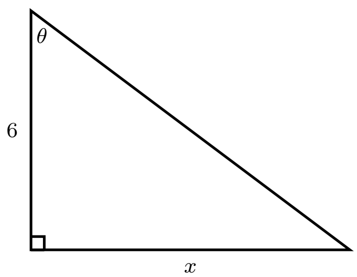
$\text{-}16t^2 + 72t + 144 = 0$

$2t^2 - 9t - 18 = 0$

$(2t + 3)(t - 6) = 0$

$t = 6$

(1) 4. An owl is sitting in a tree 6 meters above the ground. The owl is watching a mouse that is running away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 8 meters from the tree?



$\tan\theta = \frac{x}{6}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{6} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{6} \cdot \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{3}{500}$

$\frac{3}{500}$ radians per second

(1) 5. A glass in the shape of a right circular cylinder with radius 5 centimeters is being filled with water at a rate of $.05\pi$ liters per second. At what rate is the water level rising?
Recall: $V = \pi r^2 h$, 1 L = 1000 cm^3

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

$50\pi = 25\pi \frac{dh}{dt}$

$\frac{dh}{dt} = 2$

2 cm/sec

Quiz 88 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(Section 3.1: 47) **(2) 1.** Find the absolute maximum and absolute minimum of the given function on the given interval.

$f(x) = 2x^3 - 3x^2 - 12x + 1$; [-2,3]

(Section 3.1: 33) **(2) 2.** Find the critical numbers of the following function.

$f(t) = t^4 + t^3 + t^2 + 1$

(Section 3.7: 7) **(1) 3.** Find the area of a rectangle with perimeter 100 m whose area is as large as possible.

(Section 3.7: 21) **(1) 4.** Find the point on the line $y = 2x + 3$ that is closest to the origin.

(Section 2.9: 5) **(2) 5.** Find the linear approximation of $f(x) = \sqrt{1-x}$ at 0 and use it to approximate $\sqrt{0.99}$.

(Section 3.2: 19) **6.** Consider the equation $2x + \cos x = 0$.

(1) a. Show that the equation has a solution.

(1) b. Show that the equation does not have two solutions.

Quiz 89 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Calculate each of the following.

(1) a. $\sum_{i=1}^{4} 3i = 3+6+9+12 = 30$

(1) b. $\sum_{i=1}^{20}(2i^3-4i+5) = 2\sum_{i=1}^{20} i^3 - 4\sum_{i=1}^{20} i + \sum_{i=1}^{20} 5 = 2\left[\frac{20\cdot 21}{2}\right]^2 - 4\left(\frac{20\cdot 21}{2}\right) + 100 = 88{,}200 - 840 + 100 = 87{,}460$

(1) c. $\sum_{i=1}^{1000}\left[\frac{1}{i^2} - \frac{1}{(i+2)^2}\right] = 1 - \frac{1}{9} + \frac{1}{4} - \frac{1}{16} + \frac{1}{9} - \frac{1}{25} + \cdots + \frac{1}{99^2} - \frac{1}{101^2} + \frac{1}{100^2} - \frac{1}{102^2} = 1 + \frac{1}{4} - \frac{1}{101^2} - \frac{1}{102^2}$

(1) 2. Express the following using sigma notation.

$3+6+9+12+15+\cdots+300 = \sum_{i=1}^{100} 3i$

3. Use the definition of integral to evaluate each of the following.

(1) a. $\int_1^4 (2x+1)\,dx$ $= 18$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(1+\frac{3i}{n}\right)\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[2\left(1+\frac{3i}{n}\right)+1\right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\left(\frac{6i}{n}+3\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{18i}{n^2}+\frac{9}{n}\right)$$

$$= \lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{18i}{n^2}+\sum_{i=1}^{n}\frac{9}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{18}{n^2}\sum_{i=1}^{n} i+\frac{9}{n}\sum_{i=1}^{n}1\right)$$

$$= \lim_{n\to\infty}\left(\frac{18}{n^2}\cdot\frac{n(n+1)}{2}+\frac{9}{n}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\frac{9n^2+9n}{n^2}+9\right)$$

$$= 9+9$$

(1) b. $\int_0^2 \big(x^2 - x + 1\big) dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0 + \frac{2i}{n}\right) \cdot \frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^2 - \left(\frac{2i}{n}\right) + 1\right] \cdot \frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} - \frac{2i}{n} + 1\right) \cdot \frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8i^2}{n^3} - \frac{4i}{n^2} + \frac{2}{n}\right)$

$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{8i^2}{n^3} - \sum_{i=1}^{n} \frac{4i}{n^2} + \sum_{i=1}^{n} \frac{2}{n}\right)$

$= \lim_{n\to\infty} \left(\frac{8}{n^3} \sum_{i=1}^{n} i^2 - \frac{4}{n^2} \sum_{i=1}^{n} i + \frac{2}{n} \sum_{i=1}^{n} 1\right)$

$= \lim_{n\to\infty} \left(\frac{8}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{4}{n^2} \cdot \frac{n(n+1)}{2} + \frac{2}{n} \cdot n\right)$

$= \lim_{n\to\infty} \left(\frac{8}{n^3} \cdot \frac{2n^3+3n^2+n}{6} - \frac{4}{n^2} \cdot \frac{(n^2+n)}{2} + \frac{2}{n} \cdot n\right)$

$= \lim_{n\to\infty} \left(\frac{16n^3+24n^2+8n}{6n^3} - \frac{4n^2+4n}{2n^2} + 2\right)$

$= \frac{8}{3} - 2 + 2$

$= \frac{8}{3}$

4. Suppose that $f : [\text{-}20,20] \to \mathbb{R}$ and $\int_0^{20} f(x)\, dx = 4$.

(1) a. If f is even, what is $\int_{-20}^{20} f(x)\, dx$?

$\int_{-20}^{20} f(x)\, dx = 2\int_0^{20} f(x)\, dx = 8$

(1) b. If f is odd, what is $\int_{-20}^{20} f(x)\, dx$?

$\int_{-20}^{20} f(x)\, dx = 0$

Quiz 90 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. Integrate.

(1) a. $\int_0^3 \left(3x^2 + x^2 - 1\right) dx$

$= \left(x^3 + \frac{1}{3}x^3 - x\right)\Big|_0^3$

$= 27 + 9 - 3 - (0 + 0 + 0)$

$= 33$

(1) b. $\int_0^\pi \cos x \, dx = \sin x \Big|_0^\pi = \sin \pi - \sin 0 = 0$

(1) c. $\int \sec^2 x \, dx = \tan x + C$

(1) d. $\int 2x\sqrt{x^2+1}\, dx$

Set $u = x^2 + 1$ and $du = 2x\, dx$.

$\int 2x\sqrt{x^2+1}\, dx$

$= \int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}}\, du$

$= \frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{2}{3}\sqrt{\left(x^2+1\right)^3} + C$

(1) e. $\int_0^1 \left(x^3+1\right)\left(x^4+4x\right)^9 \, dx$

Set $u = x^4 + 4x$ and $du = \left(4x^3 + 4\right) dx$.

$\frac{1}{4}\, du = \left(x^3 + 1\right) dx$

$\int_0^1 \left(x^3+1\right)\left(x^4+4x\right)^9 \, dx$

$= \int_0^5 \frac{1}{4}u^9 \, du$

$= \frac{1}{40}u^{10}\Big|_0^5$

$= \frac{1}{40}\left(5^{10} - 0\right)$

$= \frac{5^{10}}{40}$

$= \frac{5^9}{8}$

f. $\int \cos x \sin^3 x \, dx = \frac{1}{4}\sin^4 + C$

(1) 2. Find the area of the region bounded by the curves $y = 0$, $x = 1$, and $y = x^2$.

$\int_0^1 x^2 \, dx = \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{3}$

(1) 3. Suppose that $F'(x) = f(x)$, $F(2) =$ -4, and $F(3) = 7$. Evaluate the integral $\int_2^3 f(x)\, dx$.

$\int_2^3 f(x)\, dx = F(x)\Big|_2^3 = F(3) - F(2) = 7 - \text{-}4 = 11$

(1) 4. Suppose that $f'(x) = x^3 - 1$ and $f(0) = 2$. Find $f(x)$.

$f(x) = \frac{1}{4}x^4 - x + C$

$f(0) = C = 2$

$f(x) = \frac{1}{4}x^4 - x + 2$

(1) 5. Set $F(x) = \displaystyle\int_0^x \left(t^3 + t^2 + t\right) dt$. Find $F'(x)$.

By the Fundamental Theorem of Calculus Part 1, $F'(x) = x^3 + x^2 + x$.

Quiz 91 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Find the area of the region bounded by the given curves. Hint: You may find it helpful to sketch the curves first.

(1) a.
$y = -x^2 - x + 6$
$y = 4$

$-x^2 - x + 6 = 4$

$-x^2 - x + 2 = 0$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$$\int_{-2}^{1} \left[(-x^2 - x + 6) - 4\right] dx$$

$$= \int_{-2}^{1} \left(-x^2 - x + 2\right) dx$$

$$= \left(-\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + 2x\right)\Big|_{-2}^{1}$$

$$= -\tfrac{1}{3} - \tfrac{1}{2} + 2 - \left(\tfrac{8}{3} - 2 - 4\right)$$

$$= \tfrac{9}{2}$$

(1) b.
$y = x^2 + x - 5$
$y = x + 4$

$x^2 + x - 5 = x + 4$

$x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$$\int_{-3}^{3} \left[x + 4 - (x^2 + x - 5)\right] dx$$

$$= \int_{-3}^{3} \left(-x^2 + 9\right) dx$$

$$= \left(-\tfrac{1}{3}x^3 + 9x\right)\Big|_{-3}^{3}$$

$$= -9 + 27 - (9 - 27)$$

$$= 36$$

2. Find the average value of the given function over the given interval.

(2) a. $f(x) = x^2 - 2x + 1$; [1,3]

$$\tfrac{1}{2}\int_{1}^{3} \left(x^2 - 2x + 1\right) dx$$

$$= \tfrac{1}{2}\left(\tfrac{1}{3}x^3 - x^2 + x\right)\Big|_{1}^{3}$$

$$= \tfrac{1}{2}\left[9 - 9 + 3 - \left(\tfrac{1}{3} - 1 + 1\right)\right]$$

$$= \tfrac{4}{3}$$

(2) b. $g(x) = \sin x + 2$; $[0,2\pi]$

$$\tfrac{1}{2\pi}\int_{0}^{2\pi} \left(\sin x + 2\right) dx$$

$$= \tfrac{1}{2\pi}\left(-\cos x + 2x\right)\Big|_{0}^{2\pi}$$

$$= \tfrac{1}{2\pi}\left[-1 + 4\pi - (-1 + 0)\right]$$

$$= 2$$

3. Suppose that S is a 3-dimensional solid whose base is the triangle bounded by $y = 0$, $x = 0$, and $y = 1 - x$. Give the volume of the solid if cross sections perpendicular to the x-axis are

(1) a. squares

$A(x) = (1 - x)(1 - x) = x^2 - 2x + 1$

$\displaystyle\int_0^1 \big(x^2 - 2x + 1\big)dx$

$= \big(\tfrac{1}{3}x^3 - x^2 + x\big)\Big|_0^1$

$= \tfrac{1}{3}$

(1) b. semicircles

$A(x) = \pi\left(\tfrac{1-x}{2}\right)^2 = \tfrac{\pi}{4}(x^2 - 2x + 1)$

$\displaystyle\int_0^1 \tfrac{\pi}{4}(x^2 - 2x + 1)\,dx$

$= \tfrac{\pi}{4}\displaystyle\int_0^1 \big(x^2 - 2x + 1\big)dx$

$= \tfrac{\pi}{12}$

Quiz 92 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(4) 1. Let $\mathcal{R}$ be the region in the plane bounded by the curves $y = 0$, $x = 1$, and $y = \sqrt{x}$. Find the volume of the solid obtained by rotating R about

a. the x-axis

$$\int_0^1 \pi x\, dx = \tfrac{1}{2}\pi x^2\Big|_0^1 = \tfrac{\pi}{2}$$

b. the line $y = 1$

$$\int_0^1 \pi\left[1 - (1-\sqrt{x})^2\right] dx = \int_0^1 \pi\left[1 - (1 - 2\sqrt{x} + x)\right] dx = \pi\int_0^1 \left(2x^{\frac{1}{2}} - x\right) dx = \pi\left(\tfrac{4}{3}x^{\frac{3}{2}} - \tfrac{1}{2}x^2\right)\Big|_0^1 = \tfrac{5\pi}{6}$$

c. the y-axis

$$\int_0^1 2\pi x\sqrt{x}\, dx = 2\pi\int_0^1 x^{\frac{3}{2}}\, dx = \tfrac{4\pi}{5}x^{\frac{5}{2}}\Big|_0^1 = \tfrac{4\pi}{5}$$

d. the line $x =$ -1

Let $\mathcal{R}'$ be the region formed by shifting $\mathcal{R}$ right one unit. Then the volume of the solid formed by revolving $\mathcal{R}$ about the line $x =$ -1 is the same as the volume of the solid formed by revolving $\mathcal{R}'$ about the y-axis which is

$$\int_1^2 2\pi(x-1)\sqrt{x-1}\, dx = \int_1^2 2\pi x\sqrt{x-1}\, dx - \int_1^2 2\pi\sqrt{x-1}\, dx$$

Evaluate each integral separately.

$$\int_1^2 2\pi x\sqrt{x-1}\, dx$$

Use substitution.

Set $u = x - 1$.

$du = dx$

$x = u + 1$

$$\int_1^2 2\pi x\sqrt{x-1}\, dx = \int_0^1 2\pi(u+1)\sqrt{u}\, du = 2\pi\int_0^1 \left(u^{\frac{3}{2}} + u\right) du = 2\pi\left(\tfrac{2}{5}u^{\frac{5}{2}} + \tfrac{1}{2}u^2\right)\Big|_0^1 = \tfrac{9\pi}{5}$$

$$\int_1^2 2\pi\sqrt{x-1}\, dx = \tfrac{4}{3}\pi(x-1)^{\frac{3}{2}}\Big|_1^2 = \tfrac{4\pi}{3}$$

$\frac{9\pi}{5} - \frac{4\pi}{3} = \frac{7\pi}{15}$

(2) 2. Assume that the density of water is 62.5 lb/ft^3. A cylindrical water tank with a circular base has radius 3 feet and height 10 feet. Find the work required to empty the tank by pumping the water out of the top for each of the following situations.

a. The tank is full.

b. The tank is half full.

See the solutions to the final exam review problems.

(3) 3. A bucket with 24 lb of water is raised 30 feet from the bottom of a well. Find the work done in each of the following cases.

See the solutions to the final exam review problems.

a. The weight of the empty bucket is 4 lb and the weight of the rope is negligible.

b. The bucket weighs 4 lb and the rope weighs 4 oz/ft.

c. The bucket weighs 4 lb, the rope weighs 4 oz/ft, and water is leaking out of the bucket at a constant rate so that only 18 lb of water remain in the bucket when it reaches the top.

Chapter 10: Math 1571 Spring 2022

Section 10.1: Quizzes

Quiz 93 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Give the center and radius of the following circle.

$x^2 + y^2 + 2x - 8y + 13 = 0$

$(x+1)^2 + (y-4)^2 = 4$

Center: (-1, 4)

Radius: 2

$x^2 + y^2 + 8x - 2y + 13 = 0$

$(x+4)^2 + (y-1)^2 = 4$

Center: (-4, 1)

Radius: 2

(2) 2. Solve the following inequalities.

a. $|x-3| < 2$

(1,5)

b. $|x+1| \geq 5$

$(-\infty,-6] \cup [4,\infty)$

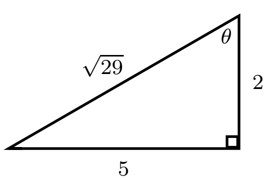
c. $|x+5| \leq 1$

[-6,-4]

(1) 3. If possible, find real numbers a and b such that $|a| + |b| = 3$ and $|a+b| = 4$. If this is not possible, explain why it is not.

This is not possible. By the triangle inequality, $|a+b| \leq |a| + |b|$

(2) 4. See the figure below. Give the six trigonometric values of θ.



$\sin\theta = \frac{5}{\sqrt{29}}$

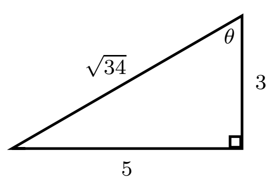
$\cos\theta = \frac{2}{\sqrt{29}}$

$\tan\theta = \frac{5}{2}$

$\cot\theta = \frac{2}{5}$

$\sec\theta = \frac{\sqrt{29}}{2}$

$\csc\theta = \frac{\sqrt{29}}{5}$



$\sin\theta = \frac{5}{\sqrt{34}}$

$\cos\theta = \frac{3}{\sqrt{34}}$

$\tan\theta = \frac{5}{3}$

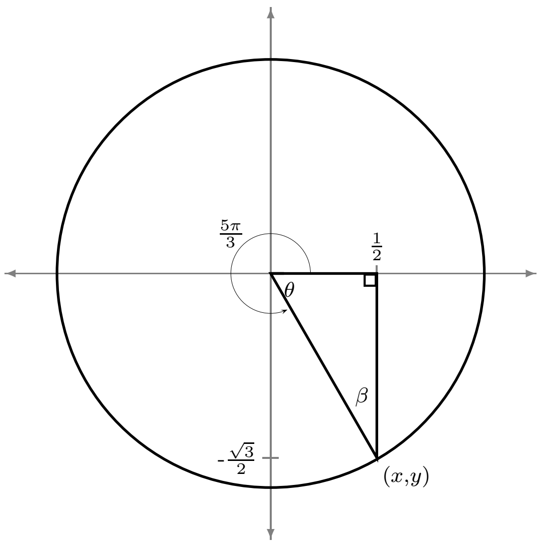
$\cot\theta = \frac{3}{5}$

$\sec\theta = \frac{\sqrt{34}}{3}$

$\csc\theta = \frac{\sqrt{34}}{5}$

(1) 5.

a. Give the six trigonometric values of $\frac{5\pi}{3}$.



$\alpha = \frac{\pi}{3}$

$\beta = \frac{\pi}{6}$

$\sin\frac{5\pi}{3} = y = \text{-}\frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{3} = x = \frac{1}{2}$

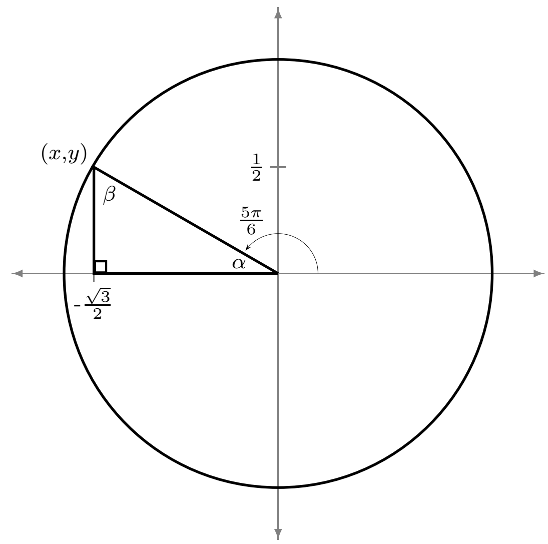
$\tan\frac{5\pi}{3} = \frac{y}{x} = \text{-}\sqrt{3}$

$\cot\frac{5\pi}{3} = \frac{x}{y} = \text{-}\frac{1}{\sqrt{3}}$

$\sec\frac{5\pi}{3} = \frac{1}{x} = 2$

$\csc\frac{5\pi}{3} = \frac{1}{y} = \text{-}\frac{2}{\sqrt{3}}$

b. Give the six trigonometric values of $\frac{5\pi}{6}$.



$\alpha = \frac{\pi}{6}$

$\beta = \frac{\pi}{3}$

$\sin\frac{5\pi}{6} = y = \frac{1}{2}$

$\cos\frac{5\pi}{6} = x = \text{-}\frac{\sqrt{3}}{2}$

$\tan\frac{5\pi}{6} = \frac{y}{x} = \text{-}\frac{1}{\sqrt{3}}$

$\cot\frac{5\pi}{6} = \frac{x}{y} = \text{-}\sqrt{3}$

$\sec\frac{5\pi}{6} = \frac{1}{x} = \text{-}\frac{2}{\sqrt{3}}$

$\csc\frac{5\pi}{6} = \frac{1}{y} = 2$

Quiz 94 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Give the equation of the line that passes through the points (1,-2) and (4,7).

$m = 3$

$y + 2 = 3(x - 1)$

$y - 7 = 3(x - 4)$

(1) 2. Give the equation of the line that passes through the point (-1,3) and is parallel (perpendicular) to the line below.

$x + 2y = 1$

$y = -\frac{1}{2}(x - 1)$

$m = -\frac{1}{2}$

Parallel: $y - 3 = -\frac{1}{2}(x + 1)$

Perpendicular: $y - 3 = 2(x + 1)$

(2) 3. Prove the following identities.

a. $\sin x \tan x + \cos x = \sec x$

$\sin x \tan x + \cos x$

$= \dfrac{\sin^2 x}{\cos x} + \cos x$

$= \dfrac{\sin^2 x + \cos^2 x}{\cos x}$

$= \dfrac{1}{\cos x}$

$= \sec x$

b. $\sin x + \cos x \cot x = \csc x$

$\sin x + \cos x \cot x$

$= \sin x + \dfrac{\cos^2 x}{\sin x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x}$

$= \dfrac{1}{\sin x}$

$= \csc x$

c. $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$\sin\left(x + \frac{\pi}{2}\right)$

$= \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right)$

$= 0 \cdot \sin x + 1 \cdot \cos x$

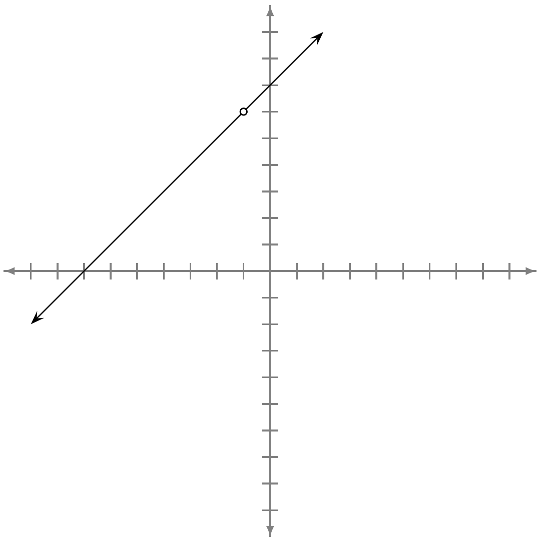
$= \cos x$

(1) 4. For the following function, give the domain and range. Sketch the graph.

$f(x) = \dfrac{x^2+8x+7}{x+1} = \dfrac{(x+7)(x+1)}{x+1} \neq x+7$

Domain: (-∞,-1) $\cup$ (-1,∞)

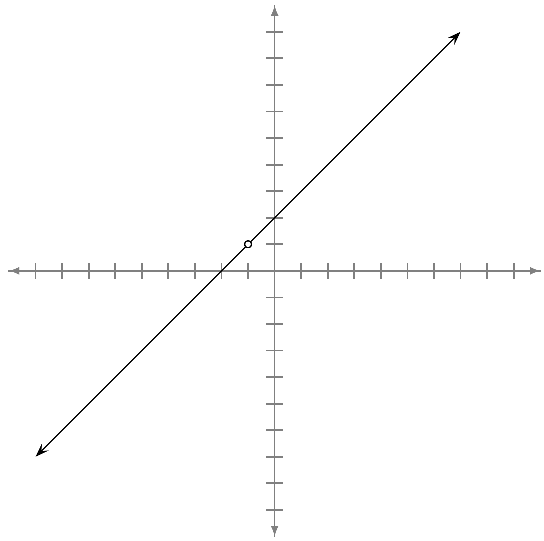
Range: (-∞,6) $\cup$ (6,∞)



$f(x) = \dfrac{x^2+3x+2}{x+1} = \dfrac{(x+2)(x+1)}{x+1} \neq x+2$

Domain: (-∞,-1) $\cup$ (-1,∞)

Range: (-∞,1) $\cup$ (1,∞)



(1) 5. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function such that $f(2) = 8$. What is f(-2) if f is

a. even?

f(-2) $= f(2) = 8$

b. odd?

f(-2) $=$ -$f(2) =$ -8

(1) 6. Set $f(x) = \sqrt{x}$ and $g(x) = \sin x$. Give each of the following.

a. $(f \circ g)(x) = \sqrt{\sin x}$

b. $(g \circ f)(x) = \sin \sqrt{x}$

(1) 7. Set $f(x) = x^2$ and $g(x) = \sin x$. Give each of the following.

a. $(f \circ g)(x) = \sin^2 x$

b. $(g \circ f)(x) = \sin x^2$

Quiz 95 Name:________________________

<u>Directions:</u> Show all of your work and justify all of your answers.

(2) 1. Solve the following quadratic equations.

a. $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

b. $x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$

No real solutions.

$x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

No real solutions.

(2) 2. For the following quadratic function, find the vertex x-intercept(s) (if any), and y-intercept. Use your answers to sketch the graph.

$f(x) = x^2 - 6x + 5$

Vertex: (3,-4)

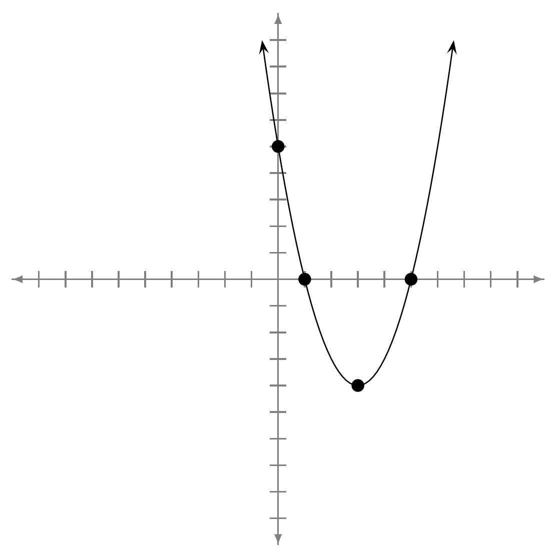
y-intercept: (0,5)

Solve: $x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 1{,}5$

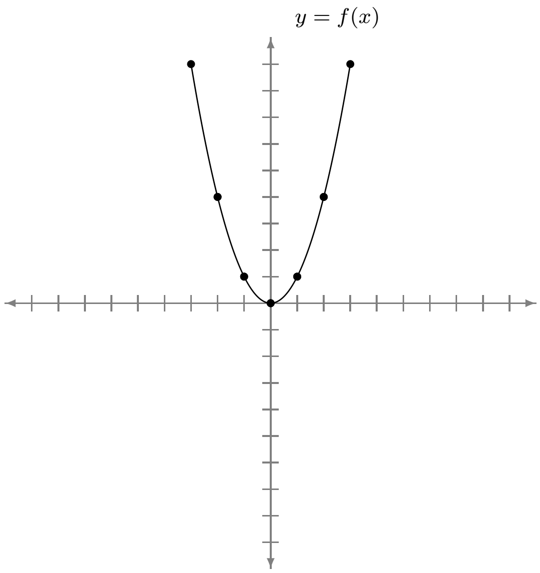
x-intercepts: (1,0), (5,0)

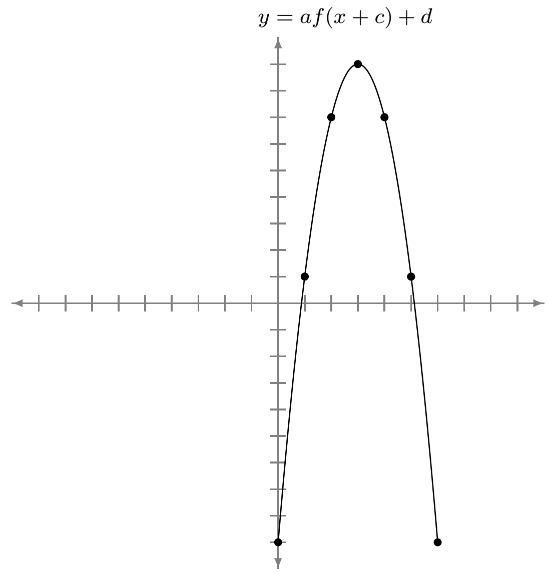


(1) 3. Find a third degree polynomial $f(x)$ such that $f(\text{-}2) = f(\text{-}1) = f(1) = 0$ and $f(2) = 36$.

Since $f(\text{-}2) = f(\text{-}1) = f(1) = 0$, $(x+2)$, $(x+1)$, and $(x-1)$ are all factors of f. So $f(x) = a(x+2)(x+1)(x-1)$ for some $a \in \mathbb{R}$. Since $f(2) = 36$, $a = 3$. Therefore, $f(x) = 3(x+2)(x+1)(x-1)$.

(2) 4. Below are the graphs of $y = f(x)$ and $y = af(x + c) + d$. Give the values of a, c, and d. Hint: First list the transformations required to transform the first graph into the second.





The second graph is obtained from the first one by applying the following transformations:

(i) shift right 3 units

(ii) vertical stretch by a factor of 2

(iii) reflect about the x-axis

(iv) shift up 9 units

Therefore, $a =$ -2, $c =$ -3, and $d = 9$.

(1) 5. Use the $\varepsilon - \delta$ definition of limit to show that $\lim\limits_{x \to 2} \big(3x + 5\big) = 11$.

Proof: Let $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta < \frac{\varepsilon}{3}$. If $|x - 2| < \delta$, then

$|x - 2| < \frac{\varepsilon}{3}$

$3|x - 2| < \varepsilon$

$|3x - 6| < \varepsilon$

$|(2x + 5) - 11| < \varepsilon$. ■

Use the $\varepsilon - \delta$ definition of limit to show that $\lim\limits_{x \to 2} \big(5x + 3\big) = 13$.

Proof: Let $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta < \frac{\varepsilon}{5}$. If $|x - 2| < \delta$, then

$|x - 2| < \frac{\varepsilon}{5}$

$5|x - 2| < \varepsilon$

$|5x - 10| < \varepsilon$

$|(5x + 3) - 13| < \varepsilon$. ■

Quiz 96 Name:________________________

Directions: Show all of your work and justify all of your answers.

(7) 1. Calculate the following limits.

a. $\lim\limits_{x\to -1} \dfrac{x^2-x-2}{x^2+4x+3}$

$= \lim\limits_{x\to -1} \dfrac{(x+1)(x-2)}{(x+1)(x+3)}$

$= \lim\limits_{x\to -1} \dfrac{x-2}{x+3}$

$= -\dfrac{3}{2}$

$\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2+2x-3}$

$= \lim\limits_{x\to 1} \dfrac{(x-1)(x+2)}{(x-1)(x+3)}$

$= \lim\limits_{x\to 1} \dfrac{x+2}{x+3}$

$= \dfrac{3}{4}$

b. $\lim\limits_{x\to 2} \dfrac{x^2-4}{x+4} = 0$

c. $\lim\limits_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$

$= \lim\limits_{x\to 9} \dfrac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3}$

$= \lim\limits_{x\to 9} \big(\sqrt{x}+3\big)$

$= 6$

$\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

$= \lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)}$

$= \lim\limits_{x\to 9} \dfrac{1}{\sqrt{x}+3}$

$= \dfrac{1}{6}$

d. $\lim\limits_{h\to 0} \dfrac{\sqrt{25+h}-5}{h}$

See class notes.

e. $\lim\limits_{x\to 1} \dfrac{x^3-1}{x-1}$

$= \lim\limits_{x\to 1} \dfrac{(x-1)(x^2+x+1)}{x-1}$

$= \lim\limits_{x\to 1} \big(x^2+x+1\big)$

$= 3$

f. $\lim\limits_{x\to 0} \dfrac{|x|}{x}$

See class notes.

g. $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right) = 0$

Note that $-x^2 \le x^2 \sin\left(\frac{1}{x^2}\right)^p \le x^2$. Since $\lim\limits_{x\to 0} -x^2 = \lim\limits_{x\to 0} x^2 = 0$, $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right) = 0$ by the Squeeze Theorem.

(1) 2. For $f(x) = x^2+x+1$, calculate the following limit.

$\lim\limits_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = \lim\limits_{h\to 0} \dfrac{(3+h)^2+(3+h)+1-13}{h} = \lim\limits_{h\to 0} \dfrac{9+6h+h^2+3+h+1-13}{h} = \lim\limits_{h\to 0} \dfrac{h^2+7h}{h} =$
$\lim\limits_{h\to 0} \big(h+7\big) = 7$

Quiz 97 Name:_________________________

<u>**Directions:**</u> Show all of your work and justify all of your answers.

(1) 1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ and $\lim_{x \to 3} f(x) = 5$.

a. What (if anything) can be asserted about $f(3)$?

Nothing can be determined about the function value.

b. If f is continuous at 3, what (if anything) can be asserted about $f(3)$?

The limit and the function value are equal.

(1) 2. Is the following function continuous at 3?

$$f(x) = \begin{cases} 3x^2 + 1, & \text{if } x \leq 3 \\ 8x + 2, & \text{if } x > 3 \end{cases}$$

No.

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (3x^2 + 1) = 28$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (8x + 2) = 26$

$\lim_{x \to 3} f(x)$ DNE

(1) 3. Is the following function continuous at 0? Is it either left continuous or right continuous at 0?

$$g(x) = \begin{cases} \dfrac{|x|}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$

$\lim_{x \to 0^+} g(x) = 1$ (See class notes)

$\lim_{x \to 0^-} g(x) = -1$ (See class notes)

Since $g(0) = 1$, g is right continuous at 0.

$$g(x) = \begin{cases} \dfrac{|x|}{x}, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$$

$\lim_{x \to 0^+} g(x) = 1$ (See class notes)

$\lim_{x \to 0^-} g(x) = -1$ (See class notes)

Since $g(0) = -1$, g is left continuous at 0.

(1) 4. Prove that the following equation has a solution.

$2x^7 + x^3 + 10x + 3 = 0$

***Proof*:** Set $f(x) = 2x^7 + x^3 + 10x + 3$ and note that f is continuous on [-1,1] since f is a polynomial. Since $f(-1) = -10$ and $f(1) = 16$, the Intermediate Value Theorem implies that there is a number $c \in$ [-1,1] such that $f(c) = 0$. ■

5. For each of the following, find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

(2) a. $f(x)$

$$= \frac{x^3+1}{x^2-1}$$

$$= \frac{(x+1)(x^2-x+1)}{(x+1)(x-1)}$$

$$\neq \frac{x^2-x+1}{x-1}$$

$$= \frac{x^2-x}{x-1} + \frac{1}{x-1}$$

$$= \frac{x(x-1)}{x-1} + \frac{1}{x-1}$$

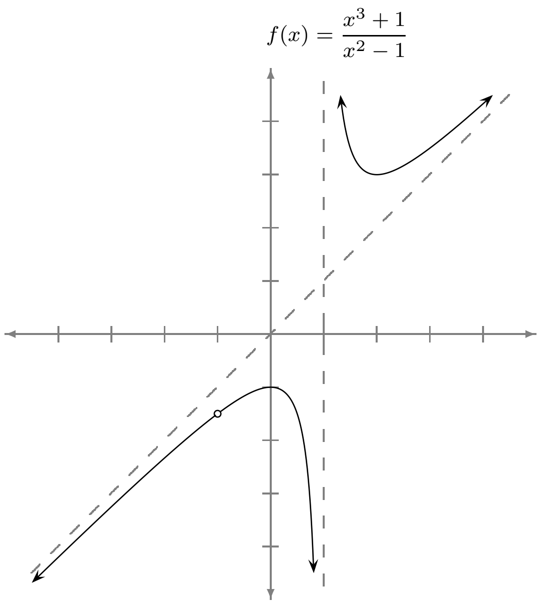
$$= x + \frac{1}{x-1}$$

$\lim_{x \to \text{-}1} f(x) = \text{-}\frac{3}{2}$

VA: $x = 1$

HA: none

OA: $y = x$



(2) b. $g(x)$

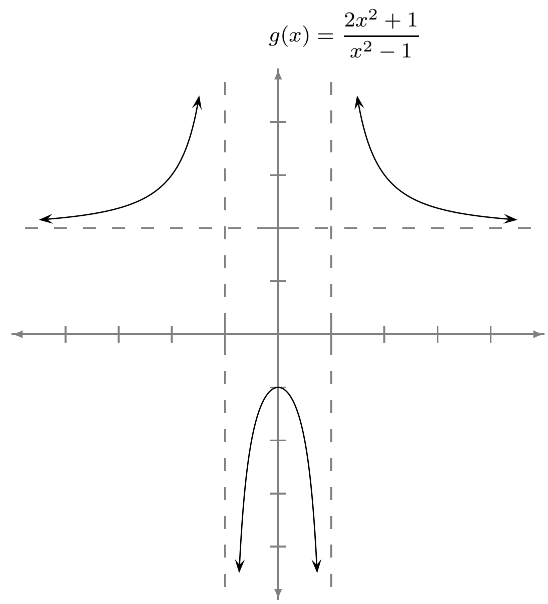
$$= \frac{2x^2+1}{x^2-1}$$

$$= \frac{2x^2+1}{(x+1)(x-1)}$$

HA: $y = 2$

VA: $x =$ -1, $x = 1$

OA: none



Quiz 98 Name:____________________

Directions: Show all of your work and justify all of your answers.

(2) 1. For each of the following, find the oblique asymptote.

a. $f(x) = \dfrac{x^4+1}{x^3+1} = x - \dfrac{-x+1}{x^3+1}$

OA: $y = x$

b. $g(x) = x + 2 + \frac{\sin x}{x}$

OA: $y = x + 2$

(2) 2. For the following, find the amplitude, period, phase shift, and average value. Sketch the graph.

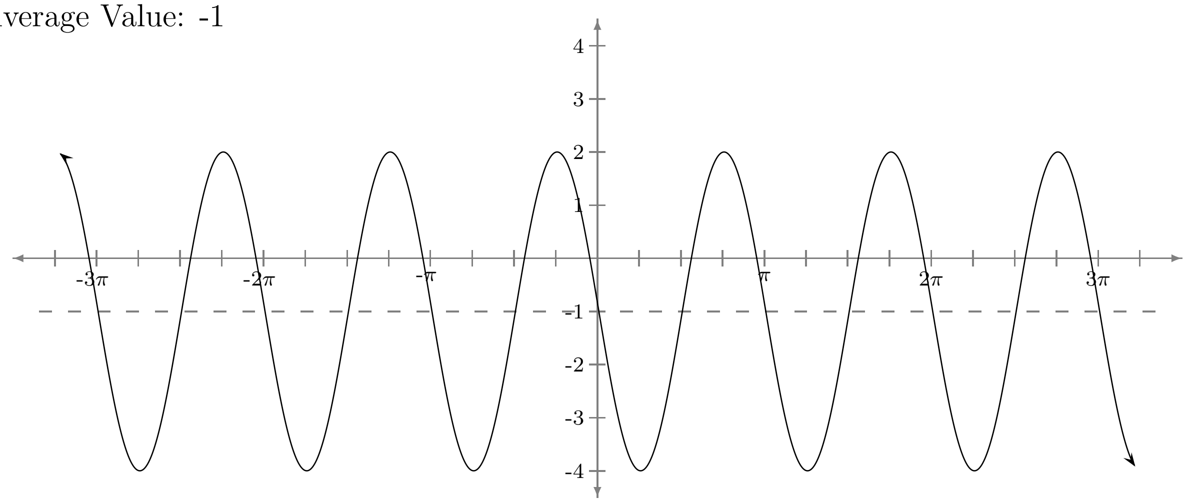
$y = 3\sin(2x - \pi) - 1$

Amplitude: 3

Period: $\frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\pi}{2}$

Average Value: -1



(2) 3. Sketch each of the following.

a. $y = \tan x$

b. $y = \sec x$

See class notes.

4. Use the definition of derivative to differentiate each of the following.

(1) a. $f(x) = 2x^2 + x - 1$

$\lim\limits_{h\to 0} \frac{1}{h}[f(x+h) - f(x)]$

$= \lim\limits_{h\to 0} \frac{1}{h}[2(x+h)^2 + (x+h) - 1 - (2x^2 + x - 1)]$

$= \lim\limits_{h\to 0} \frac{1}{h}(2x^2 + 4xh + 2h^2 + x + h - 1 - 2x^2 - x + 1)$

$= \lim\limits_{h\to 0} \frac{1}{h}(4xh + 2h^2 + h)$

$= \lim_{h \to 0}(4x + 2h + 1)$

$= 4x + 1$

(1) b. $f(x) = \sqrt{x+1}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$

$= \lim_{h \to 0} \left(\frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \right)$

$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}$

$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$

$= \frac{1}{2\sqrt{x+1}}$

(1) c. $f(x) = \frac{1}{x+1}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{h}$

$= \lim_{h \to 0} \left[\frac{(x+1)-(x+h+1)}{(x+h+1)(x+1)} \cdot \frac{1}{h} \right]$

$= \lim_{h \to 0} \frac{x+1-x-h-1}{h(x+h+1)(x+1)}$

$= \lim_{h \to 0} \frac{-h}{h(x+h+1)(x+1)}$

$= \lim_{h \to 0} \frac{-1}{(x+h+1)(x+1)}$

$= -\frac{1}{(x+1)^2}$

Quiz 99 Name:________________________

Directions: Show all of your work and justify all of your answers.

(4) 1. Differentiate each of the following.

a. $f(x) = x^{\frac{5}{2}} + x^{\frac{1}{3}}$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}} + \frac{1}{3}x^{-\frac{2}{3}}$

b. $g(x) = (x^3 + 1)(x^4 - x^3 + 2x^2 + x + 1)$

$g'(x) = (3x^2)(x^4 - x^3 + 2x^2 + x + 1) + (x^3 + 1)(4x^3 - 3x^2 + 4x + 1)$

c. $h(x) = \dfrac{x^3 + 1}{x^4 - x^3 + 2x^2 + x + 1}$

$$h'(x) = \frac{2x^2(x^4 - x^3 + 2x^2 + x + 1) - (x^3 + 1)(4x^3 - 3x^2 + 4x + 1)}{(x^4 - x^3 + 2x^2 + x + 1)^2}$$

d. $f(x) = \sqrt{x^2 + 1}$

$f(x) = (x^2 + 1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(2x)$

$f'(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

(1) 2. Suppose that f is a function such that $f'(x) = \frac{1}{x}$ and $g(x) = x^4 + 1$. Give the derivative of $(f \circ g)(x)$.

$(f \circ g)'(x) = f'[g(x)]\, g'(x) = \dfrac{4x^3}{x^4 + 1}$

(1) 3. Give the equation of the line that is tangent to the graph of the function $f(x) = x^4 + 2x^3 - 5$ at the point where $x = 1$.

$f'(x) = 4x^3 + 6x^2$

$y = f(1) = -2$

$m = f'(1) = 10$

$y + 2 = 10(x - 1)$

4. An ant is walking along a numbered line so that after t minutes its distance from the starting point is $s(t) = t^3 + t^2 + t$ inches.

(1) a. Give the ant's average rate of change from time $t = 1$ to time $t = 3$.

$$\frac{s(3) - s(1)}{3 - 1} = \frac{39 - 3}{3 - 1} = 18$$

18 inches per minute

(1) b. Give the ant's instantaneous rate of change at time $t = 2$.

$s'(t) = 3t^2 + 2t + 1$

$s'(2) = 17$

17 inches per minute

Quiz 100 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1. Differentiate each of the following.

(1) a. $f(x) = \sin(x^2+1)$

$f'(x) = 2x\cos(x^2+1)$

(1) b. $g(x) = \sin^2 x + 1$

$g'(x) = 2\sin x \cos x$

(1) c. $h(x) = \sin(\cos x)$

$h'(x) = \cos(\cos x)\cdot(\text{-}\sin x)$

(1) 2. Set $f(x) = \sin x$. Find the 203rd derivative of f, $f^{(203)}(x) = \dfrac{d^{203}y}{dx^{203}}$.

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = \text{-}\sin x$

$f'''(x) = \text{-}\cos x$

$f^{(4)}(x) = \sin x$

$\vdots$

$f^{(200)}(x) = \sin x$

$f^{(201)}(x) = \cos x$

$f^{(202)}(x) = \text{-}\sin x$

$f^{(203)}(x) = \text{-}\cos x$

(1) 3. Find a function F such that $F'(x) = \text{-}\csc x \cot x$.

$F(x) = \csc x$

(1) 4. Given the following, use implicit differentiation to find y'.

$\tan(xy) = 4$

$$\frac{d}{dx}\tan(xy) = \frac{d}{dx}4$$

$[\sec^2(xy)]\,[y + xy'] = 0$

$y\sec^2(xy) + xy'\sec^2(xy) = 0$

$y' = \frac{\text{-}y\sec^2(xy)}{x\sec^2(xy)} = \text{-}\frac{y}{x}$

5. For each of the following, give the equation of the line that is tangent to the given curve at the specified point.

(1) a. $y = \tan x$; $x = \frac{\pi}{4}$

$y' = \sec^2 x;$

$x = \frac{\pi}{4}$

$y = \tan\frac{\pi}{4}; = 1$

$m = \sec^2\frac{\pi}{4} = 2$

$y - 1 = 2\left(x - \frac{\pi}{4}\right)$

(1) b. $y^3 + x^2y + 3x = 5$; (1,1)

$\frac{d}{dx}\left(y^3 + x^2y + 3x\right) = \frac{d}{dx}5$

$3y^2y' + 2xy + x^2y' + 3 = 0$

$$y' = \frac{\text{-}2xy - 3}{3y^2 + x^2}$$

$x = 1$

$y = 1$

$m = \text{-}\frac{5}{4}$

$y - 1 = \text{-}\frac{5}{4}(x - 1)$

Quiz 101 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(1) a. $f(x) = \frac{2}{3}x^3 - 2x^2 - 6x + 7$

Asymptotes: none

$f'(x) = 2x^2 - 4x - 6 = 2(x-3)(x+1)$

CN: -1, 3

Dec: [-1,3]

Inc: (-∞,-1] ∪ [3,∞)

local max: $\frac{31}{3}$ at -1

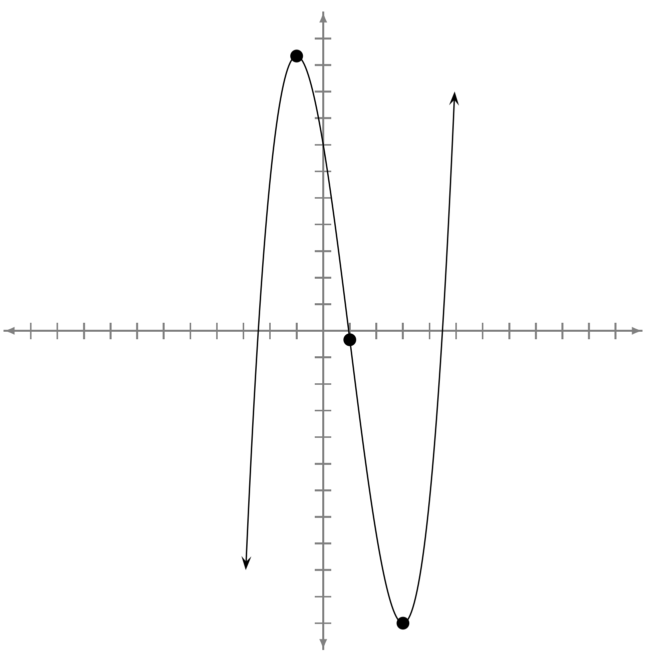
local min: -11 at 3

$f''(x) = 4x - 4 = 4(x-1)$

CD: (-∞,1)

CU: (1,∞)

IP: $\left(1,-\frac{1}{3}\right)$



(1) b. $f(x) = \dfrac{x^2+1}{x}$

$f(x) = x + \frac{1}{x}$

HA: none

VA: $x = 0$

OA: $y = x$

$f'(x) = 1 - \frac{1}{x^2} = \dfrac{x^2-1}{x^2}$

Dec: [-1,0), (0,1]

Inc: (-∞,-1], [1,∞)

Local max: -2 at -1

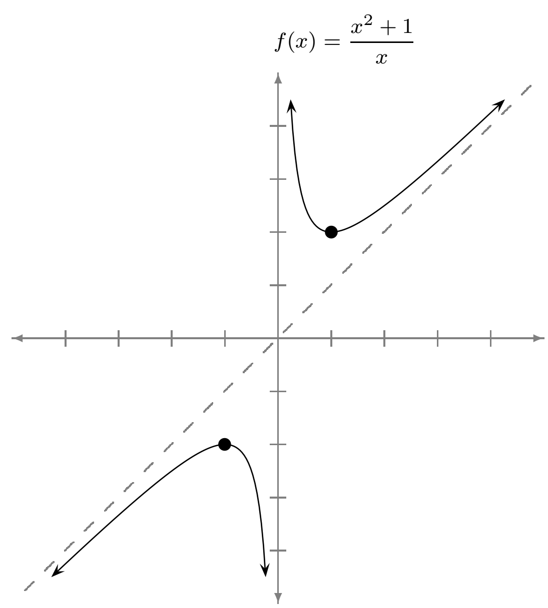
Local min: 2 at 1

$f''(x) = \frac{2}{x^3}$

CD: (-∞,0)

CU: (0,∞)

IP: none



2. A ball is thrown vertically with an initial velocity of 32 ft/sec from a height of 48 ft.

(1) a. Find the function that gives the height of the ball.

$h(t) = \text{-}16t^2 + 32t + 48$

(1) b. Find the maximum height of the ball.

$v(t) = h'(t) = \text{-}32t + 32 = \text{-}32(t-1)$

$v(t) = 0$

$\text{-}32(t-1) = 0$

$t = 1$

$h(1) = 64$

64 ft

(1) c. With what speed does the ball hit the ground?

$h(t) = 0$

$\text{-}16t^2 + 32t + 48 = 0$

$\text{-}16(t^2 - 2t - 3) = 0$

$\text{-}16(t-3)(t+1) = 0$

$t = 3$

$v(3) = \text{-}64$

64 ft/sec

(1) 3. A stone is thrown into a lake creating a circular ripple. The radius of the ripple is increasing at a rate of 3 cm per second. At what rate is the area of the ripple increasing after 2 seconds?

Solution 1:

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi \cdot 6 \cdot 3$

$\frac{dA}{dt} = 36\pi$

36π cm^2

Solution 2:

$A(t) = \pi(3t)^2 = 9\pi t^2$

$A'(t) = 18\pi t$

$A'(2) = 36\pi$

36π cm^2

(1) 4. A glass in the shape of a right circular cylinder with radius 6 centimeters is being filled with water at a rate of .01π liters per second. At what rate is the water level rising?
Recall: $V = \pi r^2 h$, 1 L = 1000 cm^3

$V = 36\pi h$

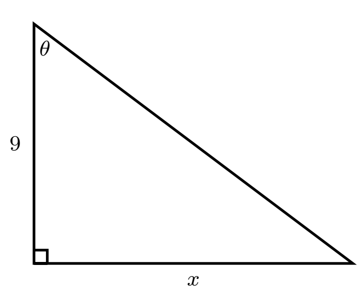
$\frac{dV}{dt} = 36\pi \frac{dh}{dt}$

$10\pi = 36\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{36}$

$\frac{5}{18}$ cm/sec

(1) 5. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is running away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?



$\tan\theta = \frac{x}{9}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{1}{250}$

$\frac{1}{250}$ radians per second

Quiz 102 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. For the following, find the absolute maximum and absolute minimum of the given function on the given interval.

$f(x) = x^3 - 3x^2 - 9x + 2$; [-4,4]

$f'(x)$

$= 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x-3)(x+1)$

CN: -1, 3

$f(\text{-}4) = \text{-}74$ (min)

$f(4) = \text{-}18$

$f(3) = \text{-}25$

$f(\text{-}1) = 7$ (max)

(1) 2. Find the minimum vertical distance between the parabola $y = x^2 - 3$ and the line $y = x - 5$ on the interval [-10,10].

$f(x)$

$= x^2 - 3 - (x - 5)$

$= x^2 - x + 2$

$f(x) = 2x - 1$

CN: $\frac{1}{2}$

$f(\text{-}10) = 102$

$f(10) = 92$

$f\left(\frac{1}{2}\right) = \frac{7}{4}$ (min)

(1) 3. Find the maximum area of a rectangle that can be built using 100 inches of material.

$A(x) = x\left[\frac{1}{2}(100 - 2x)\right] = x(50 - x) = \text{-}x^2 + 50x$; $x \in [0,50]$

$A'(x) = \text{-}2x + 50 = \text{-}2(x - 25)$

$A(0) = 0$

$A(50) = 0$

$A(25) = 625$

(1) 4. Use a linear approximation to approximate $\sqrt[3]{1001}$ $\left(\sqrt[3]{999}\right)$.

$g(x) = x^{\frac{1}{3}}$

$g'(x) = \frac{1}{3}x^{\text{-}\frac{2}{3}}$

$$g'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

$g'(1000) = \frac{1}{300}$

$g(1000) = 10$

$L(x) = \frac{1}{300}(x - 1000) + 10$

$\sqrt[3]{1001}$

$\approx L(1001)$

$= \frac{1}{300}(1001 - 1000) + 10$

$= 10 + \frac{1}{300}$

$\sqrt[3]{999}$

$\approx L(999)$

$= \frac{1}{300}(999 - 1000) + 10$

$= 10 - \frac{1}{300}$

(1) 5. Set $f(x) = x^3 - 6x^2 + 1$. Use differentials to approximate $f(2.01) - f(2)$.

$dy = f'(x)dx = (3x^2 - 12x)dx$

$dx = \Delta x = .01$

$\Delta y \approx dy = f'(2) \times 0.01 =$ -12 $\times$ 0.01 = -0.12

6. Consider the equation $x^5 + x^3 + x + 10 = 0$.

(1) a. Use the Intermediate Value Theorem to show that the equation has at least one solution.

Proof: Set $f(x) = x^5 + x^3 + x + 10$ and note that f is continuous since it is a polynomial.

$f(0) = 10$

f(-2) = -32

By the Intermediate Value Theorem, there is a solution between -2 and 0. ■

(1) b. Use the Mean Value Theorem to show that the equation has at most one solution.

Proof: Toward a contradiction, suppose that there are real numbers a and b such that $a < b$ and $f(a) = f(b)$ = 0. Since f is a polynomial, f is continuous on [a,b] and differentiable on (a,b). By the MVT, there exists $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a} = 0$. However, $f'(x) = 5x^4 + 3x^2 + 1 \geq 1$, a contradiction. Therefore, there is at most one real number c such that $f(c) = 0$. ■

(1) 7. Suppose that $f : \mathbb{R} \to \mathbb{R}$ such that $f'(x) = 16x^3 + 6x - 1$ and $f(1) = 12$ Find $f(x)$.

$f(x) = 4x^4 + 3x^2 - x + C$

$f(1) = 6 + C = 12$

$C = 6$

$f(x) = 4x^4 + 3x^2 - x + 6$

Suppose that $f : \mathbb{R} \to \mathbb{R}$ such that $f'(x) = 16x^3 + 6x - 1$ and $f(1) = 12$ Find $f(x)$.

$f(x) = 4x^4 + 3x^2 - x + C$

$f(1) = 6 + C = 21$

$C = 15$

$f(x) = 4x^4 + 3x^2 - x + 15$

Quiz 103 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Calculate each of the following. Do not simplify your answers.

(1) a. $\displaystyle\sum_{i=1}^{100}\left(2i^3 - i^2 + 3i + 4\right)$

$$= 2\sum_{i=1}^{100} i^3 - \sum_{i=1}^{100} i^2 + 3\sum_{i=1}^{100} i + \sum_{i=1}^{100} 4$$

$$= 2\left[\frac{100\cdot 101}{2}\right]^2 - \frac{100\cdot 101\cdot 201}{6} + 3\left[\frac{100\cdot 101}{2}\right] + 400$$

$= 50{,}682{,}200$

(1) b. $\sum_{j=3}^{6} j^{-1} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{19}{20}$

(1) c. $\displaystyle\sum_{n=1}^{400}\left(\frac{1}{n+1} - \frac{1}{n+3}\right) = \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \cdots + \frac{1}{399} - \frac{1}{401} + \frac{1}{400} + \frac{1}{400} - \frac{1}{402} + \frac{1}{401} - \frac{1}{403}$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{402} - \frac{1}{403}$

2. Write each of the following in sigma notation.

(1) a. $1 - 2 + 4 - 8 + \cdots + 1024 - 2048 = \displaystyle\sum_{n=0}^{11} (\text{-}2)^n$

(1) b. $3 + 6 + 9 + 12 + \cdots + 600 = \displaystyle\sum_{n=1}^{200} 3n$

(1) 3. Given that $\displaystyle\sum_{i=1}^{n} a_i = 7$ and $\displaystyle\sum_{i=1}^{n} b_i = 12$, find each of the following.

$$\sum_{i=1}^{n} (2a_i + b_i) = 2\sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i = 2\cdot 7 + 12 = 26$$

$$\sum_{i=1}^{n} (a_i + 2b_i) = \sum_{i=1}^{n} a_i + 2\sum_{i=1}^{n} b_i = 7 + 2\cdot 12 = 31$$

(1) 4. Calculate the following limit.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{8i^2}{n^3} - \frac{4i}{n^2} + \frac{2}{n}\right)$$

See Example 313 b.

(1) 5. Find a function f and numbers a and b such that

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\text{-}1+\tfrac{3i}{n}\right)^2 - 2\left(\text{-}1+\tfrac{3i}{n}\right)+4\right]\cdot\tfrac{3}{n}$$

$$\int_{\text{-}1}^{2}\left(x^2-2x+4\right)dx$$

(1) 6. Make a conjecture about the value of $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots$. Justify your answer. The only wrong answer to this question is an incomplete one.

Quiz 104 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Use the definition of integral to evaluate the following.

$$\int_0^4 \left(x^3+2x^2-3x+1\right)dx$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\tfrac{4i}{n}\right)^3+2\left(\tfrac{4i}{n}\right)^2-3\left(\tfrac{4i}{n}\right)+1\right]\tfrac{4}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\tfrac{256i^3}{n^4}+\tfrac{128i^2}{n^3}-\tfrac{48i}{n^2}+\tfrac{4}{n}\right]$$

$$= \lim_{n\to\infty}\left[\tfrac{256}{n^4}\sum_{i=1}^{n} i^3-\tfrac{128}{n^3}\sum_{i=1}^{n} i^2+\tfrac{48}{n^2}\sum_{i=1}^{n} i+\sum_{i=1}^{n}\tfrac{4}{n}\right]$$

$$= \lim_{n\to\infty}\left[\tfrac{256}{n^4}\left(\tfrac{n(n+1)}{2}\right)^2-\tfrac{128}{n^3}\left(\tfrac{n(n+1)(2n+1)}{6}\right)+\tfrac{48}{n^2}\left(\tfrac{n(n+1)}{2}\right)+4\right]$$

$$= 64-\tfrac{128}{3}+24+4$$

$$= \tfrac{148}{3}$$

(1) 2. For the following, express the given integral as a limit of a sum. Do not calculate the limit.

$$\int_{-1}^{1}\tan^{-1}x\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\tan^{-1}\left(-1+\tfrac{2i}{n}\right)\right]\tfrac{2i}{n}$$

(1) 3. Suppose that $f:\mathbb{R}\to\mathbb{R}$ and $a>0$ such that $\int_0^a f(x)\,dx = 4$. Give the value of $\int_{-a}^{a} f(x)\,dx$ under each of the following assumptions.

a. f is an even function

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx = 8$$

b. f is an odd function

$$\int_{-a}^{a} f(x)\,dx = 0$$

4. Integrate.

(1) a. $\int_0^1 \left(x^4-x^3+1\right)dx = \left(\tfrac{1}{5}x^5-\tfrac{1}{4}x^4+x\right)\Big|_0^1 = \tfrac{1}{5}-\tfrac{1}{4}+1 = \tfrac{19}{20}$

(1) b. $\int_{-1}^{2} \left(x^2+x\right)dx = \left(\tfrac{1}{3}x^3+\tfrac{1}{2}x^2\right)\Big|_{-1}^{2} = \tfrac{8}{3}+2-\left(-\tfrac{1}{3}+\tfrac{1}{2}\right) = \tfrac{9}{2}$

(1) c. $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\,dx = \text{-}\cos x\Big|_0^{\frac{\pi}{2}} = 0 - \text{-}1 = 1$

$$\int_0^{\frac{\pi}{2}} \cos x\,dx = \sin x\Big|_0^{\frac{\pi}{2}} = 1 - 0 = 1$$

(1) d. $\displaystyle\int \cos 2x\,dx = \tfrac{1}{2}\sin 2x + C$

(1) e. Use the Comparison Theorem for Integrals to give an upper bound for $\displaystyle\int_0^1 \sin^4 x\,dx$.

Since $\sin^4 x \leq 1$, $\displaystyle\int_0^1 \sin^4 x\,dx \leq \int_0^1 1\,dx = x\Big|_0^1 = 1 - 0 = 1$.

Quiz 105 Name:________________________

Directions: Show all of your work and justify all of your answers.

(4) 1. Integrate.

a. $\displaystyle\int (4x^3+2x-1)(x^4+x^2-x)^8\,dx$

Set $u = x^4+x^2-x$ and $du = (4x^2+2x-1)\,dx$.

$\displaystyle\int (4x^3+2x-1)(x^4+x^2-x)^8\,dx$

$= \displaystyle\int u^8\,du$

$= \frac{1}{9}u^9 + C$

$= \frac{1}{9}(x^4+x^2-x)^9 + C$

b. $\displaystyle\int_0^1 4x\sqrt{x^2+1}\,dx$

Set $u = x^2+1$ and $du = 2x\,dx$.

Then $2\,du = 4x\,dx$.

$\displaystyle\int_0^1 4x\sqrt{x^2+1}\,dx$

$= \displaystyle\int_1^2 2\sqrt{u}\,du$

$= \displaystyle\int_1^2 2u^{\frac{1}{2}}\,du$

$= \frac{4}{3}u^{\frac{3}{2}}\Big|_1^2$

$= \frac{4}{3}(2\sqrt{2}-1)$

$\displaystyle\int_0^1 4x\sqrt{4x^2+1}\,dx$

Set $u = 4x^2+1$ and $du = 8x\,dx$.

Then $\frac{1}{2}\,du = 4x\,dx$.

$\displaystyle\int_0^1 4x\sqrt{4x^2+1}\,dx$

$= \displaystyle\int_1^2 \tfrac{1}{2}\sqrt{u}\,du$

$= \displaystyle\int_1^2 \tfrac{1}{2}u^{\frac{1}{2}}\,du$

$= \frac{1}{3}u^{\frac{3}{2}}\Big|_1^2$

$= \frac{1}{3}(2\sqrt{2}-1)$

c. $\displaystyle\int \sec^2 x\tan x\,dx$

Set $u = \tan x$ and $du = \sec^2 x\,dx$.

$\displaystyle\int \sec^2 x\tan x\,dx$

$= \displaystyle\int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\tan^2 x + C$

d. $\displaystyle\int_{-2}^1 x\sqrt{x+3}\,dx$

Set $u = x+3$ and $du = dx$.

Then $x = u-3$.

$\displaystyle\int_{-2}^1 x\sqrt{x+3}\,dx$

$= \displaystyle\int_1^4 (u-3)\sqrt{u}\,du$

$= \displaystyle\int_1^4 \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right)du$

$= \left(\frac{2}{5}u^{\frac{5}{2}} - 2u^{\frac{3}{2}}\right)\Big|_1^4$

$= \frac{64}{5} - 16 - \left(\frac{2}{5} - 2\right)$

$= -\frac{8}{5}$

(2) 2. Find the area of the region enclosed by the given curves.

a.
$y = x^2 + 4x + 3$
$y = 3x + 5$

Solve:

$x^2 + 4x + 3 = 3x + 5$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x =$ -2, 1

$\int_{-2}^{1} \big[3x + 5 - (x^2 + 4x + 3)\big] dx$

$= \int_{-2}^{1} \big(-x^2 - x + 2\big) dx$

$= \big(-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\big)\Big|_{-2}^{1}$

$= -\frac{1}{3} - \frac{1}{2} + 2 - \big(\frac{8}{3} - 2 - 4\big)$

$= \frac{9}{2}$

b.
$y = \sqrt{x}$
$y = \frac{1}{3}x$

Solve:

$\sqrt{x} = \frac{1}{3}x$

$3\sqrt{x} = x$

$9x = x^2$

$x^2 - 9x = 0$

$x(x - 9) = 0$

$x = 0,\ 9$

$\int_0^9 \big(\sqrt{x} - \frac{1}{3}x\big) dx$

$= \int_0^9 \Big(x^{\frac{1}{2}} - \frac{1}{3}x\Big) dx$

$= \Big(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}x^2\Big)\Big|_0^9$

$= 18 - \frac{27}{2}$

$= \frac{9}{2}$

$y = \sqrt{x}$
$y = \frac{1}{2}x$

Solve:

$\sqrt{x} = \frac{1}{2}x$

$2\sqrt{x} = x$

$4x = x^2$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0,\ 4$

$\int_0^4 \big(\sqrt{x} - \frac{1}{2}x\big) dx$

$= \int_0^4 \Big(x^{\frac{1}{2}} - \frac{1}{2}x\Big) dx$

$= \Big(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{4}x^2\Big)\Big|_0^4$

$= \frac{16}{3} - 4$

$= \frac{4}{3}$

(1) 3. Find the average value of $f : [0,\pi] \to \mathbb{R}$ defined by $f(x) = \cos x + x^2$. Simplify your answer.

$\frac{1}{\pi}\int_0^{\pi} \big(\cos x + x^2\big) dx = \frac{1}{\pi}\left(\sin x + \frac{1}{3}x^3\right)\Big|_0^{\pi} = \frac{\pi^2}{3}$

(1) 4. Give the time, date, and location of the final exam.

Quiz 106 Name:__________________________

Directions: Show all of your work and justify all of your answers.

(4) 1. Let $\mathcal{R}$ be the region bounded by the curves $y = \sqrt{x}$ and $y = x$. For each of the following, find the volume of the solid formed by rotating $\mathcal{R}$ about the given line.

a. the x-axis

b. the horizontal line $y = 2$

c. the y-axis

d. the vertical line $x =$ -1

(1) 2. Let $\mathcal{R}$ be the region bounded by the curves $y = \sqrt{\sin x}$ and $y = 0$ for $0 \leq x \leq \pi$. Find the volume of the solid formed by rotating $\mathcal{R}$ about the x-axis.

(2) 3. Suppose that S is a 3-dimensional solid whose base is the triangle with vertices (0,0), (1,0), and (0,1). Give the volume of the solid if cross sections perpendicular to the x-axis are

a. squares

b. semicircles

(2) 4. Assume that the density of water is 62.5 lb/ft^3. A cylindrical water tank with a circular base has radius 3 feet and height 6 feet. Find the work required to empty the tank by pumping the water out of the top for each of the following situations.

a. The tank is full.

b. The tank is half full.

(1) 5. A spring is 1 foot long and has spring constant 4. Calculate the work required to stretch the spring to 2 feet.

(1) 6. A steel cable that is 20 ft long and weighs 20 lb is hanging over the edge of a roof. Calculate the work required to pull the cable up to the roof.

Chapter 11: Math 1571 Summer 2025

Section 11.1: Quizzes

Quiz 1 Name:__________________________

<u>Directions:</u> Show all of your work and justify all of your answers.

(1) 1. Prove that if $|x-2|<1$, then $|3x+1-7|<3$.

Proof:

$|x-2|<1$

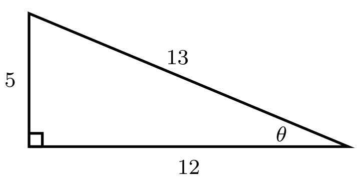
$3|x-2|<3$

$|3x-6|<3$

$|3x+1-7|<3$ ■

2. For each of the following, give the six trigonometric values of θ.

(1) a.



$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

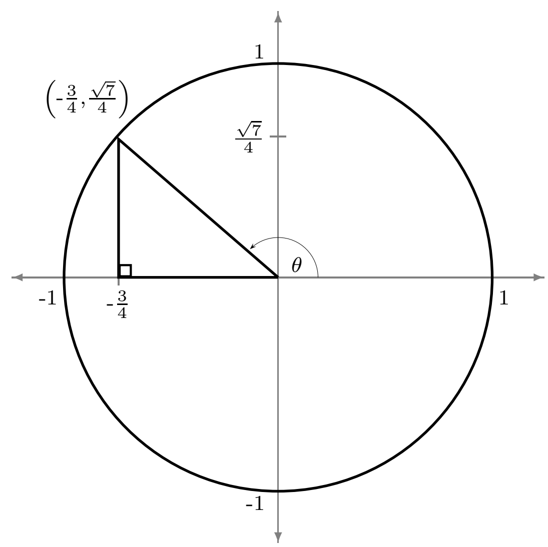
$\tan\theta = \frac{5}{12}$

$\cot\theta = \frac{12}{5}$

$\sec\theta = \frac{13}{12}$

$\csc\theta = \frac{13}{5}$

(1) b.



$\sin\theta = \frac{\sqrt{7}}{4}$

$\cos\theta = -\frac{3}{4}$

$\tan\theta = -\frac{\sqrt{7}}{3}$

$\cot\theta = -\frac{3}{\sqrt{7}}$

$\sec\theta = -\frac{4}{3}$

$\csc\theta = \frac{4}{\sqrt{7}}$

(1) 3. Give the center and radius of the following circle.

$x^2 + 2x + y^2 - 6y =$ -5

$(x+1)^2 + (y-3)^2 =$ -5 $+ 1 + 9$

$(x+1)^2 + (y-3)^2 = 5$

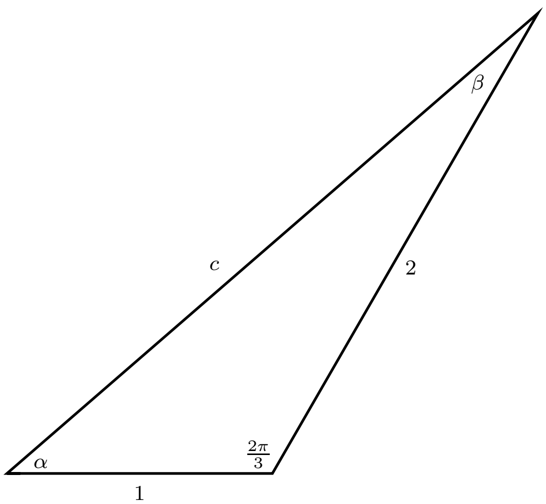
Center: (-1, 3)

Radius: $\sqrt{5}$

Quiz 2 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. In the figure below, solve for α, β, and c.



Apply the Law of Cosines.

$$c^2 = 2^2 + 1^2 - 2 \cdot 1 \cdot 2 \cdot \cos\left(\frac{2\pi}{3}\right)$$

$$c^2 = 7$$

$$c = \sqrt{7}$$

Apply the Law of Sines.

$$\frac{\sin\alpha}{2} = \frac{\sin\beta}{1} = \frac{\sin\frac{2\pi}{3}}{\sqrt{7}} = \frac{\sqrt{3}}{2\sqrt{7}} = \sqrt{\frac{3}{28}}$$

$$\sin\alpha = \sqrt{\frac{3}{7}}$$

$$\alpha = \sin^{-1}\sqrt{\frac{3}{7}}$$

$$\beta = \sin^{-1}\sqrt{\frac{3}{28}}$$

2. Prove the following identities.

(1) a. $\cos\theta\cot\theta + \sin\theta = \csc\theta$

Proof: $\cos\theta\cot\theta + \sin\theta = \frac{\cos^2\theta}{\sin\theta} + \sin\theta = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$ ■

(1) b. $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

Proof: $\sin\left(\theta + \frac{\pi}{2}\right) = \sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2} = (\sin\theta)\cdot 0 + (\cos\theta)\cdot 1 = \cos\theta$ ■

(1) 3. Give the equation of the line that contains the point (-3,5) and is perpendicular to the line $y = -\frac{1}{2}x + 7$.

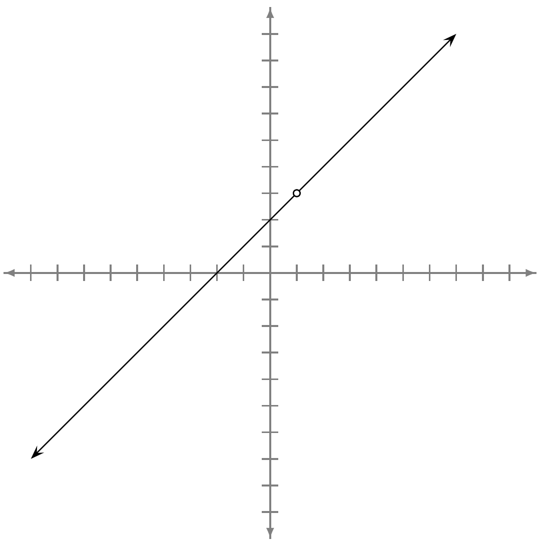
$y - 5 = 2(x + 3)$

4. For each of the following, give the domain and range and sketch the graph.

(1) a. $f(x) = \dfrac{x^2 + x - 2}{x - 1} = \dfrac{(x+2)(x-1)}{x-1} \neq x + 2$

Domain: (-∞,1) $\cup$ (1,∞)

Range: (-∞,3) $\cup$ (3,∞)

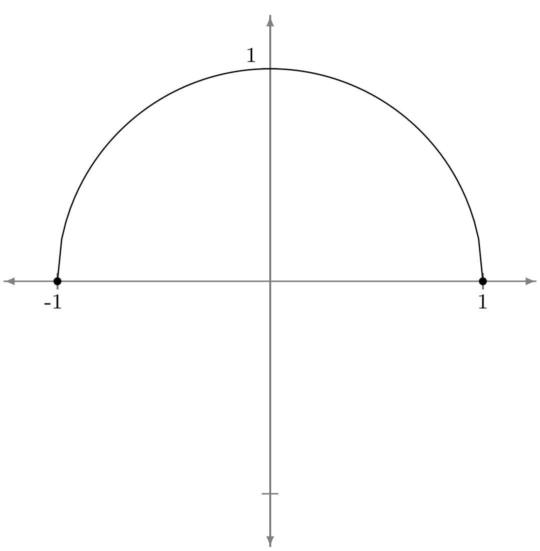


(1) b. $g(x) = \sqrt{1 - x^2}$

Domain: [-1,1]

Range: [0,1]

The graph is the top half of the unit circle.



Quiz 3 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Set $f(x) = x^2 + 4$ and $g(x) = \sqrt[3]{x}$. Find each of the following.

a. $(f \circ g)(x) = \left(\sqrt[3]{x}\right)^2 + 4 = \sqrt[3]{x^2} + 4$

b. $(g \circ f)(x) = \sqrt[3]{x^2 + 4}$

(1) 2. Prove that the composition of two odd functions is an odd function.

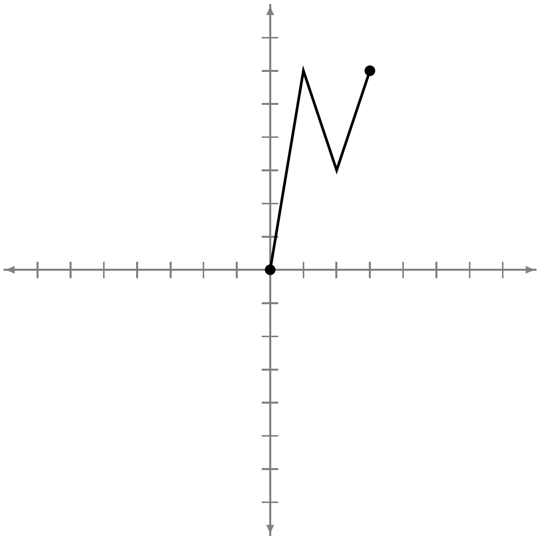
***Proof*:** If f and g are odd functions, then $(f \circ g)(\text{-}x) = f(g(\text{-}x)) = f(\text{-}g(x)) = \text{-}f(g(x)) = \text{-}(f \circ g)(x)$. ■

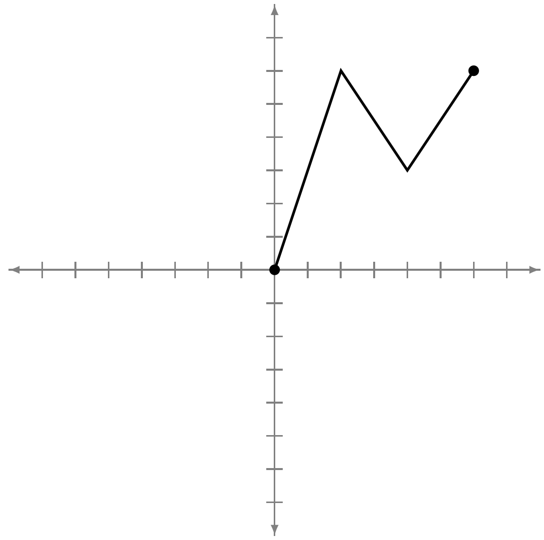
(1) 3. Find a quadratic function with x-intercepts (1,0) and (-2,0) and with y-intercept (0,4).

Begin by setting $f(x) = (x-1)(x+2)$. Compute $f(0) = \text{-}2 \neq 4$. Since $\text{-}2 \cdot \text{-}2 = 4$, set $g(x) = \text{-}2f(x) = \text{-}2(x-1)(x+2)$. Check that g is the desired function.

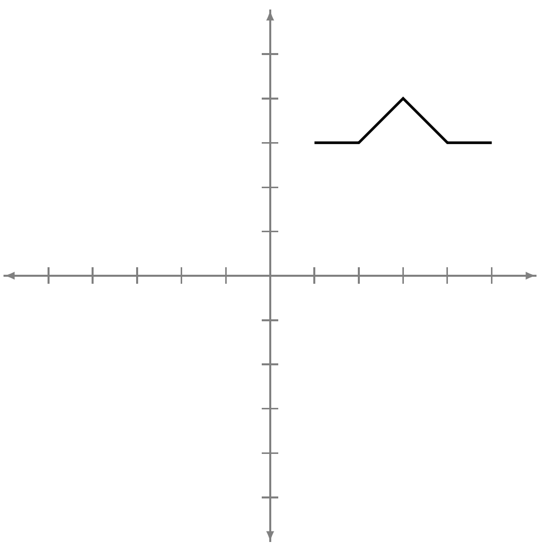
(1) 4.

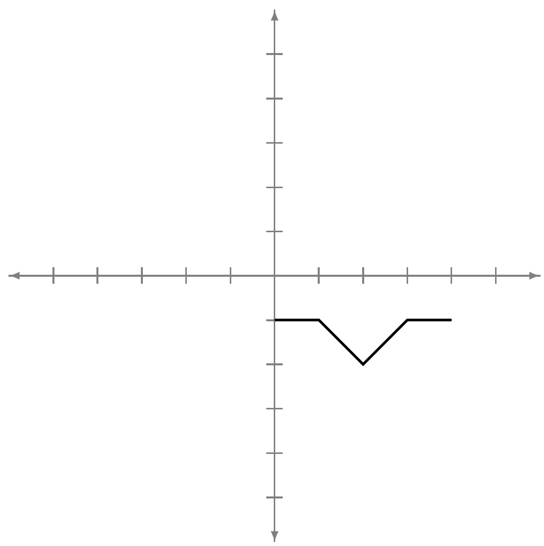
a. Below is the graph of the function $y = f(x)$. Sketch the graph of $y = f\left(\frac{1}{2}x\right)$.





b. Below is the graph of the function $y = f(x)$. Sketch the graph of $y = \text{-}f(x+1)+2$.





Quiz 4 Name:_________________________

Directions: Show all of your work and justify all of your answers.

1. Use the $\epsilon - \delta$ definition of limit to prove each of the following.

(1) a. $\lim\limits_{x\to 2}\big(4x-1\big)=7$

Proof: Let $\varepsilon > 0$ be given and set $\delta = \frac{\varepsilon}{4}$. Suppose that $0 < |x-2| < \delta$. Then

$|x-2| < \delta$

$|x-2| < \frac{\varepsilon}{4}$

$4|x-2| < \varepsilon$

$|4x-8| < \varepsilon$

$|(4x-1)-7| < \varepsilon$

as desired. ■

(1) b. $\lim\limits_{x\to 1}\big(x^2-1\big)=0$

Proof: Let $\varepsilon > 0$ be given and let $\delta < min\{\frac{\varepsilon}{3},1\}$. If $0 < |x-1| < \delta$, then

$|x-1| < \delta$

$|x-1| < \frac{\varepsilon}{3}$

$3|x-1| < \varepsilon$

$|(x+1)(x-1)| < \varepsilon$

$|x^2-1| < \varepsilon$

$|(x^2-1)-0| < \varepsilon$

as desired. ■

2. Calculate each of the following limits.

(1) a. $\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{x^2-5x+6}$

$=\lim\limits_{x\to 3}\dfrac{(x-3)(x+1)}{(x-3)(x-2)}$

$=\lim\limits_{x\to 3}\dfrac{x+1}{x-2}$

$=4$

(1) b. $\lim\limits_{h\to 0}\left(\dfrac{\sqrt{3+h}-\sqrt{3}}{h}\cdot\dfrac{\sqrt{3+h}+\sqrt{3}}{\sqrt{3+h}+\sqrt{3}}\right)$

$=\lim\limits_{h\to 0}\dfrac{3+h-3}{h(\sqrt{3+h}+\sqrt{3})}$

$=\lim\limits_{h\to 0}\dfrac{1}{\sqrt{3+h}+\sqrt{3}}$

$=\frac{1}{2\sqrt{3}}$

Quiz 5 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. For each of the following, calculate the limit or state why it does not exist.

(1) a. $\lim_{x \to 2} \frac{|x-1|}{x-1} = 1$

(1) b. $\lim_{x \to 1} \frac{|x-1|}{x-1}$

$$\lim_{x \to 1^+} \frac{|x-1|}{x-1} = \lim_{x \to 1^+} \frac{x-1}{x-1} = 1$$

$$\lim_{x \to 1^-} \frac{|x-1|}{x-1} = \lim_{x \to 1^-} \frac{1-x}{x-1} = \text{-1}$$

$$\lim_{x \to 1} \frac{|x-1|}{x-1} \text{ DNE}$$

(1) c. $\lim_{x \to 0} x^2 \left|\sin\left(\frac{1}{\sqrt[3]{x^2+x}}\right)\right|$

Since $0 \leq x^2 \left|\sin\left(\frac{1}{\sqrt[3]{x^2+x}}\right)\right| \leq x^2$ and $\lim_{x \to 0} x^2 = 0$, $\lim_{x \to 0} x^2 \left|\sin\left(\frac{1}{\sqrt[3]{x^2+x}}\right)\right| = 0$ by the Squeeze Theorem.

(1) 2. Is the following function continuous at 4?

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 4 \\ \\ 3x + 3 & \text{if } x \geq 4 \end{cases}$$

Yes.

$f(4) = 15$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \left(x^2 - 1\right) = 15$$

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \left(3x + 3\right) = 15$$

$$\lim_{x \to 4} f(x) = 15$$

$$\lim_{x \to 4} f(x) = f(4)$$

Quiz 6 Name:_________________________

Directions: Show all of your work and justify all of your answers.

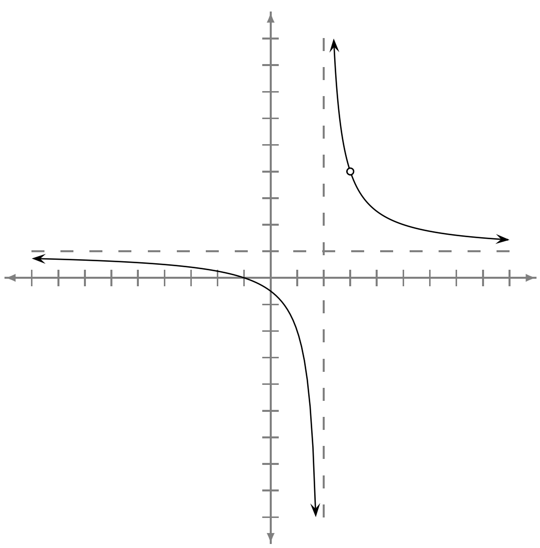
(1) 1. For the following function, find the vertical asymptotes and sketch the graph.

$$g(x) = \frac{x^2 - 2x - 3}{x^2 - 5x + 6} = \frac{(x-3)(x+1)}{(x-3)(x-2)} \neq \frac{x+1}{x-2}$$

HA: $y = 1$

VA: $x = 2$

$$\lim_{x \to 3} g(x) = \lim_{x \to 3} \frac{x+1}{x-2} = 4$$



(1) 2. Prove that the following equation has a solution.

$x^3 - \sqrt{x} + 1 = 3$

Proof: Set $f(x) = x^3 - \sqrt{x} + 1$ and note that f is continuous where it is defined. Calculate $f(1) = 1$ and $f(4) = 63$. By the Intermediate Value Theorem, there is $c \in [1,4]$ such that $f(c) = 3$. ∎

(1) 3. Suppose that $f : [0,1] \to [0,1]$ is continuous, $f(0) = 1$, and $f(1) = 0$. Prove that there exists $c \in [0,1]$ such that $f(c) = c$.

Proof: Define $g : [0,1] \to [-1,1]$ by $g(x) = f(x) - x$ and note that g is continuous on $[0,1]$ since it is the difference of continuous functions. Calculate $g(0) = 1$ and $g(1) = -1$. By the Intermediate Value Theorem, there is $c \in [0,1]$ such that $g(c) = 0$. So $f(c) - c = 0$ which means that $f(c) = c$. ∎

Quiz 7 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Find the horizontal asymptotes, vertical asymptotes, and oblique asymptotes. Use your answers to sketch the graph.

a. $f(x) = \dfrac{2x^2 - 2x - 12}{x^2 - 2x - 8}$

$= \dfrac{2(x-3)(x+2)}{(x-4)(x+2)}$

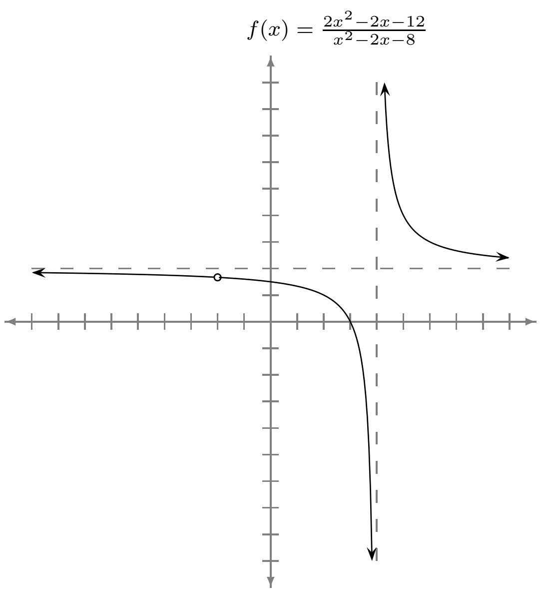
$\neq \dfrac{2(x-3)}{x-4}$

HA: $y = 2$

VA: $x = 4$

$\lim\limits_{x\to -2} f(x) = \lim\limits_{x\to -2} \frac{2(x-3)}{x-4} = \frac{5}{3}$

OA: None.



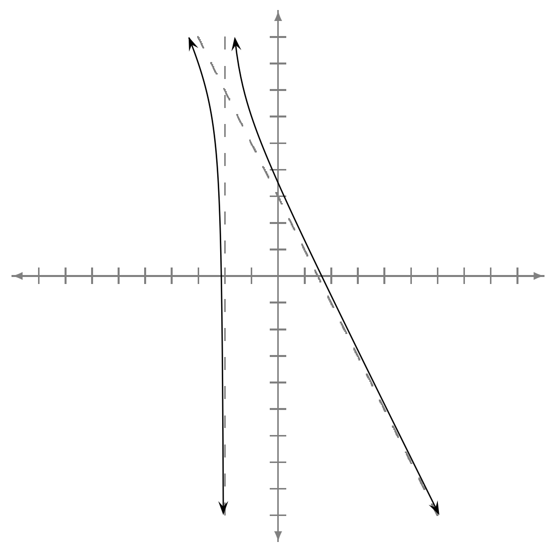
b. $g(x) = \dfrac{-2x^2 - x + 7}{x+2}$

$= -2x + 3 + \frac{1}{x+2}$

HA: None.

VA: $x = -2$

OA: $y = -2x + 3$



c. $h(x) = \dfrac{-2x^2 - x + 6}{x+2} = \dfrac{(-2x+3)(x+2)}{x+2} \neq -2x + 3$

HA: none

VA: none

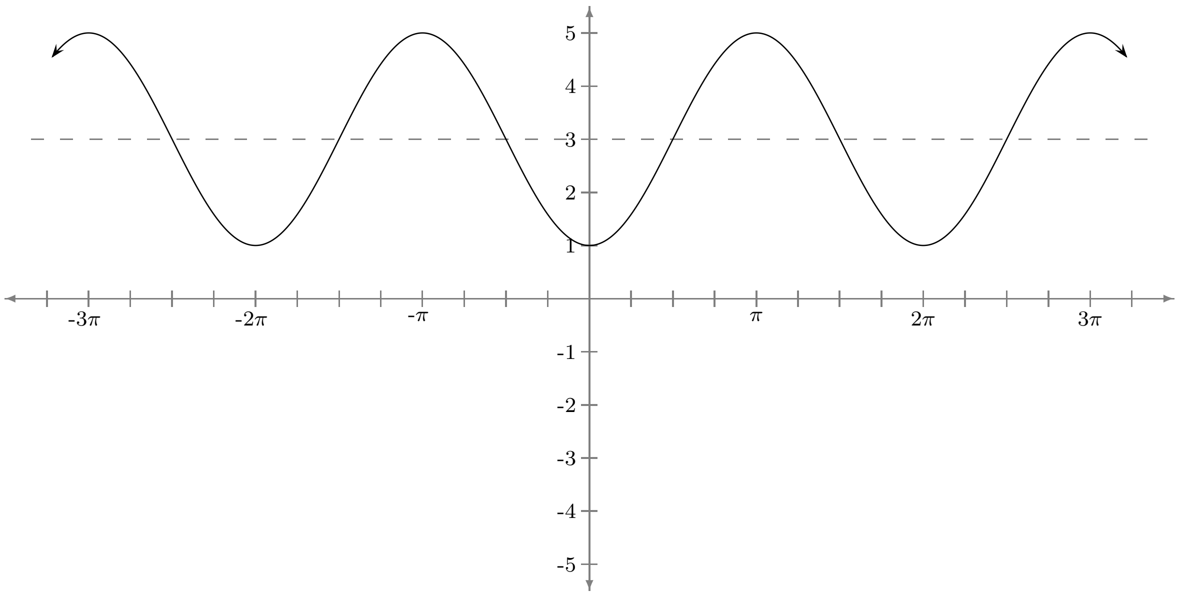
OA: $y = -2x + 3$

The graph is the line $y = -2x + 3$ with the point (-2,7) removed.

(1) 2. Find the amplitude, period, phase shift, and average value of the following. Sketch the graph.

$f(\theta) = \text{-}2\sin\left(\theta + \frac{\pi}{2}\right) + 3$

Amplitude: 2 Period: 2π Phase Shift: $\frac{\pi}{2}$ Average Value: 3



(1) 3. Calculate the following limits.

a. $\lim_{\theta \to 0} \dfrac{\sin 3\theta}{\theta} = \lim_{\theta \to 0} \dfrac{3 \sin 3\theta}{3\theta} = 3 \lim_{\theta \to 0} \dfrac{\sin 3\theta}{3\theta} = 3 \cdot 1 = 3$

b. $\lim_{\theta \to 0} \dfrac{\tan \theta}{\theta} = \lim_{\theta \to 0} \dfrac{\sin \theta}{\theta \cos \theta} = \left[\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta}\right] \cdot \left[\lim_{\theta \to 0} \dfrac{1}{\cos \theta}\right] = 1 \cdot 1 = 1$

c. $\lim_{\theta \to 0}\left(\cot \theta - \csc \theta\right) = \lim_{\theta \to 0}\left(\dfrac{\cos \theta}{\sin \theta} - \dfrac{1}{\sin \theta}\right) = \lim_{\theta \to 0} \dfrac{\cos \theta - 1}{\sin \theta} = \lim_{\theta \to 0}\left(\dfrac{\cos \theta - 1}{\theta} \cdot \dfrac{\theta}{\sin \theta}\right) = 0 \cdot 1 = 0$

Quiz 8 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. Differentiate each of the following.

a. $f(x) = x^5 - 3x^4 + 6x^2 - 7x + 8$

$f'(x) = 5x^4 - 12x^3 + 12x - 7$

b. $g(x) = \dfrac{x^8 + x + 11}{5x^3 + 1}$

$$g'(x) = \frac{(8x^7 + 1)(5x^3 + 1) - (x^8 + x + 11)(15x^2)}{(5x^3 + 1)^2}$$

c. $h(x) = (x^8 + x + 11)(5x^3 + 1)$

$h'(x) = (8x^7 + 1)(x^8 + x + 11) + (x^8 + x + 11)(15x^2)$

d. $f(x) = \sqrt{x^2 + x + 1}$

$f(x) = (x^2 + x + 1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2 + x + 1)^{-\frac{1}{2}}(2x + 1) = \frac{1}{2\sqrt{x^2+x+1}}$

(1) 2. Set $f(x) = x^4 + x^2 + x + 1$.

a. Find the equation of the line that is tangent to the graph of f at the point where $x = -1$

$f'(x) = 4x^3 + 2x + 1$

$x = -1$

$y = f(-1) = 2$

$m = f'(-1) = -5$

$y - 2 = -5(x + 1)$

b. Find $f^{(20)}(x)$.

Since f is a fourth degree polynomial, $f^{(n)}(x) = 0$ for all $n > 4$.

(1) 3. Find the function $f : \mathbb{R} \to \mathbb{R}$ such that $f'(x) = 4x^3 + x - 1$ and $f(0) = 7$.

$f(x) = x^4 + \frac{1}{2}x^2 - x + C$

Since $f(0) = 7$, $C = 7$.

$f(x) = x^4 + \frac{1}{2}x^2 - x + 7$

Quiz 9 Name:________________________

Directions: Show all of your work and justify all of your answers.

(2) 1. A particle moves according to a law of motion $s(t) = t^3 - 9t^2 + 24t + 5$ where s is measured in meters and t in seconds.

a. Find the velocity at time t.

$v(t) = s'(t) = 3t^2 - 18t + 24 = 3(t-2)(t-4)$

b. When is the particle at rest?

Times $t = 2$ and $t = 4$.

c. Find the acceleration at time t.

$a(t) = v'(t) = s''(t) = 6t - 18 =$

d. Find the total distance traveled during the first 5 seconds.

$|s(2) - s(0)| + |s(4) - s(2)| + |s(5) - s(4)| = |25 - 5| + |21 - 25| + |25 - 21| = 28$

(1) 2. Recall that the volume of a sphere $V = \frac{4}{3}\pi r^3$. A spherical snowball with radius 6 inches is melting so that its radius is decreasing at a rate of 1 inch per day.

a. Express the radius of the snowball as a function of time.

$r = 6 - t$

b. Express the volume of the snowball as a function of time.

$V = \frac{4}{3}\pi(6-t)^3$

c. Express the rate of change of the volume of the snowball as a function of time.

$\frac{dV}{dt} = 4\pi(6-t)^2(-1)$

(1) 3. Recall that acceleration due to gravity is -32 ft/sec^2. A ball is thrown upward with an initial velocity of of 40 ft/sec from a height of 200 ft. Give the height of the ball as a function of time.

See class notes.

Quiz 10 Name:________________________

Directions: Show all of your work and justify all of your answers.

(3) 1. All parts are worth one-half of a point.

2. Differentiate.

a. $f(x) = \tan x + \sec x$

$f'(x) = \sec^2 x + \sec x \tan x$

b. $g(x) = \sqrt{\sin x}$

$g'(x) = \frac{\cos x}{2\sqrt{\sin x}}$

c. $h(x) = \sin\sqrt{x}$

$h'(x) = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

3. Find the equation of the line that is tangent to the given curve at the specified point.

a. $y = \sin\left(x^3 + x + \frac{\pi}{3}\right)$; $x = 0$

$y' = (3x^2+1)\cos\left(x^3 + x + \frac{\pi}{3}\right)$

$x = 0$

$y = \frac{\sqrt{3}}{2}$

$m = y' = \frac{1}{2}$

$y - \frac{\sqrt{3}}{2} = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x + \frac{\sqrt{3}}{2}$

b. $2x^4 + y^2 + x^3y = 8$; (-1,3)

$\frac{d}{dx}\left(2x^4 + y^2 + x^3y\right) = \frac{d}{dx}8$

$8x^3 + 2yy' + 3x^2y + x^3y' = 0$

$y' = \frac{-8x^3 - 3x^2y}{2y + x^3}$

$x = -1$

$y = 3$

$m = y' = -\frac{1}{5}$

$y - 3 = -\frac{1}{5}(x+1)$

c. The circle with center (-3,1) and radius 2; (-2,$\sqrt{3}+1$)

$(x+3)^2 + (y-1)^2 = 4$

$\frac{d}{dx}\left[(x+3)^2 + (y-1)^2\right] = \frac{d}{dx}4$

$2(x+3) + 2(y-1)y' = 0$

$y' = \frac{-x-3}{y-1}$

$x = -2$

$y = \sqrt{3} + 1$

$m = y' = -\frac{1}{\sqrt{3}}$

$y - (\sqrt{3}+1) = -\frac{1}{\sqrt{3}}(x+2)$

Quiz 11 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(1) a. $g(x) = x^3 - 3x - 2$

HA: none

VA: none

OA: none

$g'(x) = 3x^2 - 3 = 3(x+1)(x-1)$

Dec: [-1,1]

Inc: (-∞,1], [1,∞)

Local max: 0 at -1

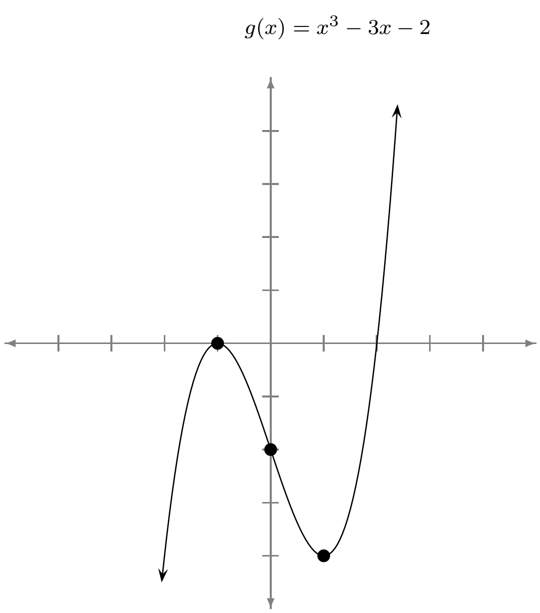
Local min: -4 at 1

$g''(x) = 6x$

CD: (-∞,0)

CU: (0,-∞)

IP: (0,-2)



(1) b. $f(x) = \dfrac{x^2+1}{x} = x + \frac{1}{x}$

HA: none

VA: $x = 0$

OA: $y = x$

$f'(x) = 1 - \frac{1}{x^2} = \dfrac{x^2-1}{x^2} = \dfrac{(x+1)(x-1)}{x^2}$

Dec: [-1,0), (0,1])

Inc: (-∞,-1], [1,∞)

Local max: -2 at -1

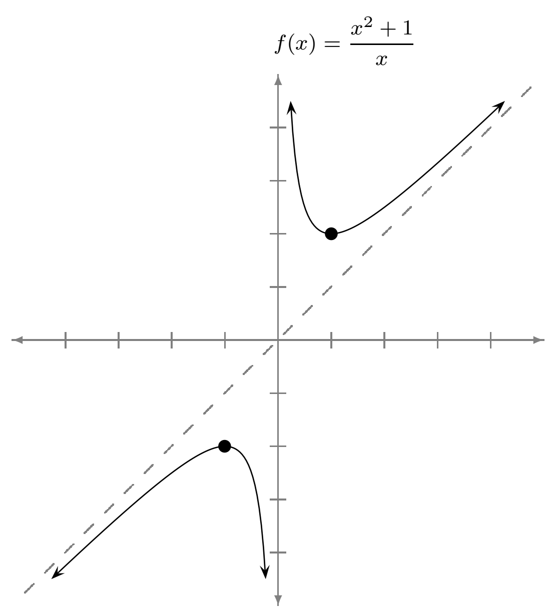
Local min: 2 at 1

$f''(x) = \frac{2}{x^3}$

CD: (-∞,0)

CU: (0,∞)

IP: none



(2) 2. A ball is thrown upward with an initial velocity of of 48 ft/sec from a height of 64 ft.

a. Give the height of the ball as a function of time.

$h(t) = -16t^2 + 48t + 64$

b. When does the ball reach its maximum height?

$v(t) = h'(t) = -32t + 48 = -16(2t - 3)$

$t = \frac{3}{2}$

$\frac{3}{2}$ sec.

c. Find the maximum height of the ball.

$h\left(\frac{3}{2}\right) = 100$

100 ft

d. When does the ball hit the ground?

$-16t^2 + 48t + 64 = 0$

$-16(t^2 - 3t - 4) = 0$

$(t - 4)(t + 1) = 0$

$t = 4$

4 sec.

e. Find the velocity of the ball when it hits the ground.

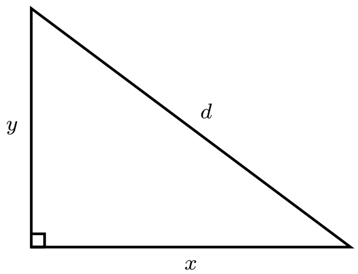
$v(4) = -80$

-80 ft/sec.

Quiz 12 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. A cat and a dog begin walking from the same point. The dog walks east at a rate of $\frac{4}{3}$ ft/sec. The cat walks north at a rate of 1 ft/sec. At what rate is the distance between them changing after 30 seconds?



$x^2 + y^2 = d^2$

$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}\left(d^2\right)$

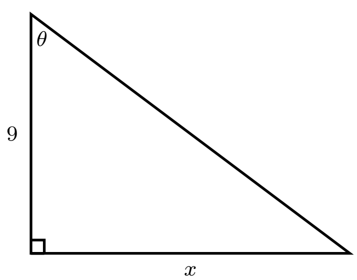
$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2d \cdot \frac{dd}{dt}$

$2(40)\left(\frac{4}{3}\right) + 2(30)(1) = 2(50)\frac{dd}{dt}$

$2(50)\frac{dd}{dt} = \frac{5}{3}$

$\frac{5}{3}$ ft/sec

(1) 2. An owl is sitting in a tree 9 meters above the ground. The owl is watching a mouse that is running away from the tree at a rate of 10 centimeters per second. At what angular rate is the owl raising its eyes when the mouse is 12 meters from the tree?



$\tan\theta = \frac{x}{9}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{dx}{dt}$

$\frac{25}{9} \cdot \frac{d\theta}{dt} = \frac{1}{9} \cdot \frac{1}{10}$

$\frac{d\theta}{dt} = \frac{1}{250}$

$\frac{1}{250}$ radians per second

(1) 3. A balloon is being inflated so that its volume is increasing at a rate of $\frac{16\pi}{3}$ cm^3 per second. At what rate is the radius of the balloon increasing after 2 seconds?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{16\pi}{3} = 4\pi(4)\frac{dr}{dt}$ [When $t = 2$, $V = \frac{32\pi}{3}$ and $r = 2$.]

$\frac{dr}{dt} = \frac{1}{3}$

$\frac{1}{3}$ cm/sec

Quiz 13 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Find the absolute extrema of the function $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 1$ on the interval [0,3].

$f'(x) = 12x^3 - 24x^2 - 12x + 24 = 12(x+1)(x-1)(x-2)$

$f(0) = 1$

$f(3) = 46$ (max)

$f(1) = 14$

$f(\text{-}1) = \text{-}18$ (min)

$f(2) = 9$

(1) 2. What is the maximum area of a rectangle that can be inscribed in the unit circle?

For $x \in [0,1]$, set $A(x) = (2x) \cdot 2\sqrt{1-x^2}$.

$A(x) = 4x\sqrt{1-x^2}$

$$A'(x) = 4\sqrt{1-x^2} + \frac{4x(\text{-}2x)}{2\sqrt{1-x^2}} = 4\sqrt{1-x^2} - \frac{4x^2}{\sqrt{1-x^2}} = \frac{4(1-2x^2)}{\sqrt{1-x^2}}$$

$A(0) = 0$

$A(1) = 1$

$A\left(\frac{1}{\sqrt{2}}\right) = 2$

Quiz 14 Name:_________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Use either a linear approximation (suggested) or differentials to approximate $\sqrt[3]{999}$.

We use the linear approximation of $f(x) = \sqrt[3]{x}$ at 1000.

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f'(1000) = \frac{1}{300}$

$L(x) = 10 + \frac{1}{300}(x - 1000)$

$L(999) = 10 - \frac{1}{300}$

(1) 2. Recall that the volume of a sphere is $V = \frac{4}{3}\pi r^3$ and the surface area of a sphere is $S = 4\pi r^2$. The radius of a sphere is measured to be 30 cm with a possible error of 1 mm.

a. Use a linear approximation or differentials (suggested) to approximate the possible error in calculating the volume of the sphere.

$dV = 4\pi r^2 dr$

$dV = 4\pi \cdot 900 \cdot \frac{1}{10} = 360\pi$

360π cm^3

b. Use a linear approximation or differentials (suggested) to approximate the possible error in calculating the surface area of the sphere.

$dS = 8\pi r dr$

$dS = 8\pi \cdot 30 \cdot \frac{1}{10} dr = 24\pi$

24π cm^2

(1) 3. Prove that the equation $x^9 + x^5 + x + 1 = 0$ has exactly one solution.

Proof: Set $f(x) = x^9 + x^5 + x + 1$ and note that f is continuous on $\mathbb{R}$. Since $f(0) = 1$ and $f(1) = 4$, $f(x) = 0$ has a solution between 0 and 1 by the Intermediate Value Theorem.

To see that $f(x) = 0$ has only one solution, suppose that it has two. Then there are $a,b \in \mathbb{R}$ with $a < b$ such that $f(a) = f(b) = 0$. By either the Mean Value Theorem or Rolle's Theorem, there is $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a} = 0$. However, this is a contradiction since $f'(c) = 9c^8 + 5c^4 + 1 > 0$. Therefore, $f(x) = 0$ has only one solution. ■

(1) 4. Calculate the following sums.

a. $\sum_{i=1}^{10}(i^3+i^2+i)$

$= \sum_{i=1}^{10} i^3 + \sum_{i=1}^{10} i^2 + \sum_{i=1}^{10} i$

$= \left(\frac{10\cdot 11}{2}\right)^2 + \frac{10\cdot 11\cdot 21}{6} + \frac{10\cdot 11}{2}$

$= 3465$

b. $\sum_{n=1}^{30}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

$= 1-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{5}+\cdots+\frac{1}{28}-\frac{1}{30}+\frac{1}{29}-\frac{1}{31}+\frac{1}{30}-\frac{1}{32}$

$= 1+\frac{1}{2}-\frac{1}{31}-\frac{1}{32}$

(1) 5. Make a conjecture about the following.

$$\sum_{n=0}^{\infty}\frac{1}{2^n} = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

The sum is 2.

Each term adds one-half of the difference between 2 and the sum of the previous terms. So the sum will not exceed 2. However, for any number $c < 2$, the sum will exceed c.

Imagine a 2 gallon bucket being filled as follows.

Put 1 gallon of water in the bucket.

Put $\frac{1}{2}$-gallon of water in the bucket.

Put $\frac{1}{4}$-gallon of water in the bucket.

Put $\frac{1}{8}$-gallon of water in the bucket.

etc.

The bucket will never overflow but will be filled after an infinite number of additions.

Quiz 15 Name:________________________

Directions: Show all of your work and justify all of your answers.

1. Use the definition of integral to evaluate each of the following.

(1) a. $\int_1^3 (2x+1)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(1+\frac{2i}{n}\right)+1\right]\frac{2}{n}$

$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} \left(\frac{4i}{n}+3\right)$

$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4}{n}\sum_{i=1}^{n} i+\sum_{i=1}^{n} 3\right)$

$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4}{n}\cdot\frac{n(n+1)}{2}+3n\right)$

$= \lim_{n\to\infty} \left(\frac{4n(n+1)}{n^2}+6\right)$

$= 10$

(1) b. $\int_0^1 (x^3+x^2)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0+\frac{i}{n}\right)\cdot\frac{1}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{i}{n}\right)^3+\left(\frac{i}{n}\right)^2\right]\frac{1}{n}$

$= \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \left(\frac{i^3}{n^3}+\frac{i^2}{n^2}\right)$

$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{n^3}\sum_{i=1}^{n} i^3+\frac{1}{n^2}\sum_{i=1}^{n} i^2\right)$

$= \lim_{n\to\infty} \left[\frac{1}{n^4}\cdot\left(\frac{n(n+1)}{2}\right)^2+\frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$

$= \frac{1}{4}+\frac{2}{6}$

$= \frac{7}{12}$

(1) 2. Given that $\int_0^4 f(x)\,dx = 7$, evaluate $\int_{-4}^4 f(x)\,dx$ given that

a. f is even.

$\int_{-4}^4 f(x)\,dx = 2\int_0^4 f(x)\,dx = 14$

b. f is odd.

$\int_{-4}^4 f(x)\,dx = 0$

(1) 3. Given that $\int_1^2 f(x)\,dx = 3$, $\int_1^2 g(x)\,dx = 2$, and $\int_1^4 f(x)\,dx = 12$, evaluate each of the following.

a. $\int_1^2 \left[2f(x)+g(x)\right]dx = 2\int_1^2 f(x) + \int_1^2 g(x)\ dx = 2\cdot 3+2 = 8$

b. $\int_2^4 f(x)\,dx = \int_1^4 f(x)\,dx - \int_2^4 f(x)\,dx = 12-3 = 9$

c. $\int_1^1 f(x)\,dx = 0$

d. $\int_2^1 f(x)\,dx = -\int_1^2 f(x)\,dx = -3$

Quiz 16 Name:________________________

Directions: Show all of your work and justify all of your answers.

(4) 1. Integrate.

a. $\displaystyle\int_1^2 \left(x^3+2x-1\right)dx$

$= \left(\frac{1}{4}x^4+x^2-x\right)\Big|_1^2$

$= 4+4-2-\left(\frac{1}{4}+1-1\right)$

$= \frac{23}{4}$

b. $\displaystyle\int \left[(x-3)(x+1)\right]dx$

$= \displaystyle\int \left(x^2-2x-3\right)dx$

$= \frac{1}{3}x^3-x^2-3x+C$

c. $\displaystyle\int \sec^2 x\,dx = \tan x + C$

d. $\displaystyle\int \frac{x^2+x+\frac{1}{x}}{x}\,dx$

$= \displaystyle\int \left(x+1+\frac{1}{x^2}\right)dx$

$= \frac{1}{2}x^2+x-\frac{1}{x}+C$

e. $\displaystyle\int (10x^4+6x^2)\sqrt[3]{x^5+x^3+7}\,dx$

Use substitution.

$u = x^5+x^3+7$

$du = (5x^4+3x^2)\,dx$

$2\,du = (10x^4+6x^2)\,dx$

$\displaystyle\int 2\sqrt[3]{u}\,du$

$= \displaystyle\int 2u^{\frac{1}{3}}\,du$

$= \frac{3}{2}u^{\frac{4}{3}}+C$

$= \frac{3}{2}\left(x^5+x^3+7\right)+C$

f. $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Use substitution.

$u = \sqrt{x} = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{-\frac{1}{2}}\,dx = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$\displaystyle\int 2\cos u\,du = 2\sin u + C = 2\sin\sqrt{x}+C$

g. $\displaystyle\int_0^5 x\sqrt{x+4}\,dx$

Use substitution.

$u = x+4$

$du = dx$

$x = u-4$

$\displaystyle\int_4^9 (u-4)\sqrt{u}\,du$

$= \displaystyle\int_4^9 \left(u^{\frac{3}{2}}-4u^{\frac{1}{2}}\right)du$

$= \left(\frac{2}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}\right)\Big|_4^9$

$= \frac{486}{5}-72-\left(\frac{64}{5}-\frac{64}{3}\right)$

$= \frac{506}{15}$

h. $\displaystyle\int_{-1}^{1} \sqrt{1-x^2}\, dx = \frac{\pi}{2}$

The integral represents the area of the top half of the unit circle.

Quiz 17 Name:________________________

Directions: Show all of your work and justify all of your answers.

(1) 1. Find the area of the region bounded by the given curves.

a.

$y = 1 - x^2$
$y = x^2 - 1$

$x^2 - 1 = 1 - x^2$

$2x^2 = 2$

$x = \pm 1$

$\int_{-1}^{1} \left[1 - x^2 - (x^2 - 1)\right] dx$

$= \int_{-1}^{1} \left(-2x^2 + 2\right) dx$

$= \left(-\frac{2}{3}x^3 + 2x\right)\Big|_{-1}^{1}$

$= -\frac{2}{3} + 2 - \left(\frac{2}{3} - 2\right)$

$= \frac{8}{3}$

b.

$y = \sqrt[3]{x}$
$y = x$

$\sqrt[3]{x} = x$

$x = x^3$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0,\ x = -1,\ x = 1$

$\int_{-1}^{0} \left(x - \sqrt[3]{x}\right) dx + \int_{0}^{1} \left(\sqrt[3]{x} - x\right) dx$

$= 2\int_{0}^{1} \left(\sqrt[3]{x} - x\right) dx$

$= 2\int_{0}^{1} \left(x^{\frac{1}{3}} - x\right) dx$

$= 2\left(\frac{3}{4}x^{\frac{4}{3}} - \frac{1}{2}x^2\right)\Big|_{0}^{1}$

$= \frac{1}{2}$

(1) 2. Compute the average value of the function $f(x) = x^3 + x - 2$ on the interval [0,2].

$$\frac{1}{2}\int_{0}^{2} \left(x^3 + x - 2\right) dx = \frac{1}{2}\left(\frac{1}{4}x^4 + \frac{1}{2}x^2 - 2x\right)\Big|_{0}^{2} = 1$$

3. Suppose that S is a 3-dimensional solid whose base is the unit circle. Give the volume of S if cross sections perpendicular to the base are

(1) a. squares. $\int_{-1}^{1} \left(2\sqrt{1 - x^2}\right)^2 dx = 8\int_{0}^{1} \left(1 - x^2\right) dx = 8\left(x - \frac{1}{3}x^3\right)\Big|_{0}^{1} = \frac{16}{3}$

(1) b. equilateral triangles.

$$\int_{-1}^{1} \frac{1}{2} \cdot 2\sqrt{1 - x^2} \cdot \sqrt{3}\sqrt{1 - x^2}\, dx = 2\sqrt{3}\int_{0}^{1} \left(1 - x^2\right) dx = 2\sqrt{3}\left(x - \frac{1}{3}x^3\right)\Big|_{0}^{1} = \frac{4\sqrt{3}}{3}$$

Points: 65

Section 11.2: Exam 1

Exam 1 Math 1571 Summer 2025 Name:__________________________

Directions: Show all of your work and justify all of your answers.

1.
(10) a. Give the equation of the line that contains the points (-3,1) and (2,4).

$m = \frac{3}{5}$

$y - 1 = \frac{3}{5}(x + 3)$

$y - 4 = \frac{3}{5}(x - 2)$

$y = \frac{3}{5}x + \frac{14}{5}$

(10) b. Give the equation of a line that is perpendicular to the line in the previous part.

Any line with slope -$\frac{5}{3}$ is a correct answer.

2. Set $f(x) = x^2 + x$ and $g(x) = \sin x$. Find each of the following.

(10) a. $(f \circ g)(x) = \sin^2 x + \sin x$

(10) b. $(g \circ f)(x) = \sin(x^2 + x)$

(10) 3. List the transformations needed to transform the graph of $y = f(x)$ into the graph of $y = -\frac{1}{2}f(x+3) - 1$.

(i) Vertical compression by a factor of 2.

(ii) Reflect about the x-axis.

(iii) Shift left 3 units.

(iv) Shift down 1 unit.

4. For each of the following, give the domain, range, horizontal asymptotes (if any), vertical asymptotes (if any), and oblique asymptotes (if any). Sketch the graph.

(10) a. $f(x)$

$= \dfrac{x^2 - x - 1}{x - 2}$

$= \dfrac{(x-2)(x+1)+1}{x-2}$

$= x + 1 + \frac{1}{x-2}$

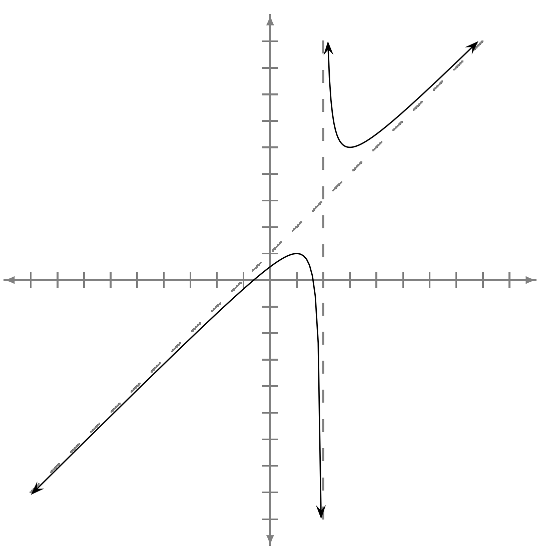
Domain: (-∞,2) $\cup$ (2,∞)

Range: (-∞,1] $\cup$ [5,∞)

HA: none

HA: $x = 2$

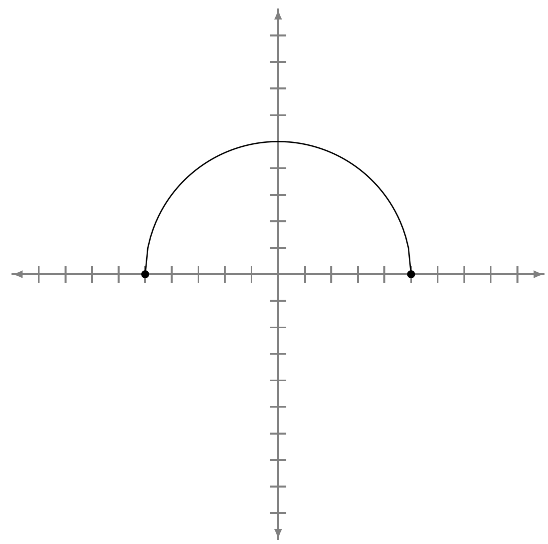
HA: $y = x + 1$



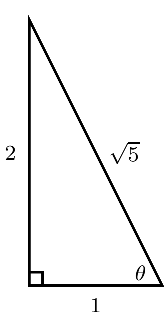
(10) b. $g(x) = \sqrt{25 - x^2}$

Domain: [-5,5]

Range: [0,5]



(30) 5. Give the six trigonometric values of θ.



$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

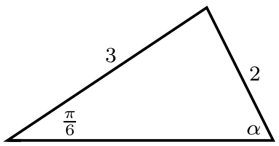
$\tan\theta = 2$

$\cot\theta = \frac{1}{2}$

$\sec\theta = \sqrt{5}$

$\csc\theta = \frac{\sqrt{5}}{2}$

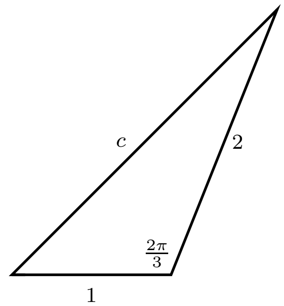
(10) 6. See the figure below. Find $\sin\alpha$.



$\frac{\sin\alpha}{3} = \frac{\sin\frac{\pi}{6}}{2}$

$\sin\alpha = \frac{3}{4}$

(10) 7. See the figure below. Find c.



$c^2 = 1^2 + 2^2 - 2(1)(2)\cos\frac{2\pi}{3} = 7$

$c = \sqrt{7}$

8. Prove the following identities.

(10) a. $\sin\theta\tan\theta + \cos\theta = \sec\theta$

***Proof*:**

$\sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$ ■

(10) b. $(\sin\theta + \cos\theta)^2 = \sin 2\theta + 1$

***Proof*:**

$(\sin\theta + \cos\theta)^2$

$= \sin^2\theta + 2\sin\cos\theta + \cos^2\theta$

$= 2\sin\theta\cos\theta + 1$

$= \sin 2\theta + 1$ ■

(10) 9. Use the $\epsilon - \delta$ definition of limit to prove the following.

$\lim_{x\to 1}\big(3x - 1\big) = 2$

***Proof*:**

Let $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta < \frac{\varepsilon}{3}$. If $0 < |x-1| < \delta$, then

$|x-1| < \frac{\varepsilon}{3}$

$3|x-1| < \varepsilon$

$|3x-3| < \varepsilon$

$|(3x-1) - 2| < \varepsilon$. ■

(10) 10. Use the $\epsilon - \delta$ definition of limit to prove **one** of the following. Make your choice clear.

a. $\lim_{x \to 2} \left(x^2 + 1\right) = 5$

Proof: Let $\varepsilon > 0$ be given and let $\delta < min\{\frac{\varepsilon}{5}, 1\}$. If $0 < |x - 2| < \delta$, then

$|x - 2| < \delta)$

$|x - 2| < \frac{\varepsilon}{5}$

$5|x - 2| < \varepsilon$

$|(x + 2)(x - 2)| < \varepsilon$

$|x^2 - 4| < \varepsilon$

$|(x^2 + 1) - 5| < \varepsilon.$

Therefore, $\lim_{x \to 6}(x^2 - 1) = 35.$ ■

b. $\lim_{x \to 0^+} \sqrt{x} = 0$

Proof: Let $\varepsilon > 0$ be given and set $\delta = \epsilon^2$.

If $0 < |x - 0| < \delta$, then

$|x| < \delta)$

$|x| < \epsilon^2$

$\sqrt{x} < \epsilon$

$|\sqrt{x} - 0| < \varepsilon.$ ■

11. Calculate the following limits.

(20) a. $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2-1} = \lim\limits_{x\to 1} \dfrac{(x+2)(x-1)}{(x+1)(x-1)} = \lim\limits_{x\to 1} \dfrac{x+2}{x+1} = \frac{3}{2}$

(10) b. $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\sin 2\theta} = \lim\limits_{\theta\to 0} \dfrac{\sin\theta}{2\sin\theta\cos\theta} = \lim\limits_{\theta\to 0} \dfrac{1}{2\cos\theta} = \frac{1}{2}$

b. $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\sin 2\theta} = \lim\limits_{\theta\to 0} \dfrac{2\theta\sin\theta}{2\theta\sin 2\theta} = \lim\limits_{\theta\to 0}\left(\dfrac{2\theta}{\sin 2\theta}\cdot\dfrac{\sin\theta}{\theta}\cdot\frac{1}{2}\right) = 1\cdot\frac{1}{2} = \frac{1}{2}$

(10) c. $\lim\limits_{x\to 0} x^2\cos^4\left(\frac{1}{\sqrt[3]{x}}\right)$

Since $0 \le x^2\cos^4\left(\frac{1}{\sqrt[3]{x}}\right) \le x^2$, $\lim\limits_{x\to 0} x^2\cos^4\left(\frac{1}{\sqrt[3]{x}}\right) = 0$ by the Squeeze Theorem.

(10) d. $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}+1}{x+1} = \lim\limits_{x\to\infty} \dfrac{\sqrt{x}\left(1+\frac{1}{\sqrt{x}}\right)}{\sqrt{x}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)} = \lim\limits_{x\to\infty} \dfrac{1+\frac{1}{\sqrt{x}}}{\sqrt{x}+\frac{1}{\sqrt{x}}} = 0$

$\lim\limits_{x\to\infty} \dfrac{\sqrt{x}+1}{x+1} = \lim\limits_{x\to\infty}\left(\dfrac{\sqrt{x}+1}{x+1}\cdot\dfrac{\sqrt{x}-1}{\sqrt{x}-1}\right) = \lim\limits_{x\to\infty} \dfrac{x-1}{(x+1)(\sqrt{x}-1)} = \lim\limits_{x\to\infty}\left(\dfrac{x-1}{x+1}\cdot\dfrac{1}{\sqrt{x}-1}\right) = 1\cdot 0 = 0$

(10) 12. For the following, assign a value to α to make the given function continuous (if possible).

$$f(x) = \begin{cases} x^2+1, & \text{if } x < 3 \\ \alpha, & \text{if } x = 3 \\ 4x-2, & \text{if } x > 3 \end{cases}$$

The function is continuous if $\alpha = 10$

(10) 13. Prove that the following equation has a solution.

$x^5 + x^3 - 3 = 0$

***Proof*:** Set $f(x) = x^5+x^3-3$. Note that $f(1) =$ -1, $f(2) = 37$, and f is continuous. So by the Intermediate Value The
there exists $c \in$ [1,2] such that $f(c) = 0$. ■

14. Use the definition of derivative to differentiate each of the following.

(10) a. $f(x) = x^3$

$\lim_{h\to 0}\big[f(x+h) - f(x)\big]\,\frac{1}{h}$

$= \lim_{h\to 0}\big[(x+h)^3 - x^3 +\big]\,\frac{1}{h}$

$= \lim_{h\to 0}\big[x^3 + 3x^2h + 3xh^2 + h^3 - x^3\big]\,\frac{1}{h}$

$= \lim_{h\to 0}\big[3x^2h + 3xh^2 + h^3\big]\,\frac{1}{h}$

$= \lim_{h\to 0}\big[3x^2 + 3xh + h^2\big]$

$= 3x^2$

(10) b. $f(x) = \sqrt{x+1}$

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$

$= \lim_{h\to 0}\left(\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}\right)$

$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h+1}+\sqrt{x+1})}$

$= \lim_{h\to 0}\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$

$= \frac{1}{2\sqrt{x+1}}$

Points: 250

Section 11.3: Exam 2

Exam 2 Math 1571 Summer 2025 Name:____________________

Directions: Show all of your work and justify all of your answers.

1. Differentiate.

(20) a.

$f(x) = x^4 + 6x^3 - x^2 + 17$

$f'(x) = 4x^3 + 18x^2 - 2x$

(10) b.

$g(x) = \sqrt[3]{x^4 + 6x^3 - x^2 + 17} = \left(x^4 + 6x^3 - x^2 + 17\right)^{\frac{1}{3}}$

$$g'(x) = \tfrac{1}{3}\left(x^4 + 6x^3 - x^2 + 17\right)^{\text{-}\frac{2}{3}}\left(4x^3 + 18x^2 - 2x\right) = \frac{4x^3 + 18x^2 - 2x}{3\sqrt[3]{\left(x^4 + 6x^3 - x^2 + 17\right)^2}}$$

(10) c.

$h(x) = \sin(x^4 + 6x^3 - x^2 + 17)$

$h'(x) = (4x^3 + 18x^2 - 2x)\cos(x^4 + 6x^3 - x^2 + 17)$

(10) 2. Find a function $f(x)$ such that $f'(x) = x^3 + x^{\frac{1}{2}} + \cos x$ and $f(0) = 1$.

$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^{\frac{3}{2}} + \sin x + 1$

3. For each of the following, give the equation of the line that is tangent to the given curve at the given point.

(20) a. $f(x) = \tan x$; $x = \frac{\pi}{4}$

$f'(x) = \sec^2 x$

$y = f\left(\frac{\pi}{4}\right) = 1$

$m = f'\left(\frac{\pi}{4}\right) = 2$

$y - 1 = 2\left(x - \frac{\pi}{4}\right)$

(10) b. $x^4y^2 + x^3 + y^2 + xy = 16$; (1,-3)

$\frac{d}{dx}\left(x^4y^2 + x^3 + y^2 + xy\right) = \frac{d}{dx}16$

$4x^3y^2 + 2x^4yy' + 3x^2 + 2yy' + y + xy' = 0$

$$y' = \frac{\text{-}4x^3y^2 - 3x^2 - y}{2x^4y + 2y + x}$$

$x = 1$

$y = \text{-}3$

$m = \frac{\text{-}36}{\text{-}11} = \frac{36}{11}$

$y + 3 = \frac{36}{11}(x - 1)$

4. A particle moves according to a law of motion $s(t) = \frac{2}{3}t^3 - 6t^2 + 16t$.

(20) a. Find the velocity at time t.

$v(t) = s'(t) = 2t^2 - 12t + 16 = 2(t-4)(t-2)$

(10) b. When is the particle moving in a positive direction? When is the particle moving in a negative direction?

Pos: $[0,2) \cup (4,\infty)$

Neg: $(2,4)$

(10) c. Find the total distance traveled during the first five seconds.

$|s(2) - s(0)| + |s(4) - s(2)| + |s(5) - s(4)| = |\frac{40}{3} - 0| + |\frac{32}{3} - \frac{40}{3}| + |\frac{40}{3} - \frac{32}{3}| = \frac{56}{3}$

(10) d. Find the acceleration at time t.

$a(t) = v'(t) = 4t - 12$

5. A ball is thrown upward with an initial velocity of of 64 ft/sec from a height of 80 ft.

(10) a. Give the height of the ball as a function of time.

$h(t) = -16t^2 + 64t + 80$

(10) b. Find the maximum height of the ball.

$v(t) = h'(t) = -32t + 64$

Solve $v(t) = 0$.

$t = 2$

$h(2) = 144$

144 ft

(10) c. What is the velocity of the ball when the ball hits the ground?

Solve $h(t) = 0$.

$-16t^2 + 64t + 80 = 0$

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

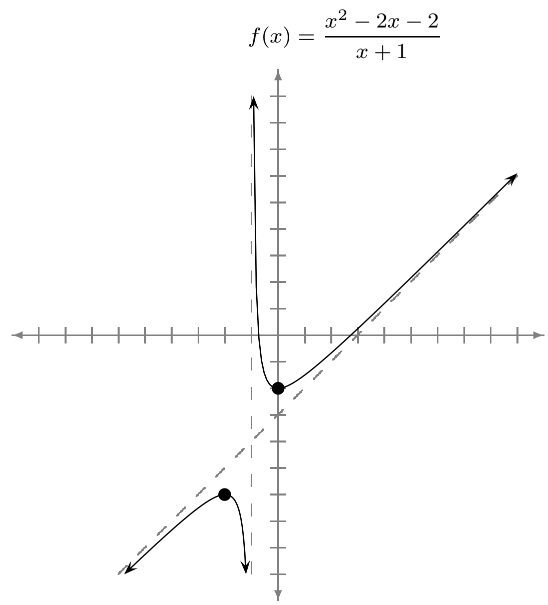
$t = 5$

$v(5) = -104$

-96 ft/sec

6. For each of the following, find the horizontal asymptotes, vertical asymptotes, oblique asymptotes, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points. Use these answers to sketch the graph.

(10) a. $f(x)$



$= \dfrac{x^2 - 2x - 2}{x + 1}$

$= \dfrac{(x + 1)(x - 3) + 1}{x + 1}$

$= x - 3 + \frac{1}{x+1}$

HA: none

VA: $x =$ -1

OA: $y = x - 3$

$f'(x)$

$= 1 - \frac{1}{(x+1)^2}$

$= \dfrac{(x + 1)^2 - 1}{(x + 1)^2}$

$= \dfrac{x^2 + 2x}{(x + 1)^2}$

$= \dfrac{x(x + 2)}{(x + 1)^2}$

Dec: [-2,-1), (-1,0]

Inc: (-∞,-2], [0,∞]

Local max: -6 at -2

Local min: -2 at 0

$f''(x) = \frac{2}{(x+1)^3}$

CD: (∞,-1)

CU: (-1,∞)

IP: none

(10) b. $g(x) = \frac{1}{3}x^3 - 9x$

Note that the function is odd.

HA: none

VA: none

OA: none

$g'(x) = x^2 - 9 = (x+3)(x-3)$

Dec: [-3,3]

Inc: (-∞,-3], [3,∞)

Local max: 18 at -3

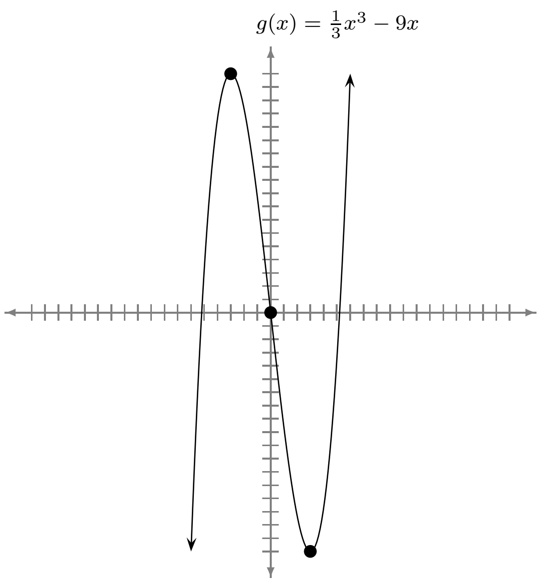
Local min: -18 at 3

$g''(x) = 2x$

CD: (-∞,0)

CU: (0,∞)

IP: (0,0)



(10) c. $f(x) = \dfrac{5x^2}{x^2+1}$

HA: $y = 5$

VA: none

OA: none

$f'(x)$

$$= \frac{10x(x^2+1) - 5x^2(2x)}{(x^2+1)^2}$$

$$= \frac{10x}{(x^2+1)^2}$$

Dec: (-∞,0]

Inc: [0,∞)

Local min: 0 at 0

$f''(x)$

$$= \frac{10(x^2+1)^2 - 10x(2)(x^2+1)(2x)}{(x^2+1)^4}$$

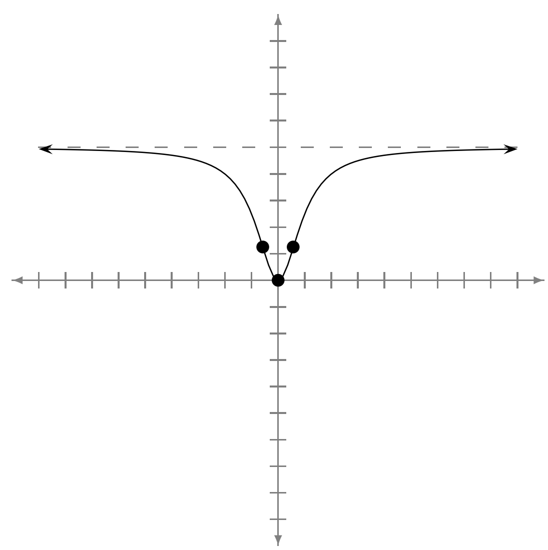
$$= \frac{10(x^2+1)[(x^2+1) - 4x^2]}{(x^2+1)^4}$$

$$= \frac{10(1-3x^2)}{(x^2+1)^3}$$

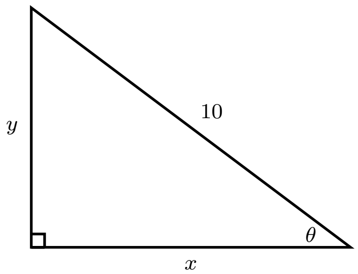
CD: $\left(-\infty, \frac{1}{-\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{3}}, \infty\right)$

CD: $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

IP: $\left(-\frac{1}{\sqrt{3}}, \frac{5}{4}\right)$, $\left(\frac{1}{\sqrt{3}}, \frac{5}{4}\right)$



7. The bottom of a 10-foot ladder is being pulled away from a wall at a rate of 3 ft /sec. When the bottom of the ladder is 6 ft from the wall



(10) a. at what rate is the top of the ladder sliding down the wall?

$x^2 + y^2 = 100$

$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}100$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(6)(3) + 2(8)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{9}{4}$

$\frac{9}{4}$ ft/sec

(10) b. at what rate is the angle between the floor and the bottom of the ladder changing?

$\cos\theta = \frac{x}{10}$

$x = 10\cos\theta$

$\frac{dx}{dt} = \frac{d}{dt}\left(10\cos\theta\right)$

$\frac{dx}{dt} = -10\sin\theta \cdot \frac{d\theta}{dt}$

$3 = -10 \cdot \frac{4}{5} \cdot \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = -\frac{3}{8}$ rad/sec

(10) 8. Recall that the volume of a cylinder is $V = \pi r^2 h$ and that 1 L = 1000 cm^3. A jar in the shape of a circular cylinder with a base of radius 9 cm is being filled with water at rate of 3 L per minute. At what rate is the height of the water increasing?

$V = 81\pi h$

$\frac{dV}{dt} = 81\pi\frac{dh}{dt}$

$3000 = 81\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1000}{27\pi}$

$\frac{1000}{27\pi}$ cm/min

(20) 9. Find the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 12x$ on the interval [-2,3].

$f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$

$f(-2) = -4$

$f(3) = -9$

$f(-1) = 7$ (max)

$f(2) = -20$ (min)

(10) 10. Find the maximum area of a rectangle that has its bottom on the x-axis and two top corners on the graph of the curve $y = -x^2 + 9$.

$A(x) = 2x(9 - x^2)$, $x \in [0,3]$

$A(x) = -2x^3 + 18x$

$A'(x) = -6x^2 + 18 = -6(x^2 - 3)$

$A(0) = 0$

$A(3) = 0$

$A(\sqrt{3}) = 12\sqrt{3}$

Points: 250